

SOME RESULTS ON JANOWSKI CLOSE TO CONVEX MAPPINGS

Melike Aydoğın

Department of Mathematics
Isik University- Sile Kampusu
Mesrutiyet Koyu Universite Sokak, D Kapi No:2
34980, Istanbul, Turkey

Abstract

Let $J_C(A, B)$ denote the class of functions $\phi(z) = z + \sum_{k=2}^{\infty} a_k z^k$ are analytic in the open unit disc D such that

$$\frac{z\phi'(z)}{s(z)} \prec \frac{1 + Az}{1 + Bz}, -1 \leq B < A \leq 1 \quad (1)$$

where $s(z) = z + \sum_{k=2}^{\infty} b_k z^k$ is convex in D . In this paper we determine the coefficient estimates distortion theorems for this class.

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1 Introduction

Let $H(D)$ be the linear space of all analytic functions defined in the open unit disc D . Let $w(z) = \sum_{k=1}^{\infty} c_k z^k$ be an analytic function in the open unit disc and satisfying the conditions $w(0) = 0$ and $|w(z)| < 1$, $z \in D$.

Let C denote the class of functions such as

$$s(z) = z + \sum_{k=2}^{\infty} b_k z^k \quad (2)$$

analytic and convex in D .

Let J_C denote the class of functions such as

$$\psi(z) = \int_0^z \frac{s(z)}{z} \quad (3)$$

is Janowski and convex in D .

Let $J_C(A, B)$ denote the class of functions such as

$$\phi(z) = z + \sum_{k=2}^{\infty} a_k z^k \quad (4)$$

analytic in D and satisfying the conditions;

$$\frac{z\phi'(z)}{s(z)} \prec \frac{1 + Az}{1 + Bz}, -1 \leq B < A \leq 1, z \in D, s(z) \in C \quad (5)$$

If we use the definition of subordination principle for $\phi(z) \in J_C(A, B)$ if and only if $\phi(z)$ can be represented in the form:

$$\frac{z\phi'(z)}{s(z)} = \frac{1 + Aw(z)}{1 + Bw(z)}, w(z) \in H(D), -1 \leq B < A \leq 1, z \in D \quad (6)$$

We study the $J_C(A, B)$ and obtain coefficient estimates, distortion theorems.

2 Some Preliminary Lemmas

We need the following lemmas.

Lemma 2.1 Let $\frac{z\phi'(z)}{s(z)} = p(z) = 1 + \sum_{k=1}^{\infty} p_k z^k$ then $|p_n| \leq (A - B)$, $n \geq 1$. The bounds are sharp, being attained for the functions

$$p_n(z) = \frac{1 + A\delta z^n}{1 + B\delta z^n}, |\delta| = 1 \quad (7)$$

[Goel and Mehrok].

Lemma 2.2 If $w(z) \in H(D)$ then for $|z| = r < 1$

$$|zw'(z) - w(z)| \leq \frac{r^2 - |w(z)|^2}{1 - r^2} \quad (8)$$

[Singh and Goel].

Lemma 2.3 Let $p(z) = \frac{1+Bw(z)}{1+Aw(z)}$, $w(z) \in H(D)$, then for $|z| = r < 1$

$$\operatorname{Re}\left[Ap(z) + \frac{B}{p(z)}\right] + \frac{r^2 |Ap(z) - B|^2 - |1 - p(z)|^2}{(1 - r^2) |p(z)|} \leq \frac{AB(A + B)r^2 - 4ABr + (A + B)}{(1 - Ar)(1 - Br)}; R_1 \leq R_0,$$

$$\frac{2}{(1-r^2)}(1-ABr^2) - [(1-A)(1-B)(1+Ar^2)(1+Br^2)]^{\frac{1}{2}}; R_1 \geq R_0.$$

where $A \neq 1$, $R_1 = \frac{1-Br}{1-Ar}$ and

$$R_0^2 = \frac{(1-B)(1+Br^2)}{(1-A)(1+Ar^2)}$$

The bounds are sharp.[Goel and Mehrotra].

3 Coefficient Inequalities

Theorem 3.1 If $\phi(z) \in J_C(A, B)$ then,

$$|a_n| \leq \frac{1}{n} + \frac{(n-1)(A-B)}{n}, n \geq 2 \quad (9)$$

The bounds are sharp.

Proof: Using (1.2) and (1.4) in (2.1) we get;

$$z(1 + \sum_{k=2}^{\infty} ka_k z^{k-1}) = (z + \sum_{k=2}^{\infty} b_k z^k)(1 + \sum_{k=1}^{\infty} p_k z^k) \quad (10)$$

Equating the coefficient of z^n in (3.2) we have;

$$na_n = b_n + p_1 b_{n-1} + p_2 b_{n-2} + \dots + p_{n-1} \quad (11)$$

Therefore using (3.2),

$$n|a_n| \leq |b_n| + (A-B)[|b_{n-1}| + |b_{n-2}| + \dots + |b_2| + 1] \quad (12)$$

Also it is well known that $|b_n| \leq 1$, $n \geq 2$. Hence $|a_n| \leq \frac{1}{n} + \frac{(n-1)(A-B)}{n}$ For $n = 2$ equality signs in (3.1) holds for the function $\phi(z)$

$$\phi'_n(z) = \frac{1}{(1-\delta_1 z)} \frac{1+A\delta_2 z^{n-1}}{1+B\delta_2 z^{n-1}}, |\delta_1| = 1, |\delta_2| = 1 \quad (13)$$

On putting $A = 1$, $B = -1$ in above theorem, we get the following result due to Grawod and Thomas.

Corollary 3.2 Let $\phi(z) \in J$, then $|a_n| \leq 2 - \frac{1}{n}$

