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Almost Asymptotically Statistical Equivalent of Double Difference Sequences of Fuzzy Numbers

Kuldip Raj

School of Mathematics
Shri Mata Vaishno Devi University
Katra-182320, J&K, INDIA
Email : kuldipraj68@gmail.com

Sunil K. Sharma

School of Mathematics
Shri Mata Vaishno Devi University
Katra-182320, J&K, INDIA
Email : sunilksharma42@yahoo.co.in

Abstract

In this paper we introduce the concept of almost asymptotically statistical equivalent of double difference sequences of fuzzy numbers and we shall make an effort to study some inclusion relations.

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1 Introduction and Preliminaries

The concepts of fuzzy sets and fuzzy set operations were first introduced by Zadeh [21] and subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces, similarity relations and fuzzy orderings, fuzzy measures of fuzzy events, fuzzy mathematical programming. Matloka [6] introduced bounded and convergent sequences of fuzzy numbers and studied some of their properties. Fuzzy set theory, compared to other mathematical theories, is perhaps the most easily adaptable theory to practice. The main reason is that a fuzzy set has the property of relativity, variability and inexactness in the definition of its elements. Instead of defining an entity in calculus by assuming that its role is exactly known, we can use fuzzy sets to define the same entity by allowing possible deviations and inexactness in its role. This representation suits well the uncertainties encountered in practical life, which make fuzzy sets a valuable mathematical tool. The initial works on double sequences is found in Bromwich [2]. Later on, it was studied by Hardy [3], Moricz [6], Moricz and Rhoades [7], Tripathy ([19], [20]), Başarır and Sonalcan [1] and many others. Hardy [3] introduced the notion of regular convergence for double sequences. Quite recently, Zeltser [22] in her Ph.D thesis has essentially studied both the theory of topological double sequence spaces and the theory of summability of double sequences. Mursaleen and Edely [9] have recently introduced the statistical convergence and Cauchy convergence for double sequences and given the relation between statistical convergent and strongly Cesaro summable double sequences. Nextly, Mursaleen [8] and Mursaleen and Edely [10] have defined the almost strong regularity of matrices for double sequences and applied these matrices to establish a core theorem and introduced the M-core for double sequences and determined those four dimensional matrices transforming every bounded double sequences $x = (x_{k,l})$ into one whose core is a subset of the M-core of x. By the convergence of a double sequence we mean the convergence in the Pringsheim sense i.e. a double sequence $x = (x_{k,l})$ has Pringsheim limit L (denoted by $P - \lim x = L$) provided that given $\epsilon > 0$ there exists $n \in N$ such that $|x_{k,l} - L| < \epsilon$ whenever $k, l > n$ see [13]. We shall write more briefly as P-convergent. The double sequence $x = (x_{k,l})$ is bounded if there exists a positive number M such that $|x_{k,l}| < M$ for all k and l. For more details about double sequence spaces and fuzzy numbers (see [12],[14],[15],[17],[18]) and references therein.

Let D denote the set of all closed and bounded intervals on R, the real line. For $A, B \in D$, we define $d(A, B) = \max(|a_1 - b_1|, |a_2 - b_2|)$, where $A = [a_1, a_2]$ and $B = [b_1, b_2]$. It is known that (D, d) is a complete metric space. A fuzzy real number X is a fuzzy set on R, i.e. a mapping $X : R \to I(= [0, 1])$ associating each real number t with its grade of membership $X(t)$.

The set of all upper-semi continuous, normal, convex fuzzy real number is denoted by $R(I)$. Throughout the paper, by a fuzzy real number X, we mean that $X \in R(I)$.

The α-cut or α-level set $[X]^\alpha$ of the fuzzy number X, for $0 < \alpha \leq 1$, defined by $[X]^\alpha = \{t \in R : X(t) > 0\}$; for $\alpha = 0$, it is the closure of the strong 0-cut i.e. closure of the set $\{t \in R : X(t) > 0\}$. Throughout α means, $\alpha \in (0, 1]$ unless otherwise it is stated.

The set of R real numbers can be embedded in $R(I)$ if we define $\bar{r} \in R(I)$ by

$$\bar{r}(t) = \begin{cases} 1, & \text{if } t = r \\ 0, & \text{if } t \neq r. \end{cases}$$

The additive identity and multiplicative identity of $R(I)$ are denoted by $\bar{0}$ and $\bar{1}$, respectively.
For $r \in R$ and $X \in R(I)$, the product rX is defined as follows:

$$\bar{r}X(t) = \begin{cases} X(r^{-1}t), & \text{if } r \neq 0 \\ 0, & \text{if } r = 0. \end{cases}$$

The absolute value $|X|$ of X in $R(I)$ is defined by

$$|X|(t) = \begin{cases} \max\{X(t), X(-t)\}, & \text{if } t \geq 0 \\ 0, & \text{if } t < 0. \end{cases}$$

Let $\bar{d} : R(I) \times R(I) \to R$ be defined by

$$\bar{d}(X, Y) = \sup_{0 \leq \alpha \leq 1} d([X]^\alpha, [Y]^\alpha).$$

Then $\bar{d}$ defines a metric on $R(I)$. It is well known that $R(I)$ is complete with respect to $\bar{d}$.
A subset E of N is said to have density $\delta(E)$, if

$$\delta(E) = \lim_{n\to\infty} \sum_{k=1}^{n} \chi_E(k) \text{ exists,}$$

where χ_E is the characteristic function of E.
A sequence $X = (X_k)$ of fuzzy numbers is said to be convergent to a fuzzy number X_0, written as $\lim_k X_k = X_0$, if for every $\epsilon > 0$ there exists a positive integer k_0 such that $\bar{d}(X_k, X_0) < \epsilon$ for $k \geq k_0$. Let $c(F)$ denote the set of all convergent sequences of fuzzy numbers.
A sequence $X = (X_k)$ of fuzzy numbers is said to be bounded if the set $\{X_k : k \in N\}$ of fuzzy numbers is bounded. We denote by $l_\infty(F)$ the set of all bounded sequences of fuzzy numbers. It is straightforward to see that $c(F) \subset l_\infty(F)$ and Nanda [11] studied the spaces of bounded and convergent sequences of fuzzy numbers and showed that they are complete metric spaces.
Nuray and Savaş [12] defined the notion of statistical convergence for fuzzy real numbers sequences and studied some properties. A sequence of fuzzy real numbers (X_k) is said to be statistically convergent to a fuzzy real number X_0, if for every $\epsilon > 0$, $\delta\Big(\Big\{k \in N : \bar{d}(X_k, X_0) \geq \epsilon\Big\}\Big) = 0$. Savaş [16] defined the notion of almost convergence of fuzzy real numbers.
The notion of difference sequence spaces was introduced by Kızmaz [5], who studied the difference sequence spaces $l_\infty(\Delta)$, $c(\Delta)$ and $c_0(\Delta)$. By a lacunary sequence $\theta = (k_r)$, $r = 0, 1, 2, \cdots$, where $k_0 = 0$, we shall mean an increasing sequence of non-negative integers $h_r = (k_r - k_{r-1}) \to \infty \quad (r \to \infty)$. The intervals determined by θ are denoted by $I_r = (k_{r-1}, k_r]$ and the ratio k_r/k_{r-1}

will be denoted by q_r.
The double sequence $\theta_{r,s} = \{(k_r, l_s)\}$ is called double lacunary if there exist two increasing sequences of integers such that

$$k_0 = 0,\ h_r = k_r - k_{r-1} \to \infty \text{ as } r \to \infty$$

and

$$l_0 = 0,\ \bar{h}_s = l_s - l_{s-1} \to \infty \text{ as } s \to \infty.$$

Let $k_{r,s} = k_r l_s$, $h_{r,s} = h_r \bar{h}_s$ and $\theta_{r,s}$ is determined by $I_{r,s} = \{(k,l) : k_{r-1} < k \leq k_r \text{ and } l_{s-1} < l \leq l_s\}$, $q_r = \frac{k_r}{k_{r-1}}$, $\bar{q}_s = \frac{l_s}{l_{s-1}}$ and $q_{r,s} = q_r \bar{q}_s$. By $l_\infty(\Delta F)$ we denote bounded double difference sequences of fuzzy numbers.

Definition 1.1 *The double difference sequence $X = (\Delta X_{k,l})$ of fuzzy numbers is said to be almost convergent to a fuzzy number X_0 if*

$$\lim_k \bar{d}(t_{km}(\Delta X), X_0) = 0, \quad \textit{uniformly in } m,$$

where

$$t_{km}(\Delta X) = \frac{1}{k+1} \sum_{i=0}^{k,l} \Delta X_{m+i}.$$

This means that for every $\epsilon > 0$, there exists $k_0 \in N$ such that $\bar{d}(t_{km}(\Delta X), X_0) < \epsilon$ whenever $k \geq k_0$ and for all m.

Definition 1.2 *Two double difference sequences X and Y of fuzzy numbers are said to be almost asymptotically equivalent if*

$$\lim_{k,l} \bar{d}\Big(\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})}, \bar{1}\Big) = 0, \quad \textit{uniformly in } m \textit{ (denoted by } X \sim^{(\Delta F)} Y).$$

Definition 1.3 *Let $(\theta_{r,s})$ be a double lacunary sequence. A double difference sequence of fuzzy numbers $X = (\Delta X_{k,l})$ is said to be almost statistically convergent or $S_\theta(\Delta)$-convergent to the fuzzy number L if for every $\epsilon > 0$*

$$\lim_r \frac{1}{h_{r,s}} \Big|\Big\{k, l \in I_{r,s} : \bar{d}(t_{km}(\Delta X_{k,l}), L) \geq \epsilon\Big\}\Big| = 0, \ \textit{uniformly in } m.$$

In this case we write $S_\theta(\Delta) - \lim X = L$ or $X_k \to L(S_\theta(\Delta))$.

Definition 1.4 *Let $(\theta_{r,s})$ be a double lacunary sequence. Two double difference sequences X and Y of fuzzy numbers are said to be almost asymptotically $S_\theta^L(\Delta)$-statistical equivalent of multiple L provided that for every $\epsilon > 0$*

$$\lim_{r,s} \frac{1}{h_{r,s}} \Big\{k, l \in I_{r,s} : \bar{d}\Big(\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})}, L\Big) \geq \epsilon\Big\}\Big| = 0, \quad \textit{uniformly in } m,$$

(denoted by $X \sim^{S_\theta^L(\Delta F)} Y$) and simply almost asymptotically $S_\theta(\Delta F)$-statistical equivalent if $L = \bar{1}$.

Definition 1.5 *Two double difference sequences X and Y of fuzzy numbers are said to be almost asymptotically statistical equivalent of multiple L provided that for every $\epsilon > 0$*

$$\lim_{m,n} \frac{1}{mn}\Big|\Big\{k \leq m, l \leq n : \bar{d}\Big(\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})}, L\Big) \geq \epsilon\Big\}\Big| = 0, \quad \textit{uniformly in } m$$

(denoted by$X \sim^{S^L(\Delta F)} Y$) and simply almost asymptotically statistical equivalent if $L = \bar{1}$.

Definition 1.6 *Let $(\theta_{r,s})$ be a double lacunary sequence. Two double difference sequences X and Y of fuzzy numbers are said to be strong $V_\theta^L(\Delta F)$-asymptotically equivalent of multiple L provided that*

$$\lim_{r,s} \frac{1}{h_{r,s}} \sum_{k,l \in I_{r,s}} \bar{d}\Big(\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})}, L\Big) = 0, \quad \textit{uniformly in } m$$

(denoted by $X \sim^{V_\theta^L(\Delta F)} Y$) and simply strong $V_\theta(\Delta F)$-asymptotically statistical equivalent if $L = \bar{1}$.

Definition 1.7 *Two double difference sequences X and Y of fuzzy numbers are said to be strong Cesaro $C_1^L(\Delta F)$-asymptotically equivalent of multiple L provided that*

$$\lim_{n} \frac{1}{mn} \sum_{k,l=1,1}^{m,n} \bar{d}\Big(\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})}, L\Big) = 0, \quad \textit{uniformly in } m \ (\textit{denoted by } X \sim^{C_1^L(\Delta F)} Y)$$

and simply strong Cesaro $C_1(\Delta F)$-asymptotically equivalent if $L = \bar{1}$.

2 Main Results

Theorem 2.1 *Let $(\theta_{r,s})$ be a double lacunary sequence. The following conditions are satisfied:*
(i) If $X \sim^{V_\theta^L(\Delta F)} Y$, then $X \sim^{S_\theta^L(\Delta F)} Y$.
(ii) If $X \in l_\infty(\Delta F)$ and $X \sim^{S_\theta^L(\Delta F)} Y$, then $X \sim^{V_\theta^L(\Delta F)} Y$.
(iii) If $X, Y \in l_\infty(\Delta F)$ then $X \sim^{V_\theta^L(\Delta F)} Y$ if and only if $X \sim^{S_\theta^L(\Delta F)} Y$.

Proof. (i) Let $\epsilon > 0$ and $X \sim^{V_\theta^L(\Delta F)} Y$, then

$$\begin{aligned}\sum_{k,l \in I_{r,s}} \bar{d}\Big(\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})}, L\Big) &\geq \sum_{k,l \in I_{r,s}, \bar{d}\big(\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})}, L\big) \geq \epsilon} \bar{d}\Big(\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})}, L\Big) \\ &\geq \epsilon\Big|\Big\{k, l \in I_{r,s} : \bar{d}\Big(\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})}, L\Big) \geq \epsilon\Big\}\Big|.\end{aligned}$$

Therefore $X \sim^{S_\theta^L(\Delta F)} Y$.
(ii) Let X and Y are in $l_\infty(\Delta F)$ and $X \sim^{S_\theta^L(\Delta F)} Y$. Then we can assume that

$$\bar{d}\Big(\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})}, L\Big) \leq T, \text{ for all } k \text{ and } m.$$

Given $\epsilon > 0$, then we have

$$\begin{aligned}
\frac{1}{h_{r,s}} \sum_{k,l\in I_{r,s}} \bar{d}\Big(\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})}, L\Big) &= \frac{1}{h_{r,s}} \sum_{k,l\in I_{r,s}, \bar{d}\left(\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})}, L\right)\geq\epsilon} \bar{d}\Big(\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})}, L\Big) \\
&+ \frac{1}{h_{r,s}} \sum_{k,l\in I_{r,s}, \bar{d}\left(\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})}, L\right)<\epsilon} \bar{d}\Big(\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})}, L\Big) \\
&\geq \frac{T}{h_{r,s}} \Big|\Big\{k, l \in I_{r,s} : \bar{d}\Big(\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})}, L\Big) \geq \epsilon\Big\}\Big| + \epsilon.
\end{aligned}$$

Therefore $X \sim^{V_\theta^L(\Delta F)} Y$.
(iii) It directly follows from (i) and (ii).

Theorem 2.2 *Let $(\theta_{r,s})$ be a double lacunary sequence with* $\liminf q_{r,s} > 1$,

$$X \sim^{C_1^L(\Delta F)} Y \ \textit{ implies } \ X \sim^{V_\theta^L(\Delta F)} Y.$$

Proof. Suppose that $\liminf q_{r,s} > 1$, then there exists a $\delta > 0$ such that $q_{r,s} \geq 1 + \delta$ for sufficiently large r, s which implies

$$\frac{h_{r,s}}{k_{r,s}} \geq \frac{\delta}{1+\delta}.$$

If $X \sim^{C_1^L(\Delta F)} Y$, then for every $\epsilon > 0$ and for sufficiently large r, s, we have
$\frac{1}{(k_{r,s})}\Big|\Big\{(k,l) \leq k_{r,s} : \Big|\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})} - L\Big| \geq \epsilon\Big\}\Big|$

$$\begin{aligned}
&\geq \frac{1}{(k_{r,s})}\Big|\Big\{k, l \in I_{r,s} : \Big|\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})} - L\Big| \geq \epsilon\Big\}\Big| \\
&\geq \frac{\delta}{1+\delta}\frac{1}{h_{r,s}}\Big|\Big\{k, l \in I_{r,s} : \Big|\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})} - L\Big| \geq \epsilon\Big\}\Big|,
\end{aligned}$$

this completes the proof.

Theorem 2.3 *Let $(\theta_{r,s})$ be a double lacunary sequence with* $\limsup q_{r,s} < \infty$,

$$X \sim^{V_\theta^L(\Delta F)} Y \ \textit{ implies } \ X \sim^{C_1^L(\Delta F)} Y.$$

Proof. Suppose that $\limsup q_{r,s} < \infty$, then there exists $B > 0$ such that $q_{r,s} < B$ for all $r, s \geq 1$. Let $X \sim^{V_\theta^L(\Delta F)} Y$ and $\epsilon > 0$. There exists $R > 0$ such that for every $i, j \geq R$

$$A_{i,j} = \frac{1}{h_{i,j}}\Big|\Big\{k, l \in I_{i,j} : \Big|\frac{t_{km}(X_{k,l})}{t_{km}(Y_{k,l})}, L\Big| \geq \epsilon\Big\}\Big| < \epsilon.$$

We can find $K > 0$ such that $A_{i,j} < K$ for all $i, j = 1, 2, \cdots$. Now let n be any integer with $k_{r-1} < n < k_r$, where $r > R$. Then
$\frac{1}{mn}\Big|\Big\{k \leq m, l \leq n : \Big|\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})} - L\Big| \geq \epsilon\Big\}\Big|$

$$\begin{aligned}
&\geq \frac{1}{k_{r-1,s}}\Big|\Big\{k, l \leq k_{r,s} : \Big|\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})} - L\Big| \geq \epsilon\Big\}\Big| \\
&= \frac{1}{k_{r-1,s}}\Big|\Big\{k, l \in I_{1,s} : \Big|\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})} - L\Big| \geq \epsilon\Big\}\Big| \\
&+ \frac{1}{k_{r-1,s}}\Big|\Big\{k, l \in I_{2,s} : \Big|\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})} - L\Big| \geq \epsilon\Big\}\Big| + \cdots \\
&+ \frac{1}{k_{r-1,s}}\Big|\Big\{k, l \in I_{r,s} : \Big|\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})} - L\Big| \geq \epsilon\Big\}\Big| \\
&= \frac{k_1}{k_{r-1,s}k_1}\Big|\Big\{k, l \in I_{1,s} : \Big|\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})} - L\Big| \geq \epsilon\Big\}\Big| \\
&+ \frac{k_2 - k_1}{k_{r-1,s}(k_2 - k_1)}\Big|\Big\{k, l \in I_{2,s} : \Big|\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})} - L\Big| \geq \epsilon\Big\}\Big| + \cdots \\
&+ \frac{k_R - k_{R-1}}{k_{r-1,s}(k_R - k_{R-1})}\Big|\Big\{k, l \in I_{r,s} : \Big|\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})} - L\Big| \geq \epsilon\Big\}\Big| + \cdots \\
&+ \frac{k_r - k_{r-1}}{k_{r-1,s}(k_r - k_{r-1})}\Big|\Big\{k, l \in I_{r,s} : \Big|\frac{t_{km}(\Delta X_{k,l})}{t_{km}(\Delta Y_{k,l})} - L\Big| \geq \epsilon\Big\}\Big| \\
&= \frac{k_1}{k_{r-1,s}k_1}A_{1,j} + \frac{k_2 - k_1}{k_{r-1,s}(k_2 - k_1)}A_{2,j} + \cdots + \frac{k_R - k_{R-1}}{k_{r-1,s}(k_R - k_{R-1})}A_{R,j} + \cdots \\
&+ \frac{k_r - k_{r-1}}{k_{r-1,s}(k_r - k_{r-1})}A_{r,j} \\
&\leq (\sup_{i,j\geq 1})\frac{k_R}{k_{r-1}} + (\sup_{i,j\geq R})\frac{k_r - k_R}{k_{r-1}} \leq K\frac{k_R}{k_{r-1,s}} + \epsilon B.
\end{aligned}$$

This completes the proof of the theorem.

Theorem 2.4 *Let* $(\theta_{r,s})$ *be a double lacunary sequence with* $1 < \liminf q_{r,s} \leq \limsup q_{r,s} < \infty$, $X \sim^{V_\theta^L(\Delta F)} Y \Leftrightarrow X \sim^{C_1^L(\Delta F)} Y$.

Proof. The proof follows from Theorem 2.2 and Theorem 2.3.

References

[1] M. Başarır, and O. Sonalcan, *On some double sequence spaces*, J. Indian Acad. Math., **21** (1999), 193-200.

[2] T. J. Bromwich, *An introduction to the theory of infinite series* , Macmillan and co. Ltd., New York (1965).

[3] G. H. Hardy, *On the convergence of certain multiple series*, Proc. Camb. Phil., Soc., **19** (1917), 86-95.

[4] H. Kızmaz, *On certain sequence spaces*, Can Math Bull, **24** (1981), 169-176.

[5] M. Matloka, *Sequences of fuzzy numbers*, BUSEFAL, **28** (1986), 28-37.

[6] F. Moricz, *Extension of the spaces c and c_0 from single to double sequences*, Acta Math. Hungarica, **57** (1991), 129-136.

[7] F. Moricz and B. E. Rhoades, *Almost convergence of double sequences and strong reqularity of summability matrices*, Math. Proc. Camb. Phil. Soc., **104** (1988), 283-294.

[8] M. Mursaleen, *Almost strongly regular matrices and a core theorem for double sequences*, J. Math. Anal. Appl., **293**(2),(2004), 523-531.

[9] M. Mursaleen and O. H. H. Edely, *Statistical convergence of double sequences*, J. Math. Anal. Appl., **288**(1),(2003), 223-231.

[10] M. Mursaleen and O. H. H. Edely, *Almost convergence and a core theorem for double sequences*, J. Math. Anal. Appl., **293**(2),(2004), 532-540.

[11] S. Nanda, *On sequence of fuzzy numbers*, Fuzzy sets and systems, **33** (1995), 123-126.

[12] F. Nuray and E. Savaş, *Statistical convergence of sequences of fuzzy numbers*, Math. Slovaca, **45** (3),(1995), 269-273.

[13] A. Pringsheim, *Zur Theori der zweifach unendlichen zahlenfolgen*, Math. Ann. **53**(1900), 289-321.

[14] K. Raj , S. K. Sharma and A. K. Sharma, *Some difference sequence spaces in n-normed spaces defined by Musielak-Orlicz function*, Armenian J. Math., **3** (2010), 127-141.

[15] K. Raj and S. K. Sharma, *Some sequence spaces in 2-normed spaces defined by Musielak-Orlicz functions*, Acta Univ. Sapientiae Math., **3** (2011), 97-109.

[16] E. Savaş, *Some almost convergent sequence spaces of fuzzy numbers generated by infinite matrices*, New Math. Nat. Comput., **2** (2006), 115-121.

[17] E. Savaş, *New double sequence spaces of fuzzy numbers*, Quaest. Math. **33** (2010), 449-456.

[18] E. Savaş and M. Mursaleen, *On statistically convergent double sequences of fuzzy numbers*, Inform. Sci. **162** (2004), 183-192.

[19] B. C. Tripathy, *Statistically convergent double sequences*, Tamkang J. Math., **34** (2003), 231-237.

[20] B. C. Tripathy, *Statistically convergent double sequences*, Tamkang J. Math., **34** (2003), 231-237.

[21] L. A. Zadeh, *Fuzzy sets*, Information and control, **8** (1965), 338-353.

[22] M. Zeltser, *Investigation of double sequence spaces by soft and hard analytical methods*, Diss. Math. Univ. Tartu. **25**, Tartu University Press, Univ. of Tartu, Faculty of Mathematics and Computer Science, Tartu (2001).

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