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ON INEXTENSIBLE FLOWS OF DUAL TANGENT DEVELOPABLE SURFACES IN THE DUAL SPACE $\mathbb{D}^3$

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Abstract

In this paper, we study inextensible flows of dual tangent developable surfaces of a curve in $\mathbb{D}^3$.

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1 Introduction

Dual numbers were introduced by W. K. Clifford (1849-1879) as a tool for his geometrical investigations [6]. After him E. Study used dual numbers and dual vectors in his research on the geometry of lines and kinematics [12]. He devoted special attention to the representation of directed lines by dual unit vectors and defined the mapping that is known by his name. There exists one-to-one correspondence between the points of dual unit sphere S^2 and the directed lines in R^3.

Surfaces representations are crucial to computer graphics, numerical simulation and computational geometry. Sampled representations, such as triangle meshes, have long served as simple, but effective, smooth surfaces approximations. The approximation of a smooth surfaces from a sampled geometric model, whether explicit or not, requries consistent notions of first-order and second-order differential geometric attributes, such as principal curvatures and principal directions. Typically, differential geometric properties are derived from surfaces vertices, mesh connectivity and, occasionally, by considering externally specified vertex normals.

Physically, inextensible curve and surface flows give rise to motions in which no strain energy is induced. The swinging motion of a cord of fixed length, for example, or of a piece of paper carried by the wind, can be described by inextensible curve and surface flows.

In this paper, we study inextensible flows of dual tangent developable surfaces of a curve in $\mathbb{D}^3$. We research inextensible flows of dual tangent developable surfaces in dual space $\mathbb{D}^3$.

2 Inextensible Flows of Dual Tangent Developable Dual Surfaces in the $\mathbb{D}^3$

Let

$$\hat{\gamma} : I \subset \mathbb{R} \to \mathbb{D}^3$$

$$s \to \hat{\gamma}(s) = \gamma(s) + \varepsilon\gamma^*(s)$$

be differential unit speed dual curve in dual space $\mathbb{D}^3$. Denote by $\{\hat{\mathbf{T}}, \hat{\mathbf{N}}, \hat{\mathbf{B}}\}$ the moving dual frenet frame along the dual space curve $\hat{\gamma}(s)$ in the dual space $\mathbb{D}^3$. Then $\hat{\mathbf{T}}, \hat{\mathbf{N}}$ and $\hat{\mathbf{B}}$ are the dual tangent, the dual principal normal and the dual binormal vector fields, respectively. Then for the dual curve $\hat{\gamma}$ the frenet formulae are given by,

$$\hat{\mathbf{T}}' = \kappa\mathbf{N} + \varepsilon(\kappa^*\mathbf{N} + \kappa\mathbf{N}^*),$$

$$\mathbf{N}' = -\kappa\mathbf{T} + \tau\mathbf{B} + \varepsilon\left(-\kappa^*\mathbf{T} - \kappa\mathbf{T}^* + \tau^*\mathbf{B} + \tau\mathbf{B}^*\right),$$

$$\hat{\mathbf{B}}' = \tau\mathbf{N} + \varepsilon\left(\tau^*\mathbf{N} + \tau\mathbf{N}^*\right),$$

where $\hat{\kappa} = \kappa + \varepsilon\kappa^*$ is nowhere pure dual natural curvatures and $\hat{\tau} = \tau + \varepsilon\tau^*$ is nowhere pure dual torsion.

The dual tangent developable of $\hat{\gamma}$ is a dual ruled surface

$$\hat{\mathbf{X}}(s,u) = \hat{\gamma}(s) + u\hat{\gamma}'(s). \tag{2.1}$$

Definition 2.1. *A dual surface evolution* $\hat{\mathbf{X}}(s,u,t)$ *and its flow* $\frac{\partial \hat{\mathbf{X}}}{\partial t}$ *are said to be inextensible if its first fundamental form* $\{\hat{\mathbf{E}}, \hat{\mathbf{F}}, \hat{\mathbf{G}}\}$ *satisfies*

$$\frac{\partial \hat{\mathbf{E}}}{\partial t} = \frac{\partial \hat{\mathbf{F}}}{\partial t} = \frac{\partial \hat{\mathbf{G}}}{\partial t} = 0.$$

This definition states that the dual surface $\hat{\mathbf{X}}(s,u,t)$ is, for all time t, the isometric image of the original surface $\hat{\mathbf{X}}(s,u,t_0)$ defined at some initial time t_0. For a developable surface, $\hat{\mathbf{X}}(s,u,t)$ can be physically pictured as the parametrization of a waving flag. For a given surface that is rigid, there exists no nontrivial inextensible evolution.

Definition 2.2. *We can define the following one-parameter family of dual* *tangent developable ruled surface*

$$\hat{\mathbf{X}}(s,u,t)=\hat{\gamma}(s,t)+u\hat{\gamma}'(s,t).$$

Theorem 2.3. *Let* $\hat{\mathbf{X}}$ *is the dual tangent developable surface in* D^3. *If* $\frac{\partial \hat{\mathbf{X}}}{\partial t}$ *is inextensible, then*

$$\frac{\partial \hat{\kappa}}{\partial t}=0.$$

Proof. Assume that $\hat{\mathbf{X}}(s,u,t)$ be a one-parameter family of dual tangent developable surface.

Then

$$\mathbf{X}_s(s,u,t)=\mathbf{T}+u\kappa\mathbf{N},$$

$$\mathbf{X}_s^*(s,u,t)=\mathbf{T}^*+u\kappa\mathbf{N}^*+u\kappa^*\mathbf{N},$$

$$\mathbf{X}_u(s,u,t)=\gamma'(s,t)=\mathbf{T}$$

$$\mathbf{X}_u^*(s,u,t)=\gamma^{*'}(s,t)=\mathbf{T}^*.$$

If we compute first fundamental form $\{\hat{\mathbf{E}},\hat{\mathbf{F}},\hat{\mathbf{G}}\}$, we have

$$\mathbf{E}=1+u^2\kappa^2,$$

$$\mathbf{E}^*=2u^2\kappa\kappa^*,$$

$$\mathbf{F}=1,$$

$$\mathbf{F}^* = 0,$$

$$\mathbf{G} = 1,$$

$$\mathbf{G}^* = 0,$$

where

$$\hat{\mathbf{E}} = \mathbf{E} + \varepsilon\mathbf{E}^*, \hat{\mathbf{F}} = \mathbf{F} + \varepsilon\mathbf{F}^*, \hat{\mathbf{G}} = \mathbf{G} + \varepsilon\mathbf{G}^*.$$

Using above system; we have

$$\frac{\partial \mathbf{E}}{\partial t} = 2u^2\kappa\frac{\partial \kappa}{\partial t},$$

$$\frac{\partial \mathbf{E}^*}{\partial t} = 2u^2(\kappa\frac{\partial \kappa^*}{\partial t} + \kappa^*\frac{\partial \kappa}{\partial t}),$$

$$\frac{\partial \mathbf{F}}{\partial t} = \frac{\partial \mathbf{F}^*}{\partial t} = 0,$$

$$\frac{\partial \mathbf{G}}{\partial t} = \frac{\partial \mathbf{G}^*}{\partial t} = 0.$$

Also, $\frac{\partial \hat{\mathbf{X}}}{\partial t}$ inextensible if

$$\frac{\partial \kappa}{\partial t} = \frac{\partial \kappa^*}{\partial t} = 0.$$

Corollary 2.4. *Let* $\hat{\mathbf{X}}$ *is the dual tangent developable surface in* $\mathbb{D}^3$*.If flow of this developable dual surface is inextensible then this dual surface is minimal if and only if*

$$\hat{\tau} = 0.$$

Proof. Using $\hat{\mathbf{X}}_s$ and $\hat{\mathbf{X}}_u$, we get

$$\mathbf{X}_{ss} = -u^2\kappa^2\mathbf{T} + (\kappa + u\frac{\partial \kappa}{\partial t})\mathbf{N} + u\kappa\tau\mathbf{B},$$

$$\mathbf{X}^*_{ss} = -u\kappa^2\mathbf{T}^* - 2u\kappa\kappa^*\mathbf{T} + \kappa\mathbf{N}^* + u\frac{\partial \kappa}{\partial t}\mathbf{N}^* + \kappa^*\mathbf{N}$$

$$+u\frac{\partial \kappa^*}{\partial t}\mathbf{N} + u\kappa\tau\mathbf{B}^* + u\kappa\tau^*\mathbf{B} + u\kappa^*\tau\mathbf{B},$$

$$\mathbf{X}_{su} = \kappa\mathbf{N},$$

$$\mathbf{X}^*_{su} = \kappa\mathbf{N}^* + \kappa^*\mathbf{N},$$

$$\mathbf{X}_{uu} = \mathbf{X}^*_{uu} = 0.$$

From above equations, $\hat{\mathbf{X}}$ is a minimal dual surface in $\mathbb{D}^3$ if and only if

$$\tau = \tau^* = 0.$$

By the use of above equation the proof is complete.

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