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INFINITESIMAL AFFINE TRANSFORMATION IN A PARA - SASAKIAN MANIFOLDS

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ABSTRACT

The purpose of this paper is to delineate an infinitesimal affine transformation in a Para-Sasakian manifolds. In section 1, we have defined and studied infinitesimal transformations in a Para-Sasakian manifolds. Section 2 is devoted to an infinitesimal automorphism in a Para-Sasakian manifolds.

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1. INTRODUCTION :

Definition 1.1:

In a Riemannian manifold, if a vector field u^α satisfies the following condition

(1.1) $$Lu\{{}^{\lambda}_{\alpha\ \beta}\} = \nabla_\alpha \nabla_\beta u^\lambda + R^\lambda{}_{\gamma\alpha\beta} u^\gamma = 0$$

is termed an **infinitesimal affine transformation** of a Para-Sasakian manifold.

Wherein Lu denotes the Lie-derivative with regard to a vector field u^α.

Definition 1.2:

A vector field u^α is called **curvature preserving infinitesimal transformation** of Para-Sasakian manifold if it satisfies the condition

(1.2) $$LuR^\lambda{}_{\gamma\alpha\beta} = 0$$

Wherein $R^\lambda{}_{\gamma\alpha\beta}$ is an Riemannian curvature tensor.

Definition 1.3:

A vector field u^α is called an **infinitesimal homothetic transformation** of Para - Sasakian manifold if u^α satisfies the condition

(1.3) $$Lug_{\alpha\beta} = \lambda g_{\alpha\beta}$$

Wherein λ is any constant.

Definition 1.4:

If $\lambda = 0$ in equation (1.3) then the vector field u^{α} is called **infinitesimal isometry**.

In a Riemannian manifold, we have [4]:

(1.4) $$LuR^{\lambda}{}_{\gamma\alpha\beta} = \nabla_{\alpha}Lu\{{}^{\ \lambda}_{\beta\ \gamma}\} - \nabla_{\beta}Lu\{{}^{\ \lambda}_{\alpha\ \gamma}\}$$

(1.5) $$Lu\{{}^{\ \lambda}_{\alpha\ \beta}\} = (1/2)g^{\lambda\gamma}(\nabla_{\alpha} Lug_{\beta\gamma} + \nabla_{\beta} Lug_{\gamma\alpha} - \nabla_{\gamma} Lug_{\alpha\beta})$$

(1.6) $$\eta_{\lambda}R^{\lambda}{}_{\gamma\alpha\beta} = \eta_{\alpha}g_{\beta\gamma} - \eta_{\beta}g_{\alpha\gamma} .$$

In this regard, we have the following theorems:

Theorem 1.1:

If a vector field u^{α} be an infinitesimal affine transformation of Para-Sasakian manifold then u^{α} becomes curvature preserving infinitesimal transformation.

Proof:

Since a vector field u^{α} is an infinitesimal affine transformation of Para-Sasakian manifold then

(1.7) $$L_u\{{}^{\lambda}_{\alpha\ \beta}\} = 0$$

By virtue of equations (1.4) and (1.7), we get

$$L_u R^{\lambda}{}_{\gamma\alpha\beta} = 0$$

Hence, u^{α} is curvature preserving infinitesimal transformation of Para-Sasakian manifold.

Theorem 1.2:

If a vector field u^{α} is an infinitesimal affine transformation of Para-Sasakian manifold then the condition

$$\xi^{\beta}(\nabla_{\alpha}\nabla_{\beta}u^{\lambda}) = 0$$

holds good.

Proof:

Since u^{α} is an infinitesimal affine transformation of Para-Sasakian manifold then

(1.8) $$\nabla_\alpha \nabla_\beta u^\lambda + R^\lambda{}_{\gamma\alpha\beta} u^\gamma = 0$$

Transvecting equation (1.8) by η_λ, we get

(1.9) $$\eta_\lambda (\nabla_\alpha \nabla_\beta u^\lambda) + \eta_\lambda R^\lambda{}_{\gamma\alpha\beta} u^\gamma = 0$$

By virtue of equations (1.6) and (1.9), we obtain

(1.10) $$\eta_\lambda (\nabla_\alpha \nabla_\beta u^\lambda) + (\eta_\gamma g_{\alpha\beta} - \eta_\alpha g_{\beta\gamma}) u^\gamma = 0$$

Transvecting equation (1.10) with ξ^λ and using equation (C-2,1.9), we get

(1.11) $$(\nabla_\alpha \nabla_\beta u^\lambda) + \xi^\lambda (\eta_\gamma g_{\alpha\beta} - \eta_\alpha g_{\beta\gamma}) u^\gamma = 0$$

Again transvecting equation (1.11) by ξ^β and using equation (C-2,1.4), we obtain

(1.12) $$\xi^\beta (\nabla_\alpha \nabla_\beta u^\lambda) = 0$$

Theorem 1.3:

If a vector field u^α is an infinitesimal isometry of Para-Sasakian manifold then the condition

$$\xi^{\gamma}\eta_{\lambda}LuR^{\lambda}{}_{\alpha\beta\gamma} = \eta_{\beta}\,Lu\eta_{\alpha} - \eta_{\alpha}\,Lu\eta_{\beta}$$

holds good.

Proof:

Taking the Lie-derivative with regard to u^{α} on both sides of equation (1.6), we get

(1.13) $$R^{\lambda}{}_{\alpha\beta\gamma}\,Lu\eta_{\lambda} + \eta_{\lambda}\,LuR^{\lambda}{}_{\alpha\beta\gamma} = g_{\beta\gamma}\,Lu\eta_{\alpha} + \eta_{\alpha}\,Lug_{\beta\gamma} - g_{\alpha\gamma}\,Lu\eta_{\beta} - \eta_{\beta}\,Lug_{\alpha\gamma}$$

Transvecting equation (1.13) by η_{λ} and using equation (1.6), we obtain

(1.14) $$(\eta_{\alpha}g_{\beta\gamma} - \eta_{\beta}g_{\alpha\gamma})Lu\eta_{\lambda} + \eta_{\lambda}\eta_{\lambda}\,LuR^{\lambda}{}_{\alpha\beta\gamma} = \eta_{\lambda}g_{\beta\gamma}\,Lu\eta_{\alpha} + \eta_{\lambda}\eta_{\alpha}\,Lug_{\beta\gamma} - \eta_{\lambda}g_{\alpha\gamma}\,Lu\eta_{\beta} - \eta_{\lambda}\eta_{\beta}\,Lug_{\alpha\gamma}$$

Transvecting equation (1.14) with ξ^{γ} and using equation (C-2,1.4), we get

(1.15) $$\xi^{\gamma}\eta_{\lambda}\eta_{\lambda}\,LuR^{\lambda}{}_{\alpha\beta\gamma} = \eta_{\lambda}\eta_{\beta}\,Lu\eta_{\alpha} + \xi^{\gamma}\eta_{\lambda}\eta_{\alpha}\,Lug_{\beta\gamma} - \eta_{\lambda}\eta_{\alpha}\,Lu\eta_{\beta} - \xi^{\gamma}\eta_{\lambda}\eta_{\beta}\,Lug_{\alpha\gamma}$$

Since u^{α} is an infinitesimal isometry of Para-Sasakian manifold then equation (1.15) reduces in the form

(1.16) $\xi^{\gamma}\eta_{\lambda}\, LuR^{\lambda}{}_{\alpha\beta\gamma} = \eta_{\beta}\, Lu\eta_{\alpha} - \eta_{\alpha}\, Lu\eta_{\beta}$,

2. INFINITESIMAL AUTOMORPHISM IN A PARA-SASAKIAN MANIFOLDS:

Definition 2.1:

A vector field u^{α} is said to be an **infinitesimal automorphism** if it satisfies the relations

(2.1) $Lug_{\alpha\beta} = 0$

(2.2) $Lu\xi^{\lambda} = 0$

(2.3) $Lu\eta_{\alpha} = 0$

And

(2.4) $Lu\phi^{\lambda}{}_{\alpha} = 0.$

Wherein Lu denotes the Lie-derivative with regard to a vector field u^{α}.

In this regard, we have the following theorem:

Theorem 2.1:

In a Para-Sasakian manifold, if a vector field u^{α} be an infinitesimal automorphism then u^{α} is curvature preserving infinitesimal transformation.

Proof:

If a vector field u^{α} is an infinitesimal automorphism then equation (1.13) reduces in the form

$$\eta_{\lambda} \, LuR^{\lambda}{}_{\alpha\beta\gamma} = 0 \tag{2.5}$$

Transvecting equation (2.5) by ξ^{λ} and using equation (C-2,1.9), we obtain

$$LuR^{\lambda}{}_{\alpha\beta\gamma} = 0 \tag{2.6}$$

Hence, u^{α} is curvature preserving infinitesimal transformation of Para-Sasakian manifold.

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