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Composition followed by differentiation between weighted Bergman-Nevanlinna spaces

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Abstract

In this paper, we characterize boundedness of $C_\varphi D$ acting on weighted Bergman-Nevanlinna spaces, where C_φ is the composition operator and D is the differentiation operator. We also provide a necessary condition and a sufficient condition for $C_\varphi D$ to be compact on weighted Bergman-Nevanlinna spaces.

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1 Introduction

Let $\mathbf{D}$ be the open unit disk in the complex plane $\mathbf{C}$, $H(\mathbf{D})$ be the algebra of all functions holomorphic on $\mathbf{D}$ and $\lambda \in (-1, \infty)$ be a real number. Let $dA(z) = \frac{1}{\pi} dxdy = \frac{1}{\pi} r dr d\theta$ be the normalized area measure on $\mathbf{D}$. For each $\lambda \in (-1, \infty)$, we set $d\nu_\lambda(z) = (\lambda+1)(1-|z|^2)^\lambda dA(z),\ z \in \mathbf{D}$. Then $d\nu_\lambda$ is a probability measure on $\mathbf{D}$. The weighted Bergman Nevanlinna space $\mathcal{A}^0_\lambda(\mathbf{D})$ consists of all $f \in H(\mathbf{D})$ such that

$$||f||_{\mathcal{A}^0_\lambda(\mathbf{D})} = \int_{\mathbf{D}} \log^+ |f(z)| d\nu_\lambda(z) < \infty,$$

where

$$\log^+ x = \begin{cases} \log x & if \;\; x \geq 1 \\ 0 & if \;\; x < 1. \end{cases}$$

In fact, $||f||_{\mathcal{A}^0_\lambda(\mathbf{D})}$ fails to be a norm, but $(f, g) \to ||f - g||_{\mathcal{A}^0_\lambda(\mathbf{D})}$ defines a translation invariant metric on $\mathcal{A}^0_\lambda(\mathbf{D})$ and this turns $\mathcal{A}^0_\lambda(\mathbf{D})$ into a complete metric space. The space $\mathcal{A}^0_\lambda(\mathbf{D})$ appears in the limit as $p \to 0$ of the weighted Bergman space

$$\mathcal{A}^p_\lambda(\mathbf{D}) = \left\{ f \in H(\mathbf{D}) : ||f||_{\mathcal{A}^p_\lambda(\mathbf{D})} = \left(\int_{\mathbf{D}} |f(z)|^p d\nu_\lambda(z) \right)^{1/p} < \infty \right\},$$

in the sense of

$$\lim_{p \to 0} \frac{t^p - 1}{p} = \log^+ t, \quad 0 < t < \infty.$$

The Bergman-Nevanlinna space $\mathcal{A}^0_\lambda(\mathbf{D})$ contains all the Bergman spaces $A^p_\lambda(\mathbf{D})$ for all $p > 0$. Obviously, the inequality

$$\log^+(x) \leq \log(1+x) \leq 1 + \log^+(x); \quad x \geq 0$$

implies that $f \in \mathcal{A}^0_\lambda(\mathbf{D})$ if and only if

$$||f||_{\mathcal{A}^0_\lambda(\mathbf{D})} \asymp \int_{\mathbf{D}} \log(1 + |f(z)|) d\nu_\lambda(z) < \infty,$$

where $X \asymp Y$ means that there is a positive constant C such that $C^{-1}X \leq Y \leq CX$. See [3] for more about weighted Bergman spaces and weighted Bergman-Nevanlinna spaces. By the subharmonicity of $\log(1 + |f(z)|)$, we have

$$\log(1 + |f(z)|) \leq C_\lambda \frac{||f||_{\mathcal{A}^0_\lambda(\mathbf{D})}}{(1-|z|^2)^{\lambda+2}}, \quad z \in \mathbf{D} \tag{1.1}$$

for all $f \in \mathcal{A}^0_\lambda(\mathbf{D})$. In particular, (1.1) tells us that if $f_n \to f$ in $\mathcal{A}^0_\lambda(\mathbf{D})$, then $f_n \to f$ locally uniformly. Here locally uniform convergence means the uniform convergence on every compact subset of $\mathbf{D}$.

Let φ be a holomorphic self-map $\mathbf{D}$ of itself. The composition operator C_φ is defined as follows $C_\varphi(f)(z) = f(\varphi(z))$ for all $f \in H(\mathbf{D})$. Let D be the differentiation operator. We know that on a general space of holomorphic functions, the differentiation operator D is typically unbounded. On the other hand, the composition operator C_φ is bounded on most of the spaces of holomorphic functions (see [1] and [6] for details), though the product is possibly still unbounded there. Hibschweiler and Portnoy [4] defined DC_φ and $C_\varphi D$ and investigated boundedness and compactness of the operators DC_φ and $C_\varphi D$ between weighted Bergman spaces. S. Ohno [5] discussed boundedness and compactness of $C_\varphi D$ between Hardy spaces. Recently, there are some papers that deal with these operators from a particular domain space of holomorphic functions into another space (see for example, [4],[5] and [7]-[18]. In this paper, we characterize boundedness of $C_\varphi D : \mathcal{A}^0_\lambda(\mathbf{D}) \to \mathcal{A}^0_\lambda(\mathbf{D})$. We also provide a necessary condition and a sufficient condition for $C_\varphi D$ to be compact on weighted Bergman-Nevanlinna spaces.

2 Preliminary Notes

Denote by $D(z,r)$ the pseudohyperbolic disk whose pseudohyperbolic centre is z and whose pseudo hyperbolic radius is r, that is:

$$D(z,r) = \Big\{\omega \in \mathbf{D} : \Big|\frac{(z-\omega)}{(1-\overline{z}\omega)}\Big| < r\Big\}.$$

For $z, \omega \in \mathbf{D}$ with

$$\rho(z,\omega) = \Big|\frac{(z-\omega)}{(1-\overline{z}\omega)}\Big| < r; \quad 0 < r < 1,$$

we have

$$\frac{(1-|z|^2)}{|1-z\overline{\omega}|} \asymp \frac{(1-|z|^2)}{(1-|\omega|^2)} \asymp 1 \quad \text{and} \quad \nu_\lambda(D(z,r)) \asymp (1-|z|^2)^{(\lambda+2)}.$$

See [1] for more information on pseudohyperbolic disks. The next two lemmas can also be found in [1] (see [2] also).

Lemma 2.1 *Let $0 < r < 1$. Then there is a sequence $\{a_n\}$ in $\mathbf{D}$ and a positive integer M such that*

(i) $\cup_{n=1}^\infty D(a_n, r) = \mathbf{D}$;

(ii) Each $z \in \mathbf{D}$ is in atmost M of the pseudohyperbolic disks $D(a_1, 2r), D(a_2, 2r), D(a_3, 2r) \cdots$;

(iii) If $n \neq m$, then $\rho(a_n, a_m) \geq r/2$.

Lemma 2.2 *Let $\lambda \in (-1, \infty)$ and $\beta > 0$, then there exists a constant $C = C(\lambda, \beta)$ such that*

$$(1-|z|^2)^\beta \int_{\mathbf{D}} \frac{d\nu_\lambda(\omega)}{|1-\overline{z}\omega|^{2+\lambda+\beta}} \asymp 1, \quad z \in \mathbf{D}.$$

Definition 2.3 *A positive Borel measure μ on $\mathbf{D}$ is called an λ-Carleson measure if and only if*

$$\sup_{z \in \mathbf{D}} \frac{\mu(D(z,r))}{(1-|z|^2)^\lambda} < \infty,$$

and it is called a vanishing λ-Carleson measure if

$$\lim_{|z| \to 1} \frac{\mu(D(z,r))}{(1-|z|^2)^\lambda} = 0.$$

The next lemma is proved in [2].

Lemma 2.4 *Let $\lambda \in (-1, \infty)$ and $0 < r < 1$, then there exists a constant $C = C(\lambda, r)$ such that the following inequality holds:*

$$\log(1+|f'(z)|) \leq C \int_{D(z,r)} \frac{\log(1+|f(\omega)|)}{(1-|\omega|)^{\lambda+3}} d\nu_\lambda(\omega).$$

Lemma 2.5 *Let $\lambda \in (-1, \infty)$ and $0 < r < 1$, be fixed. If μ is $(\lambda+3)$-Carleson measure on $\mathbf{D}$, then there exists a constant $C = C(\lambda, r)$ such that the following inequality holds:*

$$\int_{\mathbf{D}} \log(1+|f'(\omega)|) d\mu(\omega) \leq C \int_{\mathbf{D}} \log(1+|f(\omega)|) d\mu(\omega).$$

Proof. Let $0 < r < 1$, be fixed . Pick a sequence $\{a_n\}$ in $\mathbf{D}$ satisfying the conditions of Lemma 2.1. For $f \in \mathcal{A}^0_\lambda(\mathbf{D})$, we have

$$\begin{aligned}
\int_{\mathbf{D}} \log(1+|f'(\omega)|) d\mu(\omega) &\leq \sum_{n=1}^{\infty} \int_{D(a_n,r)} \log(1+|f'(\omega)|) d\mu(\omega) \\
&\leq \sum_{n=1}^{\infty} \mu(D(a_n,r)) \sup_{\omega \in D(a_n,r)} \log(1+|f'(\omega)|) \\
&\leq \sum_{n=1}^{\infty} \frac{\mu(D(a_n,r))}{(1-|a|)^{\lambda+3}} \int_{D(a_n,2r)} \log(1+|f(\omega)|) d\mu(\omega).
\end{aligned}$$

Now μ is $(\lambda+3)$ -Carleson measure on $\mathbf{D}$, so we have

$$\begin{aligned}
\int_{\mathbf{D}} \log(1+|f'(\omega)|) d\mu(\omega) &\leq C \sum_{n=1}^{\infty} \int_{D(a_n,2r)} \log(1+|f(\omega)|) d\mu(\omega) \\
&= CM \int_{\mathbf{D}} \log(1+|f(\omega)|) d\mu(\omega).
\end{aligned}$$

3 Boundedness and compactness of $C_\varphi D$ on $\mathcal{A}^0_\lambda(\mathbf{D})$

In this section, we characterize boundedness of $C_\varphi D : \mathcal{A}^0_\lambda(\mathbf{D}) \to \mathcal{A}^0_\lambda(\mathbf{D})$. We also provide a necessary condition and a sufficient condition for $C_\varphi D$ to be compact on $\mathcal{A}^0_\lambda(\mathbf{D})$.

Theorem 3.1 *Let φ be a holomorphic self-map of $\mathbf{D}$. Then the following are equivalent:*

1. *$C_\varphi D : \mathcal{A}^0_\lambda(\mathbf{D}) \to \mathcal{A}^0_\lambda(\mathbf{D})$ is bounded.*
2. *The pull-back measure $\nu_\lambda \circ \varphi^{-1}$ is a $(\lambda+3)$-Carleson measure on $\mathbf{D}$.*

Proof. Suppose that $C_\varphi D : \mathcal{A}^0_\lambda(\mathbf{D}) \to \mathcal{A}^0_\lambda(\mathbf{D})$ is bounded. Consider the function

$$f_z(\omega) = \frac{(1-|z|^2)^{\lambda+4}}{(1-\overline{z}\omega)^{\lambda+3}}, \quad z \in \mathbf{D}.$$

By Lemma 2.2, we have

$$||f_z||_{\mathcal{A}^0_\lambda(\mathbf{D})} \le ||f_z||_{A^1_\lambda(\mathbf{D})} \asymp (1-|z|)^{\lambda+3}$$

for all $z \in \mathbf{D}$. Also

$$f_z'(\omega) = (\lambda+3)\overline{z}\frac{(1-|z|^2)^{\lambda+4}}{(1-\overline{z}\omega)^{\lambda+4}}.$$

Therefore ,

$$|f_z'(\omega)| \le |z|(\lambda+3)\frac{(1-|z|^2)^{\lambda+4}}{(1-\overline{z}\omega)^{\lambda+4}},$$

and so we have $|f_z'(\omega)| \le C$ for some constant $C = C(\lambda)$. Thus $\log(1+|f_z'(\omega)|) \asymp |f_z'(\omega)|$ for all $z, \omega \in \mathbf{D}$. In addition, we have

$$\frac{(1-|z|^2)}{|1-\overline{z}\omega|} \asymp \frac{(1-|z|^2)}{(1-|\omega|^2)} \asymp 1$$

for $\omega \in D(z,r)$. Thus $|f_z'(\omega)| \asymp |z|$ for $\omega \in D(z,r)$. Since $C_\varphi D : \mathcal{A}^0_\lambda(\mathbf{D}) \to \mathcal{A}^0_\lambda(\mathbf{D})$ is bounded, there exists $C > 0$ such that

$$||C_\varphi D f_z||_{\mathcal{A}^0_\lambda(\mathbf{D})} \le C||f_z||_{\mathcal{A}^0_\lambda(\mathbf{D})} \asymp (1-|z|^2)^{\lambda+3}.$$

That is,

$$\begin{aligned}(1-|z|^2)^{\lambda+3} &\asymp ||C_\varphi D f_z||_{\mathcal{A}^0_\lambda(\mathbf{D})} \asymp \int_{\mathbf{D}} \log(1+|f_z'(\varphi(z)|))d\nu_\lambda(\omega) \\ &\ge C\int_{\mathbf{D}} |f_z'(\omega)|d(\nu_\lambda\circ\varphi^{-1})(\omega) \ge C\int_{D(z,r)} |f_z'(\omega)|d(\nu_\lambda\circ\varphi^{-1})(\omega)\end{aligned}$$

$$\asymp |z|\nu_\lambda \circ \varphi^{-1}D(z,r),$$

for all $z \in \mathbf{D}$. Consequently,

$$\sup_{z\in\mathbf{D}} \frac{(\nu_\lambda \circ \varphi^{-1})D(z,r)}{(1-|z|^2)^{\lambda+3}} < \infty.$$

Hence $\nu_\lambda \circ \varphi^{-1}$ is an $(\lambda+3)$ measure on $\mathbf{D}$. Conversely, suppose that $\nu_\lambda \circ \varphi^{-1}$ is an $(\lambda+3)$ measure on $\mathbf{D}$. Then by Lemma 2.4, we have for $f \in \mathcal{A}^0_\lambda(\mathbf{D})$,

$$\begin{aligned} ||C_\varphi D f_z||_{\mathcal{A}^0_\lambda(\mathbf{D})} &= \int_{\mathbf{D}} \log(1+|f'(\varphi(\omega))|)d\nu_\lambda(\omega) \\ &= \int_{\mathbf{D}} \log(1+|f'(\varphi(\omega))|)d(\nu_\lambda \circ \varphi^{-1})(\omega) \\ &\leq C\int_{\mathbf{D}} \log(1+|f(\varphi(\omega))|)d\nu_\lambda(\omega) \asymp ||f||_{\mathcal{A}^0_\lambda(\mathbf{D})}. \end{aligned}$$

Lemma 3.2 *Let φ be a holomorphic map of $\mathbf{D}$ such that $\varphi(\mathbf{D}) \subset \mathbf{D}$. Then $C_\varphi D : \mathcal{A}^0_\lambda(\mathbf{D}) \to \mathcal{A}^0_\lambda(\mathbf{D})$ is compact if and only if for every sequence $\{f_n\}$ which is bounded in $\mathcal{A}^0_\lambda(\mathbf{D})$ and converges to zero uniformly on compact subsets of $\mathbf{D}$ as $n \to \infty$, we have $||C_\varphi D f_n||_{\mathcal{A}^0_\lambda(\mathbf{D})} \to 0$.*

Proof follows on the same lines as the proof of proposition 3.11 in [1]. We omit the details.
We now present a sufficient condition for the compactness of of $C_\varphi D : \mathcal{A}^0_\lambda(\mathbf{D}) \to \mathcal{A}^0_\lambda(\mathbf{D})$.

Theorem 3.3 *Let φ be a holomorphic map of $\mathbf{D}$ such that $\varphi(\mathbf{D}) \subset \mathbf{D}$. Then $C_\varphi D : \mathcal{A}^0_\lambda(\mathbf{D}) \to \mathcal{A}^0_\lambda(\mathbf{D})$ is compact if the pull-back measure $\nu_\lambda \circ \varphi^{-1}$ is a vanishing $(\lambda+3)$-Carleson measure on $\mathbf{D}$.*

Proof. Suppose that $\nu_\lambda \circ \varphi^{-1}$ is a vanishing $(\lambda+3)$-Carleson measure on $\mathbf{D}$. Then

$$\frac{(\nu_\lambda \circ \varphi^{-1})D(a,r)}{(1-|a|^2)^{\lambda+3}} \to 0 \ \text{ as } \ |a| \to 1.$$

Suppose that $\{f_m\}$ is a bounded sequence in $\mathcal{A}^0_\lambda(\mathbf{D})$ that converges to zero uniformly on compact subsets of $\mathbf{D}$. Let $\{a_n\}$ be a sequence as in Lemma 2.1 such that $|a_1| < |a_2| < |a_3| \cdots$. Then for each $\epsilon > 0$ we have

$$(\nu_\lambda \circ \varphi^{-1})(D(a_n,r)) < \epsilon(1-|a_n|^2)^{\lambda+3}$$

for all $a_n \in \mathbf{D}$ such that $|a_n| > r$. Thus

$$||C_\varphi D f_m||_{\mathcal{A}^0_\lambda(\mathbf{D})} \asymp \int_{\mathbf{D}} \log(1+|f'_m(\varphi(z))|)d\nu_\lambda(z)$$

$$
\begin{aligned}
&= \int_{\mathbf{D}} \log(1+|f'_m(z)|)d(\nu_\lambda \circ \varphi^{-1})(z) \\
&= \int_{|z|\leq r_0} \log(1+|f'_m(z)|)d(\nu_\lambda \circ \varphi^{-1})(z) \\
&\quad + \int_{|z|>r_0} \log(1+|f'_m(z)|)d(\nu_\lambda \circ \varphi^{-1})(z).
\end{aligned}
$$

Since $\{f_m\}$ is a bounded sequence in $\mathcal{A}^0_\lambda(\mathbf{D})$ that converges to zero uniformly on compact subsets of $\mathbf{D}$,

$$\lim_{m\to\infty} \int_{|z|\leq r_0} \log(1+|f'_m(z)|)d(\nu_\lambda \circ \varphi^{-1})(z) = 0,$$

whereas the second term in the above expression is bounded by

$$
\begin{aligned}
&\sum_{n=k+1}^{\infty} \int_{D(a_n,r)} \log(1+|f'_m(z)|)d(\nu_\lambda \circ \varphi^{-1})(z) \\
&\leq \sum_{n=k+1}^{\infty} (\nu_\lambda \circ \varphi_{-1})(D(a_n,r)) \sup_{z\in D(a_n,r)} \log(1+|f'_m(z)|) \\
&\leq \sum_{n=k+1}^{\infty} \frac{(\nu_\lambda \circ \varphi_{-1})(D(a_n,r))}{(1-|a_n|^2)^{\lambda+3}} \int_{D(a_n,2r)} \log(1+|f_m(z)|)d\nu_\lambda(z) \\
&\leq \epsilon CM \int_{\mathbf{D}} \log(1+|f_m(z)|)d\nu_\lambda(z) = \epsilon CM||f_m||_{\mathcal{A}^0_\lambda}.
\end{aligned}
$$

Since $\epsilon > 0$ is arbitrary, we have $||C_\varphi D f_m||_{\mathcal{A}^0_\lambda(\mathbf{D})} \to 0$ as $m \to \infty$. Hence $C_\varphi D : \mathcal{A}^0_\lambda(\mathbf{D}) \to \mathcal{A}^0_\lambda(\mathbf{D})$ is compact.
Finally, we provide a necessary condition for compactness of $C_\varphi D : \mathcal{A}^0_\lambda(\mathbf{D}) \to \mathcal{A}^0_\lambda(\mathbf{D})$.

Theorem 3.4 *Let φ be a holomorphic map of $\mathbf{D}$ such that $\varphi(\mathbf{D}) \subset \mathbf{D}$. Then if $C_\varphi D : \mathcal{A}^0_\lambda(\mathbf{D}) \to \mathcal{A}^0_\lambda(\mathbf{D})$ is bounded, then $\nu_\lambda \circ \varphi^{-1}$ is a vanishing $(\lambda+2)$-Carleson measure on $\mathbf{D}$.*

Proof. Let $\{a_n\}$ be a sequence in $\mathbf{D}$ such that $|a_n| \to 1$ as $n \to \infty$. Consider the family of functions

$$f_n(z) = \frac{(1-|a_n|)^{\lambda+3}}{2|a_n|(\lambda+2)} \exp\Big[\frac{(1-|a_n|^2)^{\lambda+2}}{(1-\overline{a_n}z)^{2(\lambda+2)}}\Big].$$

Clearly, $f_n \to 0$ uniformly on compact subsets of $\mathbf{D}$ as $n \to \infty$. Also

$$||f_n||_{\mathcal{A}^0_\lambda(\mathbf{D})} \leq 1 + \int_{\mathbf{D}} \log^+ \Big|\frac{(1-|a_n|)^{\lambda+3}}{2|a_n|(\lambda+2)} \exp\Big[\frac{(1-|a_n|^2)^{\lambda+2}}{(1-\overline{a_n}z)^{2(\lambda+2)}}\Big]\Big| d\nu_\lambda(z)$$

$$\leq 1 + C \int_{\mathbf{D}} \frac{(1-|a_n|^2)^{\lambda+2}}{|1-\overline{a}_n z|^{2(\lambda+2)}} d\nu_\lambda(z) \leq 1 + C.$$

Moreover,

$$f_n'(z) = \frac{(1-|a_n|^2)^{2\lambda+5}}{|a_n|(1-\overline{a}_n z)^{2\lambda+5}} \overline{a}_n \exp\Big[\frac{(1-|a_n|^2)^{\lambda+2}}{(1-\overline{a_n} z)^{2(\lambda+2)}}\Big],$$

and so

$$|f_n'(z)| = \frac{(1-|a_n|^2)^{2\lambda+5}}{|1-\overline{a}_n z|^{2\lambda+5}} \exp\Big[Re\Big(\frac{(1-|a_n|^2)^{\lambda+2}}{(1-\overline{a_n} z)^{2(\lambda+2)}}\Big)\Big].$$

Now

$$Re\Big(\frac{(1-|a_n|^2)^{\lambda+2}}{(1-\overline{a_n} z)^{2(\lambda+2)}}\Big) \asymp \frac{1}{(1-|a_n|)^{\lambda+2}},$$

whenever $z \in D(a_n, r)$. Thus

$$\log(1+|f_n'(z)|) \geq \log^+ |f_n'(z)| \geq \frac{C}{(1-|a_n|^2)^{\lambda+2}}$$

if $z \in D(a_n, r)$. Therefore,

$$\frac{C}{(1-|a_n|^2)^{\lambda+2}} (\nu_\lambda \circ \varphi^{-1})(D(a_n, r))$$

$$\leq \int_{D(a_n,r)} \log^+ |f_n'(z)| d(\nu_\lambda \circ \varphi^{-1})(z) \leq ||C_\varphi D f_n||_{\mathcal{A}_\lambda^0(\mathbf{D})}.$$

But compactness of $C_\varphi D$ forces $||C_\varphi D f_n||_{\mathcal{A}_\lambda^0(\mathbf{D})}$ to tend to zero as $|a_n| \to 1$. Thus

$$\lim_{|a_n|\to 1} \frac{(\nu_\lambda \circ \varphi^{-1})(D(a_n, r))}{(1-|a_n|^2)^{\lambda+2}} = 0,$$

and so $\nu_\lambda \circ \varphi^{-1}$ is a vanishing $(\lambda+2)$-Carleson measure on $\mathbf{D}$.

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