

Generalized tanh method with the Riccati equation for solving the Sixth-Order Ramani equation

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Abstract

The generalized tanh method is one of most direct and effective algebraic method for obtaining exact solutions of nonlinear partial differential equations. The method can be applied to nonintegrable equations as well as to integrable ones.

In this paper, we will find new exact traveling wave solutions of the Sixth-Order Ramani equation by using generalized tanh method.

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1 Introduction

The search of exact solutions to nonlinear partial differential equations is of great importance, because these equations appear in complex physics phenomena, mechanics, chemistry, biology and engineering. A variety of powerful and

direct methods have been developed in this direction. The principal objective of this paper, is to present the generalized tanh method [1].

The aim of this paper is to find exact solutions of the Sixth-Order Ramani equation by generalized tanh method with the Riccati equation.

2 Generalized tanh method

Let us describe the generalized tanh method. For given a nonlinear equation

$$F(u, u_x, u_y, u_t, u_{xx}, u_{xy}, u_{xt}, \dots) = 0, \quad (1)$$

we look for its traveling wave solutions. The first step is to introduce the wave transformation $u(x, y, t) = u(\xi)$, $\xi = sx + ly + nt + d$, and change Eq.(1) to an ordinary differential equation(ODE)

$$H(u, u', u'', u''', \dots) = 0. \quad (2)$$

The next crucial step is to introduce a new variable $\phi = \phi(\xi)$, which is a solution of the Riccati equation

$$\frac{d\phi}{d\xi} = k + \phi^2. \quad (3)$$

The generalized tanh method admits the use of the finite expansion:

$$u(x, y, t) = u(\xi) = \sum_{i=0}^N a_i \phi^i(\xi), \quad (4)$$

where the positive integer N is usually obtained by balancing the highest-order linear term with the nonlinear terms in Eq.(2). Substituting (3) and (4) into Eq.(2) and then setting zero all coefficients of $\phi^i(\xi)$, we can obtain a system of algebraic equations with respect to the constants $k, s, l, n, a_0, \dots, a_N$. Then we can determine the constants $k, s, l, n, a_0, \dots, a_N$. The Riccati equation (3) has the general solutions:

If $k < 0$ then

$$\begin{aligned} \phi(\xi) &= -\sqrt{-k} \tanh(\sqrt{-k}\xi), \\ \phi(\xi) &= -\sqrt{-k} \coth(\sqrt{-k}\xi). \end{aligned} \quad (5)$$

If $k = 0$ then

$$\phi(\xi) = -\frac{1}{\xi}. \quad (6)$$

If $k > 0$ then

$$\begin{aligned} \phi(\xi) &= \sqrt{k} \tan(\sqrt{k}\xi), \\ \phi(\xi) &= -\sqrt{k} \cot(\sqrt{k}\xi). \end{aligned} \quad (7)$$

Therefore, by the sign test of k , we can obtain exact solutions of Eq.(1).

3 The Sixth-Order Ramani equation

For the Sixth-Order Ramani equation [4],

$$u_{xxxxxx} + 15u_x u_{xxxx} + 15u_{xx} u_{xxx} + 45u_x^2 u_{xx} - 5u_{xxx} - 15u_x u_{xt} - 15u_t u_{xx} - 5u_{tt} = 0. \tag{8}$$

By making the transformation

$$u(x, t) = u(\xi), \quad \xi = sx + nt + d, \tag{9}$$

where s, n and d are constants.

Substituting (9) into (8), we have

$$s^6 u^{(6)} + 15s^5 u' u^{(4)} + 15s^5 u'' u''' + 45s^4 (u')^2 u'' - 5s^3 n u^{(4)} - 30s^2 n u' u'' - 5n^2 u'' = 0. \tag{10}$$

Integrating of Eq.(10), we obtain

$$s^6 u^{(5)} + (-5s^3 n + 15s^5 u') u''' + 15s^4 (u')^3 - 15s^2 n (u')^2 - 5n^2 u' + R = 0. \tag{11}$$

where R is integration constant.

It is easy to show that $N = 1$, if balancing $u^{(5)}$ with $(u')^3$. Therefore, the generalized tanh method (4) admits the use of the finite expansion:

$$u(\xi) = a_0 + a_1 \phi(\xi). \tag{12}$$

Thus, by Eq.(3), we get

$$u'(\xi) = a_1 \phi^2(\xi) + a_1 k, \tag{13}$$

$$(u')^2(\xi) = a_1^2 \phi^4(\xi) + 2a_1^2 k \phi^2(\xi) + a_1^2 k^2, \tag{14}$$

$$(u')^3(\xi) = a_1^3 \phi^6(\xi) + 3a_1^3 k \phi^4(\xi) + 3a_1^3 k^2 \phi^2(\xi) + a_1^3 k^3, \tag{15}$$

$$u'''(\xi) = 6a_1 \phi^4(\xi) + 8a_1 k \phi^2(\xi) + 2a_1 k^2, \tag{16}$$

$$u^{(5)}(\xi) = 120a_1 \phi^6(\xi) + 240a_1 k \phi^4(\xi) + 136a_1 k^2 \phi^2(\xi) + 16a_1 k^3. \tag{17}$$

Substituting Eqs.(13) – (17) into Eq.(11), and equating the coefficients of like powers of

$\phi^i (i = -6, -4, -2, 0, 2, 4, 6)$ to zero yields the system of algebraic equations to a_1, b_1, s, n and k

$$\begin{aligned} \phi^6 & : 15s^4 a_1^3 + 120s^6 a_1 + 90s^5 a_1^2 = 0, \\ \phi^4 & : 240s^6 a_1 k - 30s^3 n a_1 + 210s^5 a_1^2 k - 15s^2 n a_1^2 + 45s^4 a_1^2 k = 0, \\ \phi^2 & : 136s^6 a_1 k^2 + 45s^4 a_1^3 k^2 - 5n^2 a_1 - 30s^2 n a_1^2 k - 40s^3 n a_1 k + 150s^5 a_1^2 k^2 = 0, \\ \phi^0 & : R - 5n^2 a_1 k + 15s^4 a_1^3 k^3 + 30s^5 a_1^2 k^3 + 16s^6 k^3 a_1 - 15s^2 n a_1^2 k^2 - 10s^3 n a_1 k^2 = 0. \end{aligned}$$

Solving the resulting system, by using Maple, we find the following solutions:

$$k = \frac{n}{2s^3}, \quad a_1 = -2s, \quad R = \frac{9n^3}{s^2}. \quad (18)$$

$$k = -\frac{n}{4s^3}, \quad a_1 = -4s, \quad R = \frac{9n^3}{s^2}. \quad (19)$$

Thus, the exact solutions of the Sixth-Order Ramani equation (8) have the following forms:

For the expression (18), we deduce for $k < 0$ that

$$u_1(x, t) = 2s\sqrt{-\frac{n}{2s^3}} \tanh\left(\sqrt{-\frac{n}{2s^3}}(sx + nt + d)\right) + a_0,$$

or

$$u_1(x, t) = 2s\sqrt{-\frac{n}{2s^3}} \coth\left(\sqrt{-\frac{n}{2s^3}}(sx + nt + d)\right) + a_0,$$

while for $k > 0$ we deduce that

$$u_1(x, t) = -2s\sqrt{\frac{n}{2s^3}} \tan\left(\sqrt{\frac{n}{2s^3}}(sx + nt + d)\right) + a_0,$$

or

$$u_1(x, t) = 2s\sqrt{\frac{n}{2s^3}} \cot\left(\sqrt{\frac{n}{2s^3}}(sx + nt + d)\right) + a_0.$$

For the expression (19), we deduce for $k < 0$ that

$$u_2(x, t) = 2s\sqrt{\frac{n}{s^3}} \tanh\left(\sqrt{\frac{n}{4s^3}}(sx + nt + d)\right) + a_0,$$

or

$$u_2(x, t) = 2s\sqrt{\frac{n}{s^3}} \coth\left(\sqrt{\frac{n}{4s^3}}(sx + nt + d)\right) + a_0,$$

while for $k > 0$ we deduce that

$$u_2(x, t) = -2s\sqrt{-\frac{n}{s^3}} \tan\left(\sqrt{-\frac{n}{4s^3}}(sx + nt + d)\right) + a_0,$$

or

$$u_2(x, t) = 2s\sqrt{-\frac{n}{s^3}} \cot\left(\sqrt{-\frac{n}{4s^3}}(sx + nt + d)\right) + a_0.$$

4 Conclusion

In this paper, the generalized tanh method has been successfully applied to find the solutions of the Sixth-Order Ramani equation. The generalized tanh method is used to find new exact traveling wave solutions. Thus, we can say that the proposed method can be extended to solve the problems of nonlinear partial differential equations which arising in the theory of solitons and other areas.

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