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Soret And Dufour Effect In Generalized MHD Couette Flow Of A Binary Mixture

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Abstract

The Soret and Dufour effects in hydromagnetic generalized couette flow of a binary mixture of incompressible viscous fluids between two non magnetic walls in porous medium have been studied numerically. The effects of various parameters entering into the problem have been examined on the flow of a binary mixture of gases with moderate molecular weight. Finally, the influences of the various parameters on the action of separation are presented in graphically.

Mathematics Subject Classification: 76S05, 76Txx

Keywords: Soret And Dufour Effect, Couette Flow, Binary Mixture

1 Introduction

Flow of fluids through porous media are quite prevalent due to their applications in the study of underground water resources in the field of agriculture engineering, seepage of water in river beds, to study movement of natural gases, water through oil reservoirs, in filtration and purification in chemical engineering, geothermal energy utilization, thermal energy storage and recoverable system, petroleum resources, industrial and agricultural water distribution [1-2]. Geophysics encounters the characteristics in the interaction conducting fluid and magnetic field. Engineers employ magnetohydrodynamic (MHD) principle, in the design of heat exchangers, pumps and flow meters, in space vehicle propulsion, control and re-entry, in creating novel power generators. Design engineers require relations between heat and mass transfer geometry, and boundary conditions to study cost benefit analysis to determine the amount of insulation that will yield maximum investment etc [3]. Origin of the flow through porous media relies heavily upon Darcy's experimental law. The flow through porous media capability enables engineers to simulate fluid flow through media such as ground rock, filters and catalyst beds. For example, simulating underground flow through porous rock can enable engineers to predict the movement of contaminated fluid from a solid waste landfill into a drinking water supply. In industrial applications, harmful particles can be filtered from a fluid stream by passing it through a porous solid whose pores are too small to permit passage of the particles. Additionally, Porous media may provide sites for chemical catalysis or absorption of components of the fluid. Several geophysical applications of fluid flow in porous media have been reported by Cunningham and Williams [2]. Darcy [5] was the first to initiate the mathematical theory of porous media and later Brinkman [6] proposed modification of Darcy's law through porous media. The subject of Magneto-hydrodynamic attracted the attention the of many research workers in view of its applications to Astrophysics, Geophysics and Engineering [7]. Magnetic fields also have been shown to have positive effects on numerous systems [8-12]. Laminar heat transfer in Hartmann flow between two parallel plates has been studied extensively by Roming [13] and Perlmutteer and Siegel [14]. The study of magneto-hydrodynamic flows through porous medium has been studied by several authors [15-18].Combined heat and mass transfer (or double diffusion) problems in fluid saturated porous media have applications in a variety of engineering problems such as heat exchanger devices, petroleum reservoirs, chemical catalytic reactor processes, geothermal and geophysical engineering, moisture migration in fibrous insulation and nuclear waste disposal and several others. Mass flux does not occur only by concentration gradient of species but also created by temperature gradient (Dufour effect). Energy flux is generated not only by temperature gradient but also generated by gradients of concentration of species (Soret effect).

Diffusion-thermo (Soret) effects are utilized for isotope separation and in mixture of gases with very low molecular weight (for example Hydrogen, Helium) and moderate molecular weight gases (Nitrogen, Air) [Raptis (1962), Kafoussias (1982), Sattar (1993) and Kim (2004)]. Sattar [19] has studied unsteady MHD flow through porous medium near a vertical plate. Kim [20] has developed a model for MHD flow of micropolar fluid through porous medium in straight channel between two fixed walls. Recently, Soret effect has been studied in MHD flow through porous media between two parallel walls with slip conditions [21]. Sharma and Singh [16] (2004), have discussed Soret effect in generalized MHD couette flow of a binary mixture.

In this paper, we have studied the Soret and Dufour effects in generalized MHD couette flow of a binary mixture of gasses with moderate molecular weights. Such problems are useful in separating one of the gasses present in small amount from the mixture.

2 Mathematical Analysis

Let us consider the steady flow of a binary mixture of thermally and electrically-conducting, incompressible viscous fluids enclosed between two parallel non-magnetic plates $y=0$ and $y=d$ Infinite in x and z directions using Cartesian coordinates (x,y,z) in presence of a uniform transverse magnetic field β_0 and porous medium is also considered. The plates $y=0$ taken as at constant temperature T_0, is at rest while the plate $y=d$, taken as at temperature $T_1\left(T_0>T_1\right)$, moves with a uniform velocity U_0 in the direction parallel to x-axis parallel to the horizontal plates. Further, the concentration C_1 of the first component of the binary mixture is taken as at constant value C_0 at $y=0$. The relevant equations governing the flow are:

$$\frac{d^2U_x}{d\,y^2}-\frac{\sigma B_0^2U_x}{\mu}-\frac{\upsilon}{K}U_x=\frac{1}{\mu}\frac{dp}{dx} \tag{1}$$

$$\frac{d^2T}{d\,y^2}+\frac{\mu}{K}\left(\frac{dU_x}{dy}\right)^2+\frac{\sigma B_0^2U_x^2}{K}+\frac{D_mK_T}{K\,C_sC_p}\frac{d^2C}{d\,y^2}=0 \tag{2}$$

$$\frac{d^2C_1}{d\,y^2}+S_T\frac{d}{dy}\left(C_1\frac{dT}{dy}\right)+\frac{D_mK_T}{D^*T_m}\frac{d^2T}{d\,y^2}=0 \tag{3}$$

Where the only non vanishing component of velocity will be in the direction of x-axis, while the velocity components in the direction of y-axis and z-axis will vanish i.e. $U_y = U_z = 0$ and $U_x = U_x(y)$. Also, the pressure p is independent of y and z i.e. $p = p(x)$. Where S_T denotes Soret coefficient.

The boundary conditions on velocity and temperature at the two plates are

$$U_x = 0, \qquad T = T_0 \qquad at \quad y = 0 \tag{4}$$

$$U_x = U_0, \qquad T = T_1 \qquad at \quad y = d \tag{5}$$

By substituting the following non-dimensional forms for velocity, temperature, concentration and other variables

$$U(\eta) = \frac{U_x}{U_0},\ \theta(\eta) = \frac{(T - T_0)}{(T_0 - T_1)},\ f(\eta) = \frac{C}{C_0},\ \eta = \frac{y}{d} \tag{6}$$

In equations (1), (2) and (3), we get

$$\frac{d^2U}{d\eta^2} - M^2U = -N \tag{7}$$

$$\frac{d^2\theta}{d\eta^2} + P_r E_c\left[\left(\frac{dU}{d\eta}\right)^2 + M^2U^2\right] + P_r D_u \frac{d^2 f}{d\eta^2} = 0 \tag{8}$$

$$\frac{d^2 f}{d\eta^2} + t\,d\,\frac{d}{d\eta}\left(f\frac{d\theta}{d\eta}\right) + S_c S_t \frac{d^2\theta}{d\eta^2} = 0 \tag{9}$$

Where $M^2 = \left(m_0^2 + \frac{1}{\alpha}\right),\ m_0^2 = \frac{\sigma B_0^2 d^2}{\mu}$,

$N = -\left(\frac{dP}{dx}\right)\frac{d^2}{\mu U},\ E_c = \frac{U_0^2}{C_p(T_0 - T_1)},\ p_r = \frac{\mu C_p}{K}$,

$t\,d = S_t(T_0 - T_1)$,

The boundary conditions (4) and (5) transforms as

$$U = 0, \qquad \theta = 0, \quad \& \quad f = 1, \qquad at \quad \eta = 0 \tag{10}$$

$$U = 1, \qquad \theta = 0, \quad \& \quad \frac{df}{d\eta} + t\,d\,f\frac{d\theta}{d\eta} = 0 \qquad at \quad \eta = 1 \tag{11}$$

The solutions of equations (7) to (9) subject to boundary conditions (10) and (11) are given by

$$U(\eta) = A_1\,CoshM\eta + A_2\,SinhM + A_3 \tag{12}$$

$$\theta(\eta)=\frac{P_r E_c\begin{bmatrix} B_1\,\eta^2+\eta\,\left(B_2+B_3+B_4-B_5+B_6+B_7+B_8-B_9-B_{10}-B_{11}\right)\\ +B_{12}\,CoshM\eta+B_{13}\,Cosh2M\eta+B_{14}\,Sinh2M\eta+B_{15}\,SinhM\eta\end{bmatrix} -B_{16}+B_{17}+B_{18}+B_{19}+B_{20}-B_{20}\,\eta}{B_{20}} \tag{13}$$

and

$$f(\eta)=\frac{P_r\,E_c\begin{bmatrix}(t\,d+S_c\,S_T)\,C_1\,\eta^2+S_c\,S_T\,\eta\begin{pmatrix}C_2+C_3-C_4-C_5-C_6-C_7-C_8\\ -C_9-C_{10}+C_{11}\end{pmatrix}\\ +t\,d\,\eta\left(C_{12}-C_{13}-C_{14}+C_{15}-C_{16}-C_{17}-C_{18}+C_{19}+C_{20}+C_{21}\right)\\ +\left(t\,d+S_c\,S_T\right)\\ \left(C_{22}\,CoshM\eta+C_{23}\,Cosh2M\eta+C_{24}\,Sinh2M\eta+C_{25}\,SinhM\eta\right)\end{bmatrix} -P_r\,E_c\left(S_c\,S_T+t\,d\right)\left[C_{26}+C_{27}+C_{28}+C_{29}\right] +t\,d\,\eta\,C_{30}+C_{30}}{C_{30}} \tag{14}$$

The constants of equations of (12), (13) and (14) see in Appendix.

3 Result and Discussion

The influences of the parameters N, S_c, D_u, m_0 on the action of separation are shown graphically.

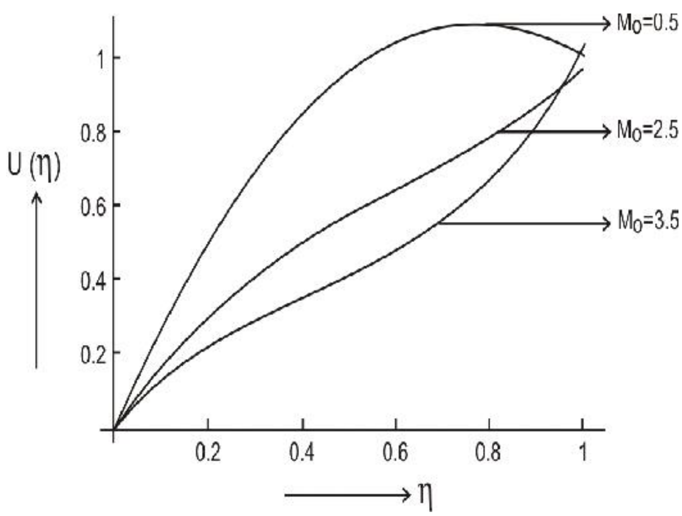

Fig:-1:- Variation of velocity (U) with applied magnetic field for Value $P_r E_c = 0.3, N = 5$, $S_c S_T = 0.06, t\,d = 0.10$, $P_r D_u = 0.142$

From this figure, it is obvious that magnetic field lowers the velocity at a point in between the plates.

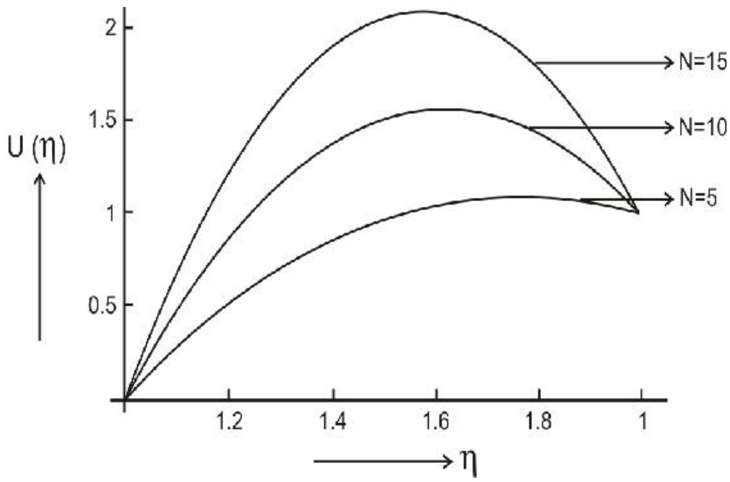

Fig:-2:- Variation of velocity with different value of Reynolds Number and for

fixed of $P_r E_c = 0.3, m_0 = 0.5,\ S_c S_T = 0.06, t\,d = 0.10,\ P_r D_u = 0.142$

Figure-2 represents the variation of velocity (U) drawn against η for different values of Reynolds number. A well known result that axial velocity U increases with the increasing value of Reynolds number is observed.

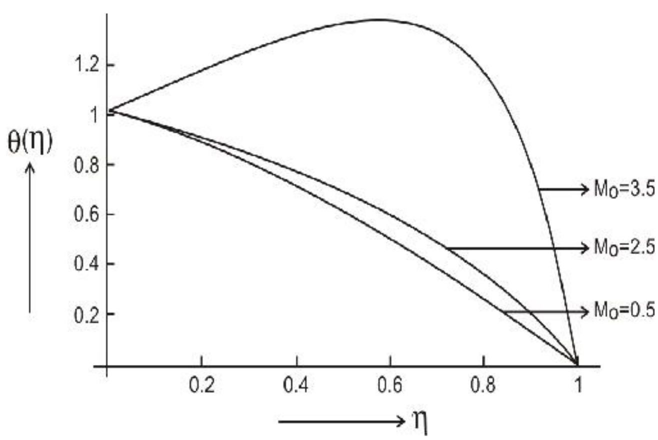

Fig:-3:-Variation of θ with different value of magnetic field and fixed of $P_r E_c = 0.3, N = 5,\ S_c S_T = 0.06, t\,d = 0.10,\ P_r D_u = 0.142$

Figure -3 presents the variation of temperature (θ) drawn against η for different value of magnetic field (M_0). Temperature (θ) at any point increases with the increasing value of applied magnetic field (M_0).

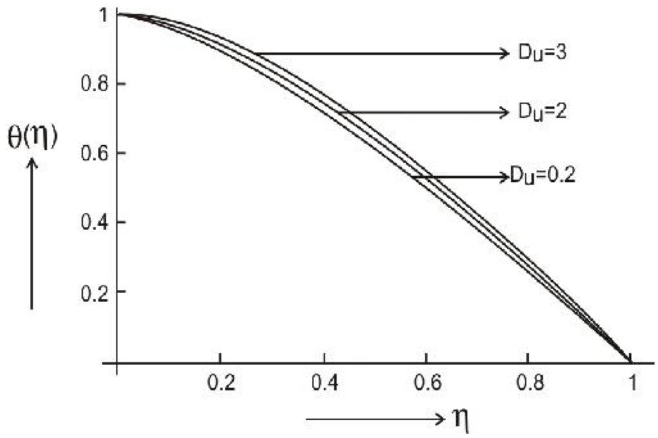

Fig:-4:- Variation of θ with (η) for different value of Dufour number (D_u) for fixed valuesof $P_r E_c = 0.3,\ N = 5,\ S_c S_T = 0.06,\ t\,d = 0.10,\ P_r = 0.71,\ m_0 = 0.5$

The effect of concentration gradient on temperature (θ) is depicted in figure-4. The coefficient of thermal diffusion (Dufour effect) is to increase the temperature at a point in the flow regime for fixed values of other parameters.

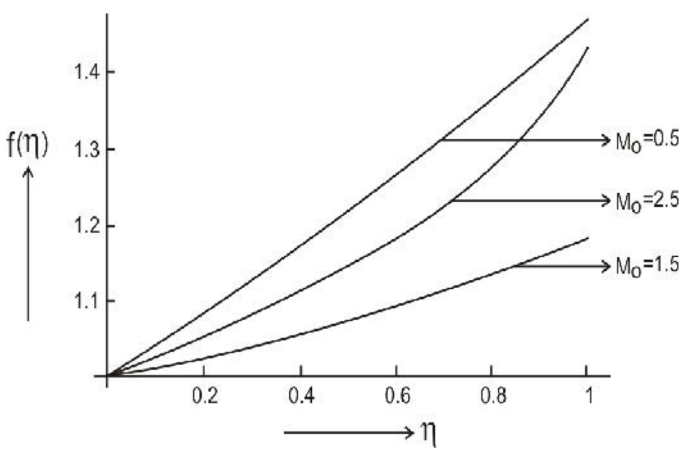

Fig:-5:- Variation of f with different value of magnetic field and fixed for $P_r E_c = 0.3, N = 5,\ S_c S_T = 0.06, t\,d = 0.10,\ P_r D_u = 0.142$

Figure -5 depicts the variation of f against η for different values of magnetic field and for fixed values of other parameters. It is observed that the applied magnetic field increases the concentration (f) for small values of magnetic field and after attaining maximum value in between $2.5 \le M_0 \le 3.0$ and there after it starts decreasing.

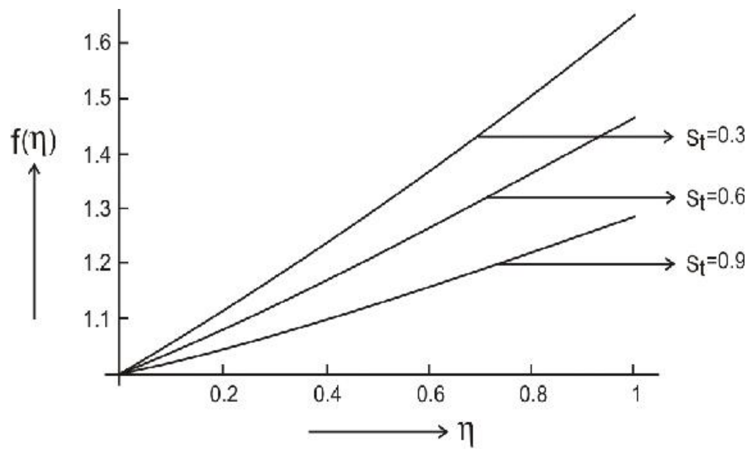

Fig:-6:- Variation of f with different value of Soret Effect and fixed values for

$$P_r E_c = 0.3, N = 5,\ S_t = 0.01, t\,d = 0.10,\ P_r D_u = 0.142,\ m_0 = 0.5$$

The effects of thermodiffusion (Soret effect) have been presented in this figure. We observe that the concentration (f) of the species in smaller quantity decreases at a point for increasing values of parameter (S_t).

5 Conclusion

In this paper, we have presented the effect of thermal-diffusion (Soret) and diffusion-thermo (Dufour) in hydro-magnetic generalized couette flow of a binary mixture of gases in presence of normal applied magnetic field in porous medium. By extending the earlier work of Sharma and Singh [16], we have introduced more realistic conditions as mentioned above in formulation and presented wide range of result.

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Appendices:

$$A_1 = \left(-\frac{N}{M^2}\right)$$

$$A_2 = \left(1 - \frac{N}{M^2} + \frac{N}{M^2} CoshM\right) Co\sec hM$$

$$A_3 = \frac{N}{M^2}$$

$$B_1 = -\frac{N^2}{2M^2}$$

$$B_2 = \frac{8NM^2 + 2N^2M^2 - N^2}{4M^4}$$

$$B_3 = \frac{N^2}{4M^4} Cosh2M$$

$$B_4 = \frac{N^2 - NM^2}{2M^4} Sinh2M \; Co\sec hM$$

$$B_5 = \frac{N^2}{2M^4} Sinh2M \; CoshM \; Co\sec hM$$

$$B_6 = \frac{\left(M^2 - N\right)^2}{4M^4} Cosh2M \; Co\sec h^2 M$$

$$B_7 = \frac{N^2}{4M^4} Cosh2M \; Cosh^2 M \; Co\sec h^2 M$$

$$B_8 = \frac{NM^2 - N^2}{2M^4} Cosh2M \; CoshM \; Co\sec h^2 M$$

$$B_9 = \frac{\left(M^2 - N\right)^2}{4M^4} Co\sec h^2 M$$

$$B_{10} = \frac{NM^2 - N^2}{2M^4} CoshM \; Co\sec h^2 M$$

$$B_{11} = \frac{N^2}{4M^4} Cosh^2 M \; Co\sec h^2 M$$

$$B_{12} = \frac{2N^2}{M^4} Cosh M\eta$$

$$B_{13} = -\frac{N^2}{4M^4} - \frac{\left(M^2 - N\right)^2}{4M^4} Co\sec h^2 M$$

$$-\frac{NM^2 - N^2}{2M^4} CoshM\, Co\sec h^2 M - \frac{N^2}{4M^4} Cosh^2 M\, Co\sec h^2 M$$

$$B_{14} = \frac{NM^2 - N^2}{2M^4} Co\sec hM + \frac{N^2}{2M^4} CoshM\, Co\sec hM$$

$$B_{15} = \frac{2N^2 - 2NM^2}{M^4} Co\sec hM - \frac{2N^2}{M^4} Co\sec hM\, CoshM$$

$$B_{16} = \left(\frac{7N^2}{4M^4}\right)$$

$$B_{17} = \frac{\left(M^2 - N\right)^2}{4M^4} Co\sec h^2 M$$

$$B_{18} = \frac{NM^2 - N^2}{2M^4} CoshM\, Co\sec h^2 M$$

$$B_{19} = \frac{N^2}{4M^4} Cosh^2 M\, Co\sec h^2 M$$

$$B_{20} = 1 - S_c\, S_T\, P_r\, D_u - td\, P_r\, D_u$$

$$C_1 = \frac{N^2}{2M^2}$$

$$C_2 = \frac{-N^2}{M^2}$$

$$C_3 = \frac{2N^2}{M^3} SinhM$$

$$C_4 = \frac{N^2}{2M^3} Sinh2M$$

$$C_5 = \frac{NM^2 - N^2}{M^3} Sinh2M\, CoshM\, Co\sec h^2 M$$

$$C_6 = \frac{N^2}{2M^3} Sinh2M\, Cosh^2M\, Co\sec h^2M$$

$$C_7 = \frac{2NM^2 - 2N^2}{M^3} CoshM\, Co\sec hM$$

$$C_8 = \frac{2N^2}{M^3} Cosh^2M\, Co\sec h^2M$$

$$C_9 = \frac{\left(M^2 - N\right)^2}{2M^3} Sinh2M\, Co\sec h^2M$$

$$C_{10} = \frac{N^2 - NM^2}{M^3} Cosh2M\, Co\sec hM$$

$$C_{11} = \frac{N^2}{M^3} Cosh2M\, CoshM\, Co\sec hM$$

$$C_{12} = \frac{N^2 - 8NM^2 - 2N^2M^2}{4M^4}$$

$$C_{13} = \frac{N^2}{4M^4} Cosh2M$$

$$C_{14} = \frac{N^2 - NM^2}{2M^4} Sinh2M\, Co\sec hM$$

$$C_{15} = \frac{N^2}{2M^4} Sinh2M\, CoshM\, Co\sec hM$$

$$C_{16} = \frac{\left(M^2 - N\right)^2}{4M^4} Cosh2M\, Co\sec h^2M$$

$$C_{17} = \frac{N^2}{4M^4} Cosh2M\, Cosh^2M\, Co\sec h^2M$$

$$C_{18} = \frac{NM^2 - N^2}{2M^4} Cosh2M\, CoshM\, Co\sec h^2M$$

$$C_{19} = \frac{\left(M^2 - N\right)^2}{4M^4} Co\sec h^2M$$

$$C_{20} = \frac{NM^2 - N^2}{2M^4} CoshM\, Co\sec h^2M$$

$$C_{21} = \frac{N^2}{4M^4} Cosh^2 M \, Co\sec h^2 M$$

$$C_{22} = \left(-\frac{2N^2}{M^4} \right)$$

$$C_{23} = \frac{N^2}{4M^4} + \frac{\left(M^2 - N\right)^2}{4M^4} Co\sec h^2 M + \frac{NM^2 - N^2}{2M^4} CoshM \, Co\sec h^2 M$$

$$+ \frac{N^2}{4M^4} Cosh^2 M \, Co\sec h^2 M$$

$$C_{24} = \frac{N^2 - NM^2}{2M^4} Co\sec hM - \frac{N^2}{2M^4} CoshM \, Co\sec hM$$

$$C_{25} = \frac{2NM^2 - 2N^2}{M^4} Co\sec hM + \frac{2N^2}{M^4} Co\sec hM \, CoshM$$

$$C_{26} = \frac{-7N^2}{4M^4}$$

$$C_{27} = \frac{N^2}{4M^4} Cosh^2 M \, Co\sec h^2$$

$$C_{28} = \frac{\left(M^2 - N\right)^2}{4M^4} Co\sec h^2 M$$

$$C_{29} = \frac{NM^2 - N^2}{2M^4} CoshM \, Co\sec h^2 M$$

$$C_{30} = 1 - S_c \, S_T \, P_r \, D_u - t \, d \, P_r \, D_u$$

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