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Ranking Trapezoidal Fuzzy Numbers Based on Apex Angles

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Abstract

In this paper, we calculation ranking of trapezoidal fuzzy numbers based on apex angles. In fact, the concept of an aggregation operator for fuzzy numbers based on the arithmetic means of the corresponding L- and R- apex angles is extended for a class of Fuzzy Numbers. This method provides the correct ordering of trapezoidal fuzzy numbers and also the this approach is very simple and easy to apply in the real life problems. For the validation, the results of the approach are compared with different existing approaches.

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1 Introduction

In most of cases in our life, the data obtained for decision making are only approximately known. In1965, Zadeh [26] introduced the concept of fuzzy set theory to meet those problems. In 1978, Dubois and Prade defined any of the fuzzy numbers as a fuzzy subset of the real line [9]. Fuzzy numbers allow us to make the mathematical model of linguistic variable or fuzzy environment. Most of the ranking procedures proposed so far in the literature cannot discriminate fuzzy quantities and some are counterintuitive. As fuzzy numbers are represented by possibility distributions, they may overlap with each other, and hence it is not possible to order them. Ranking fuzzy numbers were first proposed by Jain [11] for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Since then, various procedures to rank fuzzy quantities are proposed by various researchers. Bortolan and Degani [2] reviewed some of these ranking methods for ranking fuzzy subsets and fuzzy

numbers. Chen [3] presented ranking fuzzy numbers with maximizing set and minimizing set. Dubois and Prade [10] presented the mean value of a fuzzy number. Chu and Tsao [6] proposed a method for ranking fuzzy numbers with the area between the centroid point and original point. Deng and Liu [7] presented a centroid-index method for ranking fuzzy numbers. Liang et al. [13] and Wang and Lee [25] also used the centroid concept in developing their ranking index. Chen and Chen [4] presented a method for ranking generalized trapezoidal fuzzy numbers. Abbasbandy and Hajjari [1] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some -levels of trapezoidal fuzzy numbers. Chen and Chen [5] presented a method for fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads and Manju Pandey et al [15] proposed a New Aggregation Operator for Trapezoidal Fuzzy Numbers based on the Arithmetic Means of the Left and Right Apex Angles. Also, Some of the interesting Approach Ranking Of Trapezoidal Fuzzy Number can be found in Amit Kumar [12]. S. Rezvani ([16]-[23]) evaluated the system of ranking fuzzy numbers. Moreover, Rezvani [21] proposed a new method for ranking with Euclidean distance by the incentre of centroids.
In this paper, we calculation ranking of trapezoidal fuzzy numbers based on apex angles. In fact, the concept of an aggregation operator for fuzzy numbers based on the arithmetic means of the corresponding L- and R- apex angles is extended for a class of Fuzzy Numbers. This method provides the correct ordering of trapezoidal fuzzy numbers and also the this approach is very simple and easy to apply in the real life problems. For the validation, the results of the approach are compared with different existing approaches.

2 Preliminaries

Generally, a generalized fuzzy number A is described as any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions,

(i) μ_A is a continuous mapping from R to the closed interval [0,1],

(ii) $\mu_A(x) = 0, -\infty < x \leq a$,

(iii) $\mu_A(x) = L(x)$ is strictly increasing on $[a, b]$,

(iv) $\mu_A(x) = w, b \leq x \leq c$,

(v) $\mu_A(x) = R(x)$ is strictly decreasing on $[c, d]$,

(vi) $\mu_A(x) = 0, d \leq x < \infty$

Where $0 < w \leq 1$ and $a, b, c,$ and d are real numbers. We call this type of fuzzy number a trapezoidal fuzzy number, and it is denoted by

$$A = (a, b, c, d; w) \ . \tag{1}$$

A $A = (a, b, c, d; w)$ is a fuzzy set of the real line R whose membership function $\mu_A(x)$ is defined as

$$\mu_A(x) = \begin{cases} w\frac{x-a}{b-a} & if \quad a \leq x \leq b \\ w & if \quad b \leq x \leq c \\ w\frac{d-x}{d-c} & if \quad c \leq x \leq d \\ 0 & \quad Otherwise \end{cases} \tag{2}$$

2.1 Fuzzy Aggregation

Aggregation operations on fuzzy numbers are operations by which several fuzzy numbers are combined to produce a single fuzzy number. An excellent account of Mathematical Aggregation Operators is given in [8].

i) Arithmetic Mean

The arithmetic mean aggregation operator defined on n trapezoidal fuzzy numbers (a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2),..., (a_n, b_n, c_n, d_n) produces the result (a, b, c, d) where

$a = \frac{1}{n}\sum_1^n a_i$, $b = \frac{1}{n}\sum_1^n b_i$, $c = \frac{1}{n}\sum_1^n c_i$ and $d = \frac{1}{n}\sum_1^n d_i$.

ii) Geometric Mean

The arithmetic mean aggregation operator defined on n trapezoidal fuzzy numbers (a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2),..., (a_n, b_n, c_n, d_n) produces the result (a, b, c, d) where

$a = (\prod_1^n a_i)^{\frac{1}{n}}$, $b = (\prod_1^n b_i)^{\frac{1}{n}}$, $c = (\prod_1^n c_i)^{\frac{1}{n}}$ and $d = (\prod_1^n d_i)^{\frac{1}{n}}$.

An Aggregation Operators trapezoidal fuzzy numbers are given in [19] Consider the trapezoidal fuzzy number shown in (Figure 1). If the value of this trapezoidal fuzzy number is $v \in [b, c]$ the corresponding possibility $\mu = 1$. The left side apex angle of this trapezoidal fuzzy number is $\mathcal{L}apb$. The right side apex angle of this trapezoidal fuzzy number is $\mathcal{L}drc$. The left and right side apex angles of the trapezoid refer to the apex angles subtended to the left and the right of the interval [b,c] respectively. But

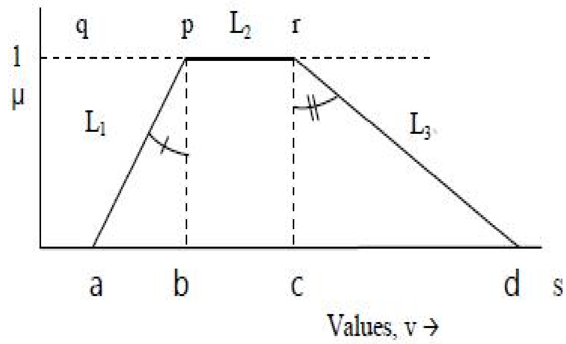

Figure 1: Trapezoidal Fuzzy Number

$$\mathcal{L}apb = \frac{\pi}{2} - \mathcal{L}bap \tag{3}$$

and

$$\mathcal{L}drc = \mathcal{L}sdr - \frac{\pi}{2} \tag{4}$$

Considering the left side and averaging over n trapezoidal fuzzy numbers we have

$$\frac{1}{n}\sum_{1}^{n}(\mathcal{L}apb)_i = \frac{1}{n}\sum_{1}^{n}(\mathcal{L}(\frac{\pi}{2} - bap)_i) \tag{5}$$

$$\frac{1}{n}\sum_{1}^{n}(\mathcal{L}apb)_i = \frac{\pi}{2} - \frac{1}{n}\sum_{1}^{n}(\mathcal{L}bap)_i \tag{6}$$

The left side of the above equation represents the contributions of the left lines(aggregate apex angle). It can be seen that

$$\tan(\frac{1}{n}\sum_{1}^{n}(\mathcal{L}apb)_i) = \frac{1}{\tan(\frac{1}{n}\sum_{1}^{n}(\mathcal{L}bap)_i)} \tag{7}$$

It can be shown that

$$\tan(\frac{1}{n}\sum_{1}^{n}(\mathcal{L}apb)_i) = [tan(\frac{1}{n}\sum_{1}^{n} tan^{-1}(b_i - a_i))]^{-1} \tag{8}$$

Under identical treatment, it can be shown that

$$b = \frac{1}{n}\sum_{1}^{n} b_i, \qquad c = \frac{1}{n}\sum_{1}^{n} c_i \quad . \tag{9}$$

Subsequently it is possible to show that

$$a = \frac{1}{n}\sum_{1}^{n} b_i - \tan[\frac{1}{n}\sum_{1}^{n} \tan^{-1}(b_i - a_i)] \tag{10}$$

and

$$d = \frac{1}{n}\sum_{1}^{n} c_i + \tan[\frac{1}{n}\sum_{1}^{n} \tan^{-1}(d_i - c_i)] \tag{11}$$

3 Proposed Approach

In this section, by modifying the Manju Pandey et al. [15] a new approach is proposed for the ranking of trapezoidal fuzzy numbers and using the proposed approach, in the Theorem 1.

Theorem 1. Let $A_1 = (a_1, b_1, c_1, d_1; w_1)$, $A_2 = (a_2, b_2, c_2, d_2; w_2)$ be two trapezoidal fuzzy numbers then

$$A_1 > A_2 \;\; if \;\; D(A_1) > D(A_2)$$

$$A_1 < A_2 \;\; if \;\; D(A_1) < D(A_2)$$

$$A_1 \sim A_2 \;\; if \;\; D(A_1) \sim D(A_2) \tag{12}$$

3.1 Method to Find the Values of $D(A_1)$ and $D(A_2)$

Let $A_1 = (a_1, b_1, \alpha_1, \beta_1; w_1)_E$ and $A_2 = (a_2, b_2, \alpha_2, \beta_2; w_2)_E$ be two generalized exponential fuzzy numbers, then use the following steps to find the values of $D(A_1)$ and $D(A_2)$

∗ Step 1

Find b_1 and b_2

$$b_1 = \frac{w_1}{n}\sum_{1}^{n} b_{1i} \quad . \tag{13}$$

and

$$b_2 = \frac{w_2}{n}\sum_{1}^{n} b_{2i} \quad . \tag{14}$$

∗ Step 2

Find c_1 and c_2

$$c_1 = \frac{w_1}{n}\sum_{1}^{n} c_{1i} \quad . \tag{15}$$

and

$$c_2 = \frac{w_2}{n}\sum_{1}^{n} c_{2i} \quad . \tag{16}$$

∗ Step 3

Find $x(A_1)$ and $x(A_2)$

$$x(A_1) = a_1 = \frac{w_1}{n}\sum_1^n b_{1i} - \tan[\frac{w_1}{n}\sum_1^n \tan^{-1}(b_{1i} - a_{1i})] \tag{17}$$

and

$$x(A_2) = a_2 = \frac{w_2}{n}\sum_1^n b_{2i} - \tan[\frac{w_2}{n}\sum_1^n \tan^{-1}(b_{2i} - a_{2i})] \tag{18}$$

∗ Step 4

Find $y(A_1)$ and $y(A_2)$

$$y(A_1) = d_1 = \frac{w_1}{n}\sum_1^n c_{1i} + \tan[\frac{w_1}{n}\sum_1^n \tan^{-1}(d_{1i} - c_{1i})] \tag{19}$$

and

$$y(A_2) = d_2 = \frac{w_2}{n}\sum_1^n c_{2i} + \tan[\frac{w_2}{n}\sum_1^n \tan^{-1}(d_{2i} - c_{2i})] \tag{20}$$

∗ Step 5

Calculation $D(A_1)$, $D(A_2)$ as following

$$D(A_1) = \sqrt{x(A_1)^2 + y(A_1)^2} \tag{21}$$

and

$$D(A_2) = \sqrt{x(A_2)^2 + y(A_2)^2} \tag{22}$$

and use of theorem 1. we have

$$A_1 > A_2 \;\; if \;\; D(A_1) > D(A_2)$$

$$A_1 < A_2 \;\; if \;\; D(A_1) < D(A_2)$$

$$A_1 \sim A_2 \;\; if \;\; D(A_1) \sim D(A_2)$$

4 Results

Example 1. Let $A = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $B = (0.1, 0.2, 0.3, 0.4; 0.7)$ be two generalized trapezoidal fuzzy number, then

∗ Step 1

Find b_A and b_B

$$b_A = \frac{w_1}{n}\sum_1^n b_{1i} = 0.35 \times 0.4 = 0.14$$

and

$$b_B = \frac{w_2}{n}\sum_1^n b_{2i} = 0.7 \times 0.2 = 0.14$$

∗ Step 2

Find c_A and c_B

$$c_A = \frac{w_1}{n}\sum_1^n c_{1i} = 0.35 \times 0.6 = 0.21$$

and

$$c_B = \frac{w_2}{n}\sum_1^n c_{2i} = 0.7 \times 0.3 = 0.21$$

∗ Step 3

Find $x(A)$ and $x(B)$

$$x(A) = \frac{w_1}{n}\sum_1^n b_{1i} - \tan[\frac{w_1}{n}\sum_1^n \tan^{-1}(b_{1i} - a_{1i})] = 0.14 - \tan[0.35(\tan^{-1}(0.4 - 0.2))] =$$

$$= 0.14 - \tan(3.96) = 0.14 - 0.0692 = 0.0708$$

and

$$x(B) = \frac{w_2}{n}\sum_1^n b_{2i} - \tan[\frac{w_2}{n}\sum_1^n \tan^{-1}(b_{2i} - a_{2i})] = 0.14 - \tan[0.7(\tan^{-1}(0.2 - 0.1))] =$$

$$= 0.14 - \tan(4) = 0.14 - 0.0699 = 0.0701$$

∗ Step 4

Find $y(A)$ and $y(B)$

$$y(A) = \frac{w_1}{n}\sum_1^n c_{1i} + \tan[\frac{w_1}{n}\sum_1^n \tan^{-1}(d_{1i} - c_{1i})] = 0.21 + \tan[0.35(\tan^{-1}(0.8 - 0.6))] =$$

$$= 0.21 + \tan(3.96) = 0.21 + 0.0692 = 0.2792$$

and

$$y(B) = \frac{w_2}{n}\sum_1^n c_{2i} + \tan[\frac{w_2}{n}\sum_1^n \tan^{-1}(d_{2i} - c_{2i})] = 0.21 + \tan[0.7(\tan^{-1}(0.4 - 0.3))] =$$

$$= 0.21 + \tan(4) = 0.21 + 0.0699 = 0.2799$$

∗ Step 5

Calculation $D(A)$, $D(B)$ as following

$$D(A) = \sqrt{x(A)^2 + y(A)^2} = \sqrt{(0.0708)^2 + (0.2792)^2} = 0.2880$$

and

$$D(B) = \sqrt{x(B)^2 + y(B)^2} = \sqrt{(0.0701)^2 + (0.2799)^2} = 0.2885$$

Then $D(A) < D(B) \Rightarrow A < B$.

Example 2. Let $A = (0.1, 0.2, 0.4, 0.5; 1)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

∗ Step 1

Find b_A and b_B

$$b_A = \frac{w_1}{n}\sum_1^n b_{1i} = 1 \times 0.2 = 0.2$$

and

$$b_B = \frac{w_2}{n}\sum_1^n b_{2i} = 1 \times 0.3 = 0.3$$

∗ Step 2

Find c_A and c_B

$$c_A = \frac{w_1}{n}\sum_1^n c_{1i} = 1 \times 0.4 = 0.4$$

and

$$c_B = \frac{w_2}{n}\sum_1^n c_{2i} = 1 \times 0.3 = 0.3$$

∗ Step 3

Find $x(A)$ and $x(B)$

$$x(A) = \frac{w_1}{n}\sum_1^n b_{1i} - \tan[\frac{w_1}{n}\sum_1^n \tan^{-1}(b_{1i} - a_{1i})] = 0.2 - \tan[(\tan^{-1}(0.2 - 0.1))] =$$

$$= 0.2 - \tan(5.7105931) = 0.2 - 0.099999999 = 0.100000001$$

and

$$x(B) = \frac{w_2}{n}\sum_1^n b_{2i} - \tan[\frac{w_2}{n}\sum_1^n \tan^{-1}(b_{2i} - a_{2i})] = 0.3 - \tan[(\tan^{-1}(0.3 - 0.1))] =$$

$$= 0.3 - \tan(11.3099324) = 0.3 - 0.199999998 = 0.100000002$$

∗ Step 4

Find $y(A)$ and $y(B)$

$$y(A) = \frac{w_1}{n}\sum_1^n c_{1i} + \tan[\frac{w_1}{n}\sum_1^n \tan^{-1}(d_{1i} - c_{1i})] = 0.4 + \tan[(\tan^{-1}(0.5 - 0.4))] =$$

$$= 0.4 + \tan(5.7105931) = 0.4 + 0.099999999 = 0.499999999$$

and

$$y(B) = \frac{w_2}{n}\sum_1^n c_{2i} + \tan[\frac{w_2}{n}\sum_1^n \tan^{-1}(d_{2i} - c_{2i})] = 0.3 + \tan[(\tan^{-1}(0.5 - 0.3))] =$$

$$= 0.3 + \tan(11.3099324) = 0.3 + 0.199999998 = 0.499999998$$

∗ Step 5

Calculation $D(A)$, $D(B)$ as following

$$D(A) = \sqrt{x(A)^2 + y(A)^2} = \sqrt{(0.100000001)^2 + (0.499999999)^2} = 0.50990195$$

and

$$D(B) = \sqrt{x(B)^2 + y(B)^2} = \sqrt{(0.100000002)^2 + (0.499999998)^2} = 0.50990194$$

Then $D(A) > D(B) \Rightarrow A > B$.

Example 3. Let $A = (0.1, 0.2, 0.4, 0.5; 1)$ and $B = (1, 1, 1, 1; 1)$ be two generalized trapezoidal fuzzy number, then

$*$ Step 1 Find b_A and b_B

$$b_A = \frac{w_1}{n}\sum_1^n b_{1i} = 1 \times 0.2 = 0.2$$

and

$$b_B = \frac{w_2}{n}\sum_1^n b_{2i} = 1 \times 1 = 1$$

$*$ Step 2

Find c_A and c_B

$$c_A = \frac{w_1}{n}\sum_1^n c_{1i} = 1 \times 0.4 = 0.4$$

and

$$c_B = \frac{w_2}{n}\sum_1^n c_{2i} = 1 \times 1 = 1$$

$*$ Step 3

Find $x(A)$ and $x(B)$

$$x(A) = \frac{w_1}{n}\sum_1^n b_{1i} - \tan[\frac{w_1}{n}\sum_1^n \tan^{-1}(b_{1i} - a_{1i})] = 0.2 - \tan[(\tan^{-1}(0.2 - 0.1))] =$$

$$= 0.2 - \tan(5.7105931) = 0.2 - 0.099999999 = 0.100000001$$

and

$$x(B) = \frac{w_2}{n}\sum_1^n b_{2i} - \tan[\frac{w_2}{n}\sum_1^n \tan^{-1}(b_{2i} - a_{2i})] = 1 - \tan[(\tan^{-1}(1 - 1))] =$$

$$= 1 - \tan(0) = 1 - 0 = 1$$

$*$ Step 4

Find $y(A)$ and $y(B)$

$$y(A) = \frac{w_1}{n}\sum_1^n c_{1i} + \tan[\frac{w_1}{n}\sum_1^n \tan^{-1}(d_{1i} - c_{1i})] = 0.4 + \tan[(\tan^{-1}(0.5 - 0.4))] =$$

$$= 0.4 + \tan(5.7105931) = 0.4 + 0.099999999 = 0.499999999$$

and

$$y(B) = \frac{w_2}{n}\sum_1^n c_{2i} + \tan[\frac{w_2}{n}\sum_1^n \tan^{-1}(d_{2i} - c_{2i})] = 1 + \tan[(\tan^{-1}(1-1))] =$$

$$= 1 + \tan(0) = 1 + 0 = 1$$

∗ Step 5

Calculation $D(A)$, $D(B)$ as following

$$D(A) = \sqrt{x(A)^2 + y(A)^2} = \sqrt{(0.100000001)^2 + (0.499999999)^2} = 0.50990195$$

and

$$D(B) = \sqrt{x(B)^2 + y(B)^2} = \sqrt{(1)^2 + (1)^2} = 1.4142$$

Then $D(A) < D(B) \Rightarrow A < B$.

Example 4. Let $A = (-0.5, -0.3, -0.3, -0.1; 1)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

∗ Step 1 Find b_A and b_B

$$b_A = \frac{w_1}{n}\sum_1^n b_{1i} = 1 \times (-0.3) = -0.3$$

and

$$b_B = \frac{w_2}{n}\sum_1^n b_{2i} = 1 \times 0.3 = 0.3$$

∗ Step 2

Find c_A and c_B

$$c_A = \frac{w_1}{n}\sum_1^n c_{1i} = 1 \times (-0.3) = -0.3$$

and

$$c_B = \frac{w_2}{n}\sum_1^n c_{2i} = 1 \times 0.3 = 0.3$$

∗ Step 3

Find $x(A)$ and $x(B)$

$$x(A)=\frac{w_1}{n}\sum_1^n b_{1i}-\tan[\frac{w_1}{n}\sum_1^n \tan^{-1}(b_{1i}-a_{1i})]=-0.3-\tan[(\tan^{-1}(-0.3+0.5))]=$$

$$=-0.3-\tan(11.3099324)=-0.3-0.199999998=-0.499999998$$

and

$$x(B)=\frac{w_2}{n}\sum_1^n b_{2i}-\tan[\frac{w_2}{n}\sum_1^n \tan^{-1}(b_{2i}-a_{2i})]=0.3-\tan[(\tan^{-1}(0.3-0.1))]=$$

$$=0.3-\tan(11.3099324)=0.3-0.199999998=0.100000002$$

∗ Step 4

Find $y(A)$ and $y(B)$

$$y(A)=\frac{w_1}{n}\sum_1^n c_{1i}+\tan[\frac{w_1}{n}\sum_1^n \tan^{-1}(d_{1i}-c_{1i})]=-0.3+\tan[(\tan^{-1}(-0.1+0.3))]=$$

$$=-0.3+\tan(11.3099324)=-0.3+0.199999998=-0.100000002$$

and

$$y(B)=\frac{w_2}{n}\sum_1^n c_{2i}+\tan[\frac{w_2}{n}\sum_1^n \tan^{-1}(d_{2i}-c_{2i})]=0.3+\tan[(\tan^{-1}(0.5-0.3))]=$$

$$=0.3+\tan(11.3099324)=0.3+0.199999998=0.499999998$$

∗ Step 5

Calculation $D(A)$, $D(B)$ as following

$$D(A)=\sqrt{x(A)^2+y(A)^2}=\sqrt{(-0.499999998)^2+(-0.100000002)^2}=0.50990194$$

and

$$D(B)=\sqrt{x(B)^2+y(B)^2}=\sqrt{(0.100000002)^2+(0.499999998)^2}=0.50990194$$

Then $D(A)\sim D(B)\Rightarrow A\sim B$.

Example 5. Let $A = (0.3, 0.5, 0.5, 1; 1)$ and $B = (0.1, 0.6, 0.6, 0.8; 1)$ be two generalized trapezoidal fuzzy number, then

$*$ Step 1 Find b_A and b_B

$$b_A = \frac{w_1}{n} \sum_1^n b_{1i} = 1 \times 0.5 = 0.5$$

and

$$b_B = \frac{w_2}{n} \sum_1^n b_{2i} = 1 \times 0.6 = 0.6$$

$*$ Step 2

Find c_A and c_B

$$c_A = \frac{w_1}{n} \sum_1^n c_{1i} = 1 \times 0.5 = 0.5$$

and

$$c_B = \frac{w_2}{n} \sum_1^n c_{2i} = 1 \times 0.6 = 0.6$$

$*$ Step 3

Find $x(A)$ and $x(B)$

$$x(A) = \frac{w_1}{n} \sum_1^n b_{1i} - \tan[\frac{w_1}{n} \sum_1^n \tan^{-1}(b_{1i} - a_{1i})] = 0.5 - \tan[(\tan^{-1}(0.5 - 0.3))] =$$

$$= 0.5 - \tan(11.3099324) = 0.5 - 0.199999998 = 0.300000002$$

and

$$x(B) = \frac{w_2}{n} \sum_1^n b_{2i} - \tan[\frac{w_2}{n} \sum_1^n \tan^{-1}(b_{2i} - a_{2i})] = 0.6 - \tan[(\tan^{-1}(0.6 - 0.1))] =$$

$$= 0.6 - \tan(26.565051) = 0.6 - 0.499999996 = 0.100000004$$

$*$ Step 4

Find $y(A)$ and $y(B)$

$$y(A) = \frac{w_1}{n} \sum_1^n c_{1i} + \tan[\frac{w_1}{n} \sum_1^n \tan^{-1}(d_{1i} - c_{1i})] = 0.5 + \tan[(\tan^{-1}(1 - 0.5))] =$$

$$= 0.5 + \tan(26.565051) = 0.5 + 0.499999996 = 0.999999996$$

and

$$y(B) = \frac{w_2}{n}\sum_1^n c_{2i} + \tan[\frac{w_2}{n}\sum_1^n \tan^{-1}(d_{2i} - c_{2i})] = 0.6 + \tan[(\tan^{-1}(0.8 - 0.6))] =$$

$$= 0.6 + \tan(11.3099324) = 0.6 + 0.199999998 = 0.799999998$$

* Step 5

Calculation $D(A)$, $D(B)$ as following

$$D(A) = \sqrt{x(A)^2 + y(A)^2} = \sqrt{(0.300000002)^2 + (0.999999996)^2} = 1.044$$

and

$$D(B) = \sqrt{x(B)^2 + y(B)^2} = \sqrt{(0.100000004)^2 + (0.799999998)^2} = 0.8062$$

Then $D(A) > D(B) \Rightarrow A > B$.

Example 6. Let $A = (0, 0.4, 0.6, 0.8; 1)$ and $B = (0.2, 0.5, 0.5, 0.9; 1)$ and $C = (0.1, 0.6, 0.7, 0.8; 1)$ be three generalized trapezoidal fuzzy number, then

* Step 1 Find b_A and b_B and b_C

$$b_A = \frac{w_1}{n}\sum_1^n b_{1i} = 1 \times 0.4 = 0.4$$

and

$$b_B = \frac{w_2}{n}\sum_1^n b_{2i} = 1 \times 0.5 = 0.5$$

and

$$b_C = \frac{w_3}{n}\sum_1^n b_{3i} = 1 \times 0.6 = 0.6$$

* Step 2

Find c_A and c_B and c_C

$$c_A = \frac{w_1}{n}\sum_1^n c_{1i} = 1 \times 0.6 = 0.6$$

and

$$c_B = \frac{w_2}{n}\sum_1^n c_{2i} = 1 \times 0.5 = 0.5$$

and

$$c_C = \frac{w_3}{n}\sum_1^n c_{3i} = 1 \times 0.7 = 0.7$$

∗ Step 3

Find $x(A)$ and $x(B)$ and $x(C)$

$$x(A) = \frac{w_1}{n}\sum_1^n b_{1i} - \tan[\frac{w_1}{n}\sum_1^n \tan^{-1}(b_{1i} - a_{1i})] = 0.4 - \tan[(\tan^{-1}(0.4 - 0))] =$$

$$= 0.4 - \tan(21.801409) = 0.4 - 0.399999990 = 0.00000001$$

and

$$x(B) = \frac{w_2}{n}\sum_1^n b_{2i} - \tan[\frac{w_2}{n}\sum_1^n \tan^{-1}(b_{2i} - a_{2i})] = 0.5 - \tan[(\tan^{-1}(0.5 - 0.2))] =$$

$$= 0.5 - \tan(16.699244) = 0.5 - 0.299999995 = 0.200000005$$

and

$$x(C) = \frac{w_3}{n}\sum_1^n b_{3i} - \tan[\frac{w_3}{n}\sum_1^n \tan^{-1}(b_{3i} - a_{3i})] = 0.6 - \tan[(\tan^{-1}(0.6 - 0.1))] =$$

$$= 0.6 - \tan(26.565051) = 0.6 - 0.499999996 = 0.100000004$$

∗ Step 4

Find $y(A)$ and $y(B)$ and $y(C)$

$$y(A) = \frac{w_1}{n}\sum_1^n c_{1i} + \tan[\frac{w_1}{n}\sum_1^n \tan^{-1}(d_{1i} - c_{1i})] = 0.6 + \tan[(\tan^{-1}(0.8 - 0.6))] =$$

$$= 0.6 + tan(11.3099324) = 0.6 + 0.199999998 = 0.799999998$$

and

$$y(B) = \frac{w_2}{n}\sum_1^n c_{2i} + \tan[\frac{w_2}{n}\sum_1^n \tan^{-1}(d_{2i} - c_{2i})] = 0.5 + \tan[(\tan^{-1}(0.9 - 0.5))] =$$

$$= 0.5 + \tan(21.801409) = 0.5 + 0.399999990 = 0.89999999$$

and

$$y(C) = \frac{w_3}{n}\sum_1^n c_{3i} + \tan[\frac{w_3}{n}\sum_1^n \tan^{-1}(d_{3i} - c_{3i})] = 0.7 + \tan[(\tan^{-1}(0.8 - 0.7))] =$$

$$= 0.7 + \tan(5.7105931) = 0.7 + 0.099999999 = 0.799999999$$

∗ Step 5

Calculation $D(A)$, $D(B)$ and $D(C)$ as following

$$D(A) = \sqrt{x(A)^2 + y(A)^2} = \sqrt{(0.00000001)^2 + (0.799999998)^2} = 0.8$$

and

$$D(B) = \sqrt{x(B)^2 + y(B)^2} = \sqrt{(0.200000005)^2 + (0.89999999)^2} = 0.9219$$

and

$$D(C) = \sqrt{x(C)^2 + y(C)^2} = \sqrt{(0.100000004)^2 + (0.799999999)^2} = 0.8062$$

Then $D(A) < D(C) < D(B) \Rightarrow A < C < B$.

Example 7. Let $A = (0.1, 0.2, 0.4, 0.5; 1)$ and $B = (-2, 0, 0, 2; 1)$ be two generalized trapezoidal fuzzy number, then

∗ Step 1 Find b_A and b_B

$$b_A = \frac{w_1}{n}\sum_1^n b_{1i} = 1 \times 0.2 = 0.2$$

and

$$b_B = \frac{w_2}{n}\sum_1^n b_{2i} = 1 \times 0 = 0$$

∗ Step 2

Find c_A and c_B

$$c_A = \frac{w_1}{n}\sum_1^n c_{1i} = 1 \times 0.4 = 0.4$$

and

$$c_B = \frac{w_2}{n}\sum_{1}^{n} c_{2i} = 1 \times 0 = 0$$

∗ Step 3

Find $x(A)$ and $x(B)$

$$x(A) = \frac{w_1}{n}\sum_{1}^{n} b_{1i} - \tan[\frac{w_1}{n}\sum_{1}^{n} \tan^{-1}(b_{1i} - a_{1i})] = 0.2 - \tan[(\tan^{-1}(0.2 - 0.1))] =$$

$$= 0.2 - \tan(5.7105931) = 0.2 - 0.099999999 = 0.100000001$$

and

$$x(B) = \frac{w_2}{n}\sum_{1}^{n} b_{2i} - \tan[\frac{w_2}{n}\sum_{1}^{n} \tan^{-1}(b_{2i} - a_{2i})] = 0 - \tan[(\tan^{-1}(0 + 2))] =$$

$$= 0 - \tan(63.434949) = 0 - 2.00000001 = -2.00000001$$

∗ Step 4

Find $y(A)$ and $y(B)$

$$y(A) = \frac{w_1}{n}\sum_{1}^{n} c_{1i} + \tan[\frac{w_1}{n}\sum_{1}^{n} \tan^{-1}(d_{1i} - c_{1i})] = 0.4 + \tan[(\tan^{-1}(0.5 - 0.4))] =$$

$$= 0.4 + \tan(5.7105931) = 0.4 + 0.099999999 = 0.499999999$$

and

$$y(B) = \frac{w_2}{n}\sum_{1}^{n} c_{2i} + \tan[\frac{w_2}{n}\sum_{1}^{n} \tan^{-1}(d_{2i} - c_{2i})] = 0 + \tan[(\tan^{-1}(2 - 0))] =$$

$$= 0 + \tan(63.434949) = 0 + 2.00000001 = 2.00000001$$

∗ Step 5

Calculation $D(A)$, $D(B)$ as following

$$D(A) = \sqrt{x(A)^2 + y(A)^2} = \sqrt{(0.100000001)^2 + (0.499999999)^2} = 0.50990195$$

and

$$D(B) = \sqrt{x(B)^2 + y(B)^2} = \sqrt{(-2.00000001)^2 + (2.00000001)^2} = 2.8284$$

Then $D(A) < D(B) \Rightarrow A < B$.

For the validation of the proposed ranking function, in Table 1, it is shown that this approach is very simple and easy to apply in the real life problems. For the validation, the results of the approach are compared with different existing approaches.

Table (1): A comparison of the ranking results for different approaches

Approaches	Ex.1	Ex.2	Ex.3	Ex.4	Ex.5	Ex.6	Ex.7
Cheng1998	$A < B$	$A \sim B$	Error	$A \sim B$	$A > B$	$A < B < C$	Error
Chu2002	$A < B$	$A \sim B$	Error	$A < B$	$A > B$	$A < B < C$	Error
Chen2007	$A < B$	$A < B$	$A < B$	$A < B$	$A > B$	$A < C < B$	$A > B$
Abbasbandy2009	Error	$A \sim B$	$A < B$	$A \sim B$	$A < B$	$A < B < C$	$A > B$
Chen2009	$A < B$	$A < B$	$A < B$	$A < B$	$A > B$	$A < B < C$	$A > B$
Kumar2010	$A > B$	$A \sim B$	$A < B$	$A < B$	$A > B$	$A < B < C$	$A > B$
Singh2010	$A < B$	$A < B$	$A < B$	$A < B$	$A > B$	$A < B < C$	$A > B$
Rezvani2013	$A > B$	$A > B$	$A < B$	$A < B$	$A < B$	$A < B < C$	$A > B$
Proposed approach	$A < B$	$A > B$	$A < B$	$A \sim B$	$A > B$	$A < C < B$	$A < B$

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