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Complex in a Simple Delayed Discrete Neural Network

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Abstract

In this paper, we show that a delayed discrete Hopfield neural network of two identical neurons with no self-connections can demonstrate chaotic behavior away from the origin. To this end, we first transform the model, by a novel way, into an equivalent system which enjoys some nice properties, and construct chaotic invariant sets of this system such that the dynamics is conjugate to the shift with two symbols. This is complementary to the results in Huang and Zou (J. Nonlinear Sci.,15(2005), 291-303), where it was shown that the same system can have chaotic behavior near the origin.

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1 Introduction

Research on chaotic behavior in neural networks has attracted more and more attentions due to its potential applications to various practical problem[1, 2, 4, 7]. For example, in[1], the non-periodic associative dynamics of the chaotic neural networks was studied by Adcachi and Aihara. In[4], Freeman proved that chaos dynamics exists in real neurons and neural networks play an important role in neural activity.

Among the most frequently used and studied neural networks is the continuous Hopfield neural network which was first considered in [5]. Its various discrete versions have also been intensively and extensively studied in literature. In particular, for the following simple discrete version

$$\begin{cases} x(n+1) = \beta x(n) + \alpha f(y(n-k)) \\ y(n+1) = \beta y(n) + \alpha f(x(n-k)) \end{cases} \tag{1}$$

where $\alpha > 0$, $\beta \in (0,1)$ and the delay $k \geq 1$, Wu and Zhang [10] showed that under some conditions on the activation function $f(x)$, for every positive integer p with $p|2k$, System 1 has several distinct asymptotically stable p-periodic solutions in a region of the x-y plane away from the origin $(0,0)$. In a recent work, Huang and Zou [6] further showed that under certain technical conditions on the nonlinear function $f(x)$, System (1) actually demonstrates Li-York type chaotic behavior in a *neighborhood of the origin.*

Model (1) is for a network consisting of two identical neurons with a uniform connection between the two neurons. In a more recent work, Kaslik and Balint [7] considered the following generalization of (1):

$$\begin{cases} x(n+1) = \beta_1 x(n) + \alpha_{12} f_2(y(n-k_2)) \\ y(n+1) = \beta_2 y(n) + \alpha_{21} f_1(x(n-k_1)) \end{cases} \tag{2}$$

where $n \in \mathbf{N}, \beta_1 \in (0,1)$ for $i = 1,2$, and α_{12} and α_{21} are constants representing connection strengths, and the delays $k_i \geq 0$, $i = 1,2$, are fixed integers. The activation functions $f_i : \mathbf{R} \to \mathbf{R}$, $i = 1,2$, are continuously differentiable. In addition to the stability and bifurcation analysis by central manifold theory, Kaslik and Balint [7] also showed that under some conditions, (1) may exhibit chaos in the *vicinity of the origin* as well, generalizing the result reported in [6].

Notice that the chaotic behaviors obtained in [6] and [7] all occur in neighborhoods of the origin $(0,0)$ in the x-y plane. Thus, one naturally wonders whether the system (1) would have chaotic behaviors in the other region. In this paper we will investigate the possibility of chaos for the system (1) outside the a neighborhood of the origin. Our method is motivated by the idea of establishing the horseshoe structure in families of generalized Henon-like maps in [8],[9].

The rest of this paper is organized as follows. In Section 2, we construct a map $\Phi(\lambda, \cdot)$ from l_∞ to l_∞; and by applying the implicit function theorem in Banach spaces to this parameterized map, we obtain a uniform result of the implicit functions on infinitely many branches. This result will be used in Section 3 to construct a conjugacy map from the full shift at certain values of the parameter to solutions of (1). To achieve this, we rewrite the model (1) as a system of difference equations by a novel way which enjoys some nice properties that the rewritings in [6] and [7] do not have. In particular, we are able to obtain an invariant set *away from the origin* for the transformed system, and show that on this invariant set the map representing the transformed system is topologically conjugate to the full shift on the symbolic dynamical system with two symbols. This conjugacy implies chaos for (1) in the sense of Devaney. The is complementary to the results in[6]. At the end, we present a particular example and its numeric simulations, which verify the theoretical prediction.

2 Preliminaries and Lemmas

First let us introduce some notations. Let l_∞ be the space of bounded real sequences endowed with the norm $\|y\| = sup\{|y_n| : n \in \mathbf{Z}\}$ for $y = (y_n), y_n \in \mathbf{R}$ and let $\sigma : l_\infty \to l_\infty$ be the shift map, i.e. $(\sigma y)_n = y_{n+1}$. Let l^2_∞ be the direct product of 2 copies of l_∞ and $\sigma \times \sigma : l^2_\infty \to l^2_\infty$ be the direct product of 2 copies of the shift map σ. Now we define a map $\Phi(\lambda, \cdot)$ from l^2_∞ to l^2_∞ as follows:

$$\Phi(\lambda, Z)_n = \lambda(-z_{n+1} + \beta z_n) + \phi(z_{n-k+1}), \forall Z = (z_n) \in l^2_\infty, \lambda > 0, \tag{3}$$

where $z_n = (x(n), y(n))^T \in \mathbf{R}^2, \phi(z) = (f(y), f(x))^T, z = (x, y)^T \in \mathbf{R}^2$,.

Lemma 2.1 *[3] Let* (Λ, d) *be a metric space,* Y, Z *be Banach spaces, and* $U \subset \Lambda \times Y$ *be open. Suppose* $F : U \to Z$ *is a continuous map and there exists a point* $(\lambda_0, y_0) \in U$ *with the following conditions:*

(1) $DF_y(\lambda, y)$ *is continuous at* (λ_0, y_0)*, where* $DF_y(\lambda, y)$ *is Fréchet partial derivative of* $F(\lambda, y)$ *with respect to* y*;*

(2) $DF_y(\lambda_0, y_0) : Y \to Z$ *is an invertible linear map;*

(3) $F(\lambda_0, y_0) = 0$.

Then there exist open ball $B_{r_1}(y_0) = \{y \mid \|y - y_0\| < r_1\}$ *and* $B_\delta(\lambda_0) = \{d(\lambda, \lambda_0) < \delta$ *where* $r_1 > 0, \delta > 0$*, such that for any* $\lambda \in B_\delta(\lambda_0)$*, the function* $F(\lambda, y) = 0$ *exists the unique continuous solution* $y = h(\lambda) \in B_{r_1}(y_0)$ *and* $y_0 = h(\lambda_0)$.

Theorem 2.1 *Suppose* $f : \mathbf{R} \to \mathbf{R}$ *is continuously differentiable and there exist two distinct points* x', x'' *in* $\mathbf{R}^2$ *such that* $f(x') = f(x'') = 0$ *with* $f'(x') \neq 0$, $f'(x'') \neq 0$*. We have*

(i) there are $0 < \lambda_0$ *and* $0 < \delta_0$ *such that for every* $\bar{Z} \in \Gamma = \{Z = (z_n = (x_n, y_n)^T)|x_n, y_n = x', orx'', n \in \mathbf{Z}\}$*, for every* $\lambda \in B_{\lambda_0}(0)$*, there is a unique* $Z(\lambda) \in B_{\delta_0}(\bar{Z})$ *with* $\Phi(\lambda, Z(\lambda)) = 0$*;*

(ii) for every $0 < \delta < \delta_0$, there is $0 < \lambda' < \lambda_0$ such that for every $\lambda \in \bar{B}_{\lambda_1}(0) = \{\lambda : d(\lambda, 0) \leq \lambda'\}$ and for every $\bar{Z} \in \Gamma$ there is a unique $Z(\lambda)$ satisfying $||Z(\lambda) - \bar{Z}|| \leq \delta$ and $\Phi(\lambda, Z(\lambda)) = 0$.

Proof. Firstly, for any $\bar{Z} \in \Gamma$, one may easily check that $\Phi(0, Z) = 0$. This verify (i) in Lemma 2.1. The differentiability (ii) in Lemma 2.1 is ensured by the differentiability of f. One has that $f(x') = f(x'') = 0$ with $f'(x') \neq 0$, $f'(x'') \neq 0$, so for each $\overline{Z} \in \Gamma$, by calculating, we have

$$D\phi(\bar{z}_n) = \begin{pmatrix} 0 & f'(x') \text{ or } f'(x'') \\ f'(x') \text{ or } f'(x'') & 0 \end{pmatrix}.$$

$D\Phi_Z(0, \overline{Z}) = (\sigma \times \sigma)^{-k} \cdot diag(\cdots, D\phi(\overline{z}_1), D\phi(\overline{z}_2), \cdots)$. Therefore $D\Phi_Z(0, \overline{Z})$ is a invertible linear operator, verifying condition (iii) of Lemma 2.1. By Lemma 2.1, for each $\overline{Z} \in \Gamma$, there exist $\lambda_0 > 0, \delta > 0$ and for any $\lambda < \lambda_0$, there exists a unique $Z(\lambda)$ satisfying $\Phi(\lambda, Z(\lambda)) = 0$ and $\|Z(\lambda) - \overline{Z}\| \leq \delta$.

To prove (i), it suffices to show that there exist the constants $0 < \lambda_0$ and $0 < \delta_0$ are independent of $\bar{Z} \in \Gamma$. For every $\bar{Z} \in \Gamma$, the constants $0 < r_{\bar{Z}}$ and $0 < \delta_{\bar{Z}}$ in the implicit function theorem are determined as follows. Giving the estimation of the norm $\|(D\Phi_Z(0, \bar{Z}))^{-1}\|$, say $\|(D\Phi_Z(0, \bar{Z}))^{-1}\| \leq M_{\bar{Z}}$, choose $0 < \lambda_{\bar{Z}}$ and $0 < \delta_{\bar{Z}}$ such that $\|(D\Phi_Z(\lambda, Z)) - (D\Phi_Z(0, \bar{Z}))\| \leq \frac{1}{2M_{\bar{Z}}}$ for $\lambda \in B_{\lambda_{\bar{Z}}}(0), W \in B_{\delta_{\bar{Z}}}(\bar{Z})$. Furthermore, $\|\Phi(\lambda, \bar{Z})\| \leq \frac{\delta_{\bar{Z}}}{2M_{\bar{Z}}}$ for $\lambda \in B_{\lambda_{\bar{Z}}}(0)$.

For showing that there exist the constants $0 < r_0$ and $0 < \delta_0$ are independent of $\bar{W}$. Let us assume

$$\frac{1}{M} = \min\{|f'(x')|, |f'(x'')|\}, b = \beta$$

in which $M, b > 0$; then for any $\bar{W} \in \Gamma$, one has that $\|(D\Phi_W(0, \bar{W}))^{-1}\| \leq M$. Since $f'(x)$ is continuous at $x = x' or x''$, there exists δ_1 such that $|f'(x) - f'(x')| \leq \frac{1}{4M}$ for $x \in B_{\delta_1}(x')$, $|f'(x) - f'(x'')| \leq \frac{1}{4M}$ for $x \in B_{\delta_1}(x'')$. Note that

$$(D\Phi_Z(\lambda, Z) - D\Phi_Z(0, \bar{Z})W)_{2n+1} = \lambda(-w_{n+1} + \beta w_n) + Bw_{n-1},$$

where

$$B = \begin{pmatrix} 0 & f'(y_{n-1}) - f'(\bar{y}_{n-1}) \\ f'(x_{n-1}) - f'(\bar{x}_{n-1}) & 0 \end{pmatrix},$$

$\bar{x}_n, \bar{y}_n = x' \text{ or } x''$, $\forall n \in \mathbf{Z}, \forall \bar{Z} = (\bar{z}_n) \in \Gamma$. Therefore one chooses $\delta_0 = \delta_1, \lambda_1 = \frac{1}{4M(1+b)}$, for every $\bar{Z} \in \Gamma$, for any $Z \in l_\infty \times l_\infty$ with $\|Z - \bar{Z}\| \leq \delta_0$ and $|\lambda| \leq \lambda_1$, we have

$$\|D\Phi_Z(\lambda, Z) - D\Phi_Z(0, \bar{Z})\| \leq |\lambda|(1 + b) + \frac{1}{4M} \leq \frac{1}{2M}.$$

Furthermore, we choose $\lambda_2 = \frac{\delta_0}{2M(1+b)}$, then for $|\lambda| \leq \lambda_2$, by the definition of $\Phi(\lambda, \cdot)$, we have

$$\|\Phi(\lambda, \bar{Z})\| \leq |\lambda|(1+b) \leq \frac{\delta_0}{2M}.$$

Let $\lambda_0 = \min\{\lambda_1, \lambda_2\}$, then the constants λ_0 and δ_0 satisfy the conditions of (i). Therefore (i) is proved.

For (ii), for every $\delta < \delta_0$, we choose $r = \min\{\frac{1}{4M(1+b)}, \frac{\delta}{2M(1+b)}\} < \lambda_0$, then the conclusion of (ii) holds on by the proof of (i).

3 Main Results

In this section, we consider the following system of a simple discrete network of two identical neurons with excitatory interactions:

$$\begin{cases} x(n) = \beta x(n-1) + \alpha f(y(n-k)) \\ y(n) = \beta y(n-1) + \alpha f(x(n-k)) \end{cases}$$

where $n \in \mathbf{N}, \alpha > 0, \beta \in (0,1)$ and $k \geq 1$ is a fixed integer. $f : \mathbf{R} \to \mathbf{R}$ is a map. Letting $\omega_j(n) = (x(n-j+1), y(n-j+1))^T \in \mathbf{R}^2$ for $j = 1, 2, \cdots, k$, we then rewrite the system (1) as the following discrete dynamical system on $\mathbf{R}^{2k}$:

$$\omega(n+1) = F_\alpha(\omega(n)),$$

where $\omega(n) = \{\omega_1(n), \omega_2(n), \cdots \omega_k(n)\}, F_\alpha : \mathbf{R}^k \times \mathbf{R}^k \to \mathbf{R}^k \times \mathbf{R}^k$ is given by

$$F_\alpha \begin{pmatrix} \omega_1(n) \\ \omega_2(n) \\ \vdots \\ \omega_{k-1}(n)\omega_k(n) \end{pmatrix} = \begin{pmatrix} \omega_2(n) \\ \omega_3(n) \\ \vdots \\ \beta\omega_k(n) + \alpha(f(y_k(n-k+1), f(x_k(n-k+1))^T \end{pmatrix}.$$

The following assumption on the nonlinear activation function f will be needed in our main results.

H_1 : $f : \mathbf{R} \to \mathbf{R}$ is continuous differentiable, and there exist two distinct points $x', x'' \in \mathbf{R}$ such that $f(x') = f(x'') = 0$, $f'(x') \neq 0$ and $f'(x'') \neq 0$.

By the above assumption we construct, based on Theorem 2.1, an invariant set Λ_α of F_α for $\alpha \geq \frac{1}{\lambda_0}$, on which is topologically conjugate to the shift map $\sigma \times \sigma$ on $\Sigma_2 \times \Sigma_2$ endowed with the product topology on Σ_2. Therefore F_α is chaotic on Λ_α according to Devaney's definition.

According to Theorem 2.1, we may well define a continuous map T_α:

$$T_\alpha(\overline{Z}) = Z(\frac{1}{\alpha}).$$

where $Z(\frac{1}{\alpha})$ is a unique solution that satisfying $\Phi(\frac{1}{\alpha}, Z(\frac{1}{\alpha})) = 0$.

Let $\Sigma_\alpha \times \Sigma_\alpha \subset l^2_\infty$ denote the image of Γ under T_α, i.e. $\Sigma_\alpha \times \Sigma_\alpha = T_\alpha(\Gamma)$. For any $\alpha \geq \frac{1}{\lambda_0}$, we have the following lemma.

Lemma 3.1 *T_α commutes with the shift map $\sigma \times \sigma$, i.e.*

$$\sigma \times \sigma \circ T_\alpha = T_\alpha \circ \sigma \times \sigma.$$

Therefore $\sigma \times \sigma(\Sigma_\alpha \times \Sigma_\alpha) = \Sigma_\alpha \times \Sigma_\alpha$.

Proof. Note that if Z is a zero point of $\Phi(1/\alpha, \cdot)$, so is $\sigma \times \sigma(Z)$. Then for any $\overline{Z} \in \Gamma, \sigma \times \sigma \circ T_\alpha(\overline{Z}) = \sigma \times \sigma(Z(\frac{1}{\alpha}))$ is a zero point of $\Phi(1/\alpha, Z)$. From Theorem 2.1 it follows that $\|Z(\frac{1}{\alpha}) - \overline{Z}\| \leq \delta$; hence$\|\sigma \times \sigma(Z(\frac{1}{\alpha})) - \sigma \times \sigma(\overline{Z})\| = \|Z(\frac{1}{\alpha}) - \overline{Z}\| \leq \delta$. Hence by the uniqueness of $Z(\lambda)$ in Theorem 2.1, we have $\sigma \times \sigma(T_\alpha(\overline{Z})) = T_\alpha(\sigma \times \sigma(\overline{Z}))$. Note that $\sigma \times \sigma(\Gamma) = \Sigma_2 \times \Sigma_2$, it follow that $\sigma \times \sigma(\Sigma_\alpha \times \Sigma_\alpha) = \Sigma_\alpha \times \Sigma_\alpha$.

Let $\omega_i(n) = z_{n-k+i}, i = 1, 2, \cdots, k$, then $\{\omega(n)\}_{n \in \mathbf{Z}}$ is a bounded global orbit of F_α if only if $Z = (z_n) \in l^2_\infty$ is a zero point of $\Phi(1/\alpha, Z)$. We define a projection map from $\Sigma_\alpha \times \Sigma_\alpha$ to $\mathbf{R}^k \times \mathbf{R}^k$ as follows:

$$\Pi(Z) = \omega(k), \forall Z \in \Sigma_\alpha \times \Sigma_\alpha.$$

Lemma 3.2 *Let $\Lambda_\alpha = \Pi(Z), \forall Z \in \Sigma_\alpha \times \Sigma_\alpha$, then Λ_α is invariant for F_α.*

Proof. By Lemma 3.1, we have $\sigma \times \sigma(\Sigma_\alpha \times \Sigma_\alpha) = \Sigma_\alpha \times \Sigma_\alpha$. On the other hand $\forall Z \in \Sigma_\alpha \times \Sigma_\alpha, \Pi(\sigma \times \sigma(Z)) = \omega(k+1) = F_\alpha(\omega(k))$. Therefore $F_\alpha(\Lambda_\alpha) = \Lambda_\alpha$.

In Section 2, the set Γ is a subset of l^2_∞. Now we treat it as $\Sigma_2 \times \Sigma_2$ and endow with the product topology on Σ_2 . We are now state and prove our main result.

Theorem 3.1 *Suppose that $f : \mathbf{R} \to \mathbf{R}$ is continuous differentiable and satisfy the condition of H_1 in the system (1), then F_α on Λ_α is topologically conjugate to the shift map $\sigma \times \sigma$ on $\Sigma_2 \times \Sigma_2$. Therefore the system (1) is chaotic in the sense of Devaney.*

Proof. Let $\Sigma_2 \times \Sigma_2$ be equipped with the usual metric: $d(Z, \overline{Z}) = max\{2^{-|n|} | z_n \neq \overline{z}_n, n \in \mathbf{Z}\}$. We define the map from $\Sigma_2 \times \Sigma_2$ to Λ_α is dented by $h = \Pi \circ T_\alpha$. Let $\|\omega - \overline{\omega}\|_{\mathbf{R}^k \times \mathbf{R}^k} = sup_{1 \leq i \leq k}\{|x_i - \overline{x}_i|, |y_i - \overline{y}_i|\}$ for $\omega = (\omega_1, \cdots, \omega_k)^T, \overline{\omega} = (\overline{\omega}_1, \cdots, \overline{\omega}_k)^T, \omega_i = (x_i, y_i)^T, \overline{\omega}_i = (\overline{x}_i, \overline{y}_i), i = 1, 2, \cdots, k$. Note that the metric on $\mathbf{R}^k \times \mathbf{R}^k$ defined above is equivalent to the Euclidean metric.

Let $\Omega = \{\xi_i | i = 1, \cdots, 2^{2k}\}$ denote the distinct points in $\mathbf{R}^k \times \mathbf{R}^k$ with components either $(x', x'')^T, (x', x')^T, (x'', x')^T$ or $(x'', x'')^T$. Let A_i be the closed neighborhood of ξ_i with radius δ. For any $\overline{Z} = (\overline{z}_n) \in \Sigma_2 \times \Sigma_2$, let $\overline{\omega}_1(n) = \overline{z}_{n+1}, \cdots, \overline{\omega}_k(n) = \overline{z}_{n+k}, n \in \mathbf{Z}$; then $\overline{\omega}(n) \in \Omega$. Let $s = (\cdots, s_{-1}, s_0, s_1, \cdots)$

be a sequence with $s_i \in \{1, \cdots, 2^{2k}\}$ having the property $\overline{\omega}(n) = \xi_{s_n}$. Obviously, the sequence s associated to $\overline{Z}$ by $\overline{\omega}(n) = \xi_{s_n}$ is unique. Let $\{\omega(n)\}_{n\in\mathbf{Z}}$ be a bounded global orbit of F_α associated to $T_\alpha(\overline{z})$ by $\omega_i(n) = z_{n-k+i}, i = 1, 2, \cdots, k$; then $\|\omega(n) - \overline{\omega}(n)\|_{\mathbf{R}^k\times\mathbf{R}^k} = \|\omega(n) - \xi_{s_n}\|_{\mathbf{R}^k\times\mathbf{R}^k} \leq \delta$ by Theorem 2.1, i.e., $\omega(n) \in A_{s_n}$.

Let $s = (\cdots, s_{-1}, s_0, s_1, \cdots)$ be a sequence with $s_i \in \{1, \cdots, 2^{2k}\}$ associated to a given $\overline{Z} = (\overline{z}_n) \in \Sigma_2 \times \Sigma_2$. Define

$$\omega_{s_{-i}\cdots s_0\cdots s_j} = F_\alpha^{-j}(A_{s_j}) \cap \cdots \cap A_{s_0} \cap \cdots \cap F_\alpha^{i}(A_{s_j})$$

for $i > 0$ and $j > 0$. Note that $\omega_{s_{-i}\cdots s_0\cdots s_j}$ may by rewritten as

$$\omega_{s_{-i}\cdots s_0\cdots s_j} = \{Z = F_\alpha^{-i}(Z) \in A_{s_i}, \cdots, Z \in A_{s_0}, \cdots, F_\alpha^{j}(Z) \in A_{s_j}.$$

It follows that $\{\omega_{s_{-i}\cdots s_0\cdots s_j}\}$ forms a nested sequence of nonempty closed sets as $i \to +\infty$ and $j \to +\infty$. By the uniqueness of continuation in Theorem 2.1 we have that $\bigcap_{i>0,j>0} \omega_{s_{-i}\cdots s_0\cdots s_j}$ consists of a unique point and thus $\mathrm{diam}(\omega_{s_{-i}\cdots s_0\cdots s_j}) \to 0$ as $i \to +\infty$ and $j \to +\infty$. Therefore $\bigcap_{i>0,j>0} \omega_{s_{-i}\cdots s_0\cdots s_j} = h(\overline{Z}) = \Pi \circ T_\alpha(\overline{Z})$. Because F_α is invertible, one has Π is one-to-one and onto. Note that T_α is bijective, and so is h. Next we show that h is continuous. Choose $\overline{Z} \in \Sigma_2 \times \Sigma_2$, then there is an unique sequence $\overline{s} = (\cdots, \overline{s}_{-1}, \overline{s}_0, \overline{s}_1, \cdots)$ corresponding to $\overline{Z}$. For any $\varepsilon > 0$, there exists an integer n such that $\mathrm{diam}(\overline{\omega}_{s_{-i}\cdots s_0\cdots s_j}) < \varepsilon$. Pick $\delta_1 = 1/2^{n+k+1}$. Then for any $Z \in \Sigma_2 \times \Sigma_2$ with $d(Z, \overline{Z}) < \delta_1$, Y agrees with $\overline{Y}$ in the terms with index $i = -n-k$ to $i = n+k$, which implies that the sequence s corresponding to Y agrees with $\overline{s}$ in the terms with index from $i = -n$ to $i = n$. We deduce that $h(Z), h(\overline{Z}) \in \omega_{\overline{s}_{-n}\cdots\overline{s}_0\cdots\overline{s}_n}$; hence $\|h(Z) - h(\overline{Z})\| < \varepsilon$. It follows that h is continuous. Therefore h is a homeomorphism.

Let $T_\alpha(\overline{Z}) = Z = (z_n)$. Then we have $h(\overline{Z}) = \Pi \circ T_\alpha(\overline{Z}) = (z_1, z_2, \cdots, z_k)^T$, and $F_\alpha(h(\overline{Z})) = (z_2, z_3, \cdots, z_{k+1})^T$. On the other hand, $h \circ \sigma \times \sigma(\overline{Z}) = \Pi \circ T_\alpha \circ \sigma \times \sigma(\overline{Z}) = \Pi \circ \sigma \times \sigma \circ T_\alpha(\overline{Z}) = (z_2, z_3, \cdots, z_{k+1})^T = F_\alpha(h(\overline{Z}))$. Consequently F_α on Λ_α is topologically conjugate to the shift map $\sigma \times \sigma$ on $\Sigma_2 \times \Sigma_2$. Therefore the system (1) is chaotic in the sense of Devaney.

Remark 3.1 *This is complementary to the result reported in Huang and Zou[6], where it was shown that the same system can have chaotic behavior near the origin.*

Remark 3.2 *One can show that , a more general case , if f have $m > 2$ simple zeros respectively, then there exists $\alpha_0 > 0$ such that for any $\alpha > \alpha_0$, F_α on Λ_α is topologically conjugate to the full shift map $\sigma \times \sigma$ on $\Sigma_m \times \Sigma_m$ in the system (1).*

Remark 3.3 *One can check that theorem 3.1 still hold for small C^1 perturbations of f .*

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