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Low Priority Customer Behavior Analysis of an M/G/1 Model with No Preemptive Priority

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Abstract

Due to the queue phenomenon different customers needing different service quality, a model is established as follows: there are two types of customers in the system and their arrival rates are different; first-class customers have no preemptive priority, the different service time for the different customers and all the service time obeys the general distribution. The following conclusions are drawn: the Laplace - Steele Kyrgyz transform of the low-priority customers waiting time stationary distribution; the average waiting time in the system of low priority customers; the Laplace - Steele Kyrgyz transform of the low-priority customers staying time stationary distribution; At last, this paper points out the problems to be solved.

Mathematics Subject Classification: 60k25

Keywords: non-preemptive, priority, general distribution

1 Introduction

Differences in quality of service requirements due to the different types of business, single service standards are often unable to meet certain business requirements, which require that the customers are divided into different priority, and according to the priority to get the services and quality. Actually, model with priority has been widely used [1−2], and have come to some better results [3−4]. However, most of the literatures are based on service time with exponentially distributed [5]. In this paper, a queuing model with no preemptive and the service time obeyed general distribution is established.

2 Model hypothesis

(1) The system contains two types of customers, each type customer reaches the system at Poisson flow, and the arrival rate of the first class customer is λ_1 , the arrival rate of the second class customer is λ_2 . The arrival processes of different type customers are independent.

(2)First-come, first-served for the same priority customers. The first type customer owns a higher priority than the second type customers.

(3)There is only one service window in the system, and its capacity is unlimited.

(4) General distribution is applied to the service time "T" for each type customers, and the service time distribution for the first type customer is denoted by $B_1(x)$, the service time distribution for the second type customer is denoted by $B_2(x)$.

(5)The arrival time of the second type customer is less than the service time of the first type customers.

(6)The service processes of the different type customers are independent.

(7)If the second type customer is being served when the first type customer arrives, the system does not interrupt existing services.

Let $\beta_i = \int_0^\infty x dB_i(x)$,$\beta_i^{(2)} = \int_0^\infty x^2 dB_i(x)$, $B_i^*(s) = \int_{0-}^\infty e^{-sx} dB_i(x)$, $i = 1, 2$

3 Mathematical model

Theorem 1 The Laplace - Steele Kyrgyz transform of the second type customers' waiting time stationary distribution

$$W_2^*(s) = D_1^*(s)\widetilde{W}_2^*(s + \lambda_1 (1 - D_1^*(s))) \tag{1}$$

among which

$$\widetilde{W}_2^*(s) = \frac{(1 - \lambda_1\beta_1 - \lambda_1\beta_2)\, s}{s - \lambda_1 + \lambda_1 B_1^*(s) B_2^*(s)} \tag{2}$$

$$D_1^*(s) = \sum_{j=1}^{\infty} \int_{0-}^{\infty} e^{-(\lambda_1+s)x} \frac{(\lambda_1 x)^{j-1}}{j!} dB_1^{(j)}(x) \tag{3}$$

the second type customers' average waiting time

$$\overline{W}_2 = \overline{\widetilde{W}}_2(1 + \lambda_1\overline{D}_1) + \overline{D}_1 \tag{4}$$

among which

$$\overline{\widetilde{W}}_2 = \frac{(\lambda_1\beta_1^{(2)} + \lambda_2\beta_2^{(2)} + 2\beta_1\beta_2)}{2\,(1 - \lambda_1\beta_1 - \lambda_2\beta_2)} \tag{5}$$

$\overline{D_1}$is the average length of the busy period of the first customers, and

$$\overline{D_1} = \frac{\beta_1}{1 - \lambda_1 \beta_1} \tag{6}$$

Proof: We credit the system above for the system s_1. Now we introduce the systems s_2 and s_3.

s_2: Only one type customer in the system, customers reach the system at Poisson flow with the parameter λ_1, general distribution is applied to the service time T for the customers, and the service time distribution is denoted by $B_1(x)$, first-come, first-served.

s_3: Only one type customer in the system, customers reach the system at Poisson flow with the parameter λ_2, general distribution is applied to the service time T for the customers, and the service time distribution is denoted by $B_2(x)$, first-come, first-served.

When the second customers reach the system , their waiting time "d" in the system is consisted by two parts:

The first part d_1 is the service time of the first type customers having been in the system, and the service time of the first type customers entering the system during the service period of the first type customers having been in the system. Obviously d_1 equals to the length of the busy period in the system s_2, and its the Laplace - Steele Kyrgyz transform of the distribution function is $D_1^*(s)$, which is determined by (1).

The second part is the service time $\widetilde{\omega}_2$ of the second type customers having been in the system, and the service time of the first type customers entering the system during $\widetilde{\omega}_2$. Obviously $\widetilde{\omega}_2$ equals to the waiting time of the customers when they enter the system in the system s_3 . Suppose there are $v(v = 0, 1, \cdots)$ first type customers entering the system within the time $\widetilde{\omega}_2$, and the delay because of their arriving equals to the sum of v independent length of the busy period $d_1, d_2, \cdots d_v$ in the system s_2 , among which d_i and v are independent. The distribution of $d_i(i = 1, 2, \cdots v)$ is $D_1(x)$, and the distribution of v is $P\{v = j\} = \int_{0-}^{\infty} e^{-\lambda_1 x} \frac{(\lambda_1 x)^j}{j!} d\widetilde{W}_2(x)$, $j = 0, 1 \cdots$. So we can get: $\omega_2 = d + (\widetilde{\omega_2} + d_1 + \cdots + d_v)$, then $W_2(x) = P\{\omega_2 \le x\} = P\{\widetilde{\omega_2} + d_1 + \cdots d_v \le x\} = \int_{0-}^{x} P\{\widetilde{\omega_2} + d_1 + \cdots d_v \le x | \widetilde{\omega_2} = y\} dP\{\widetilde{\omega_2} \le y\}$
$= \int_{0-}^{x} P\{d + d_1 + \cdots d_v \le x - y | \widetilde{\omega_2} = y\} d\widetilde{W}_2(y)$
$= \int_{0-}^{x} \sum_{j=0}^{\infty} P\{d + d_1 + \cdots d_v \le x - y | \widetilde{\omega_2} = y, v = j\} \times P\{v = j | \widetilde{\omega_2} = y\} d\widetilde{W}_2(y)$
$= \sum_{j=0}^{\infty} \int_{0-}^{x} P\{d + d_1 + \cdots + d_v \le x - y\} e^{-\lambda_1 y} \frac{(\lambda_1 y)^j}{j!} d\widetilde{W}_2(y)$
$= \sum_{j=0}^{\infty} \int_{0-}^{x} D_1^{(j+1)}(x - y) e^{-\lambda_1 y} \frac{(\lambda_1 y)^j}{j!} d\widetilde{W}_2(y)$, among which $D_1^{(j)}(x)$ is the j fold convolutions of $D_1(x)$.

The above equation can also be written as

$$W_2(x) = \sum_{j=0}^{\infty} \left[\int_{0-}^{x} e^{-\lambda_1 y} \frac{(\lambda_1 y)^j}{j!} d\widetilde{W}_2(y) \right] \otimes D_1^{(j+1)}(x) \tag{7}$$

among which "$\otimes$" is a convolution operation. Take the Laplace - Steele Kyrgyz transformation and so: $W_2^*(s) = \sum_{j=0}^{\infty} \left[\int_{0-}^{x} \sum_{j=0}^{\infty} e^{-(s+\lambda_1)} \frac{(\lambda_1 x)^j}{j!} d\widetilde{W}_2(x) \right] [D_1^*(s)]^{j+1}$, namely, $W_2^*(s) = W_2^* [s + \lambda_1 (1 - D_1^*(s))] D_1^*(s)$, namely (1), and (4) is derived from the differential of (1).

By theorem 1 we can get the following conclusion:

Theorem 2 The Laplace - Steele Kyrgyz transform of the second type customers staying time stationary distribution

$$T_2^*(s) = W_2^*(s) B_2^*(s) \tag{8}$$

among which $W_2^*(s)$ is determined by the formula (1).

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