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Seismic Wave in Non-homogeneous Intermediate Layer on Half-Space

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Abstract

The propagation of SH-wave (a type of seismic wave) in a non-homogeneous intermediate layer on half-space has been investigated. The rigidity and density of the intermediate layer are assumed as $\mu(1+e^{\varepsilon z})$ and $\rho(1+e^{\varepsilon z})$ i.e. vary exponentially as function of depth. The dispersion equation is obtained for the generated point source. The effect of nonhomogeneity on the generated SH-wave is also shown graphically for the various values of material parameters chosen for earth. The amplitude of the SH-wave falls off very sharply as the wave number increases slowly. In the absence of non-homogeneity factor ε, the dispersion equation reduces to the classical equation.

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1 Introduction

Seismic waves are energy waves which are the cause of a volcano earthquake, or explosion. The wave propagation in elastic medium having different boundaries is important for seismologists or geophysicists to predict the seismic behavior of earth. The propagation velocity of these waves depends on rigidity, density and elasticity of the earth. On the other, seismology is the study of earthquake and seismic wave that tells us about the structure of Earth and physics of earthquake. A geophysicist studies earthquakes and seismic waves. The infrastructure of the Earth's interior can be understood with the help of seismic wave phenomena.

The propagation of seismic waves in a heterogeneous elastic media is of great importance in earth-quake engineering and seismology on account of occurrence of heterogeneity in the earth crust, as the earth is made up of different layers. SH- waves are surface seismic waves that cause horizontal shifting of earth during the earthquake. The particle motion of SH- waves forms a horizontal line perpendicular to direction of propagation. As a result, the theory of seismic waves has been developed by Stoneley [1], Bullen [2], Ewing et al. [3], Hunters [4] and Jeffreys [5]. Jeffreys solved the Love-wave problem for a model earth, neglecting curvature, the layers represented by a single equivalent homogeneous layer of depth, rigidity and density. Rommel [8] presented a note for the propagation of shear waves with point source under the influence of heterogeneity and it was presented by Chattopadhyay et al. [9]. Kakar and Kakar [15] discussed Love waves in a non-homogeneous elastic media, Kakar and Gupta [16] purposed Love waves in a non-homogeneous orthotropic layer under 'P' overlying semi-infinite non-homogeneous medium. Roy [17] studied wave propagation in a thin two-layered. Datta [18] discussed surface waves in an elastic solid medium with the gravity field. Goda [19] studied the effect of non-homogeneity and anisotropy on Stoneley waves.

The Dirac delta function or δ function known as the unit impulse function is a function on the real number line i.e. 0 (zero) everywhere except at 0 (zero), with an integral of one over the whole real line [6]. It is a mathematical abstraction which is used to approximate some physical phenomenon. The δ function can be considered of as a rectangular pulse that increases narrower and narrower while simultaneously increasing larger and larger. The Dirac delta function is a defined by

$$\delta(x-x_0)=\begin{cases}0 & \text{for} \quad x\neq x_0\\ \infty & \quad\quad\quad x=x_0\end{cases} \tag{a}$$

such that, for any function $f(x)$ that possesses a Taylor series at $x=x_0$,

$$\int_{x_0-\varepsilon}^{x_0+\varepsilon} dx\delta(x-x_0)f(x)=f(x_0) \qquad \forall\varepsilon>0 \tag{b}$$

In particular, setting $f(x)=1$, we have

$$\int_{x_0-\varepsilon}^{x_0+\varepsilon} dx\delta(x-x_0)=1$$

Another way to write Eq. (b) is

$$\int_a^b dx\delta(x-x_0)f(x)=\begin{cases}f(x_0) & \text{for} \quad x_0\in[a,b]\\ 0 & \quad\quad\quad \textit{otherwise}\end{cases} \qquad \text{given by Eq. (a)}$$

Some analytic representations of δ function are

$$\delta(x)=\begin{cases}\gamma^{-1} & \text{for} \quad x\in[-\gamma,\gamma]\\ 0 & \quad\quad\quad \textit{otherwise}\end{cases} \qquad \text{as} \quad \gamma\to 0 \tag{c}$$

$$\delta(x)=\frac{1}{\pi}\lim_{\gamma\to\infty}\frac{\gamma}{x^2+\gamma^2} \qquad \text{(Lorentzian)} \tag{d}$$

$$\delta(x)=\frac{1}{\sqrt{2\pi}}\lim_{\gamma\to\infty}\exp\left(-\frac{x^2}{2\gamma^2}\right) \qquad \text{(Gaussian)} \tag{e}$$

which are simply distributions with vanishing width. An idealized point source of wave can be described using the delta function. [7].

Further, Green's function method is very useful to solve heterogeneous differential equations subject to certain boundary conditions. That is why; we take Green's function technique to solve the problem of wave propagation. Also, it is a strong mathematical tool to work out asymptotic approximations of solutions of differential equations. There are many papers on Green's function techniques available in the literatures, a few are Vaclav and Kiyoshi [10], Kazumi and Robert [11, 12], Popov [13] and George and Christos [14].

Here we discuss the influence of point source on the propagation of SH-waves in non-homogeneous elastic layer. The δ function is taken as the point source. The rigidity and density of the intermediate layer are assumed to vary exponentially as function of depth. Green's function technique and Fourier transform are used to obtain dispersion equation for the intermediate layer. The equation of transmitted wave in the lower medium is also calculated. Various curves are plotted for dispersion equation to show the effects of inhomogeneity on SH-wave in the intermediate layer.

2 Formulation of the problem

The medium to be considered is contained between parallel plane surfaces, infinite in extent. The upper plane surface is supposed free from stress, and the lower surface rigidly fixed. We shall assume that the seismic wave is travelling along x-axis and z-axis is taken vertically downwards. P is point source of disturbance and is taken at the line of intersection of the interface and z- axis (Fig. 1). Let μ_1, ρ_1 be the rigidity and density of the first half-space layer. Let μ_3, ρ_3 be the rigidity and density of the lower half-space layer.

The variations of inhomogeneous parameters in the intermediate layer are

$$\mu_2 = \mu(1+e^{\varepsilon z}), \qquad \rho_2 = \rho(1+e^{\varepsilon z}). \tag{1}$$

where $'\varepsilon'$ is small positive real constant and having dimensions m^{-1}.

The equation of motion for line source can be written as

$$\tau_{ij,j} + F_i = \rho \ddot{u}_i, \tag{2}$$

where τ_{ij} are the stress components, ρ is the density of the medium and F_i are body forces.

For shear wave propagation along the x-axis, we have

$$u = 0, \qquad w = 0, \qquad v = v(x,z,t), \tag{3}$$

Therefore, the equation of motion for upper homogeneous isotropic medium is $(-\infty < x, y < \infty, -\infty < z \le 0)$

$$\frac{\partial}{\partial x}\left(\mu_1 \frac{\partial v_1}{\partial x}\right) + \frac{\partial}{\partial z}\left(\mu_1 \frac{\partial v_1}{\partial z}\right) - \rho_1 \frac{\partial^2 v_1}{\partial t^2} = 0, \tag{4}$$

or

$$\mu_1 \left(\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial z^2}\right) - \rho_1 \frac{\partial^2 v_1}{\partial t^2} = 0, \tag{5}$$

Taking $v_1(x,z,t) = v_1(x,z)e^{i\omega t}$ in Eq. (5)

$$\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial z^2} + \frac{\rho_1}{\mu_1}\omega^2 v_1 = 0, \tag{6}$$

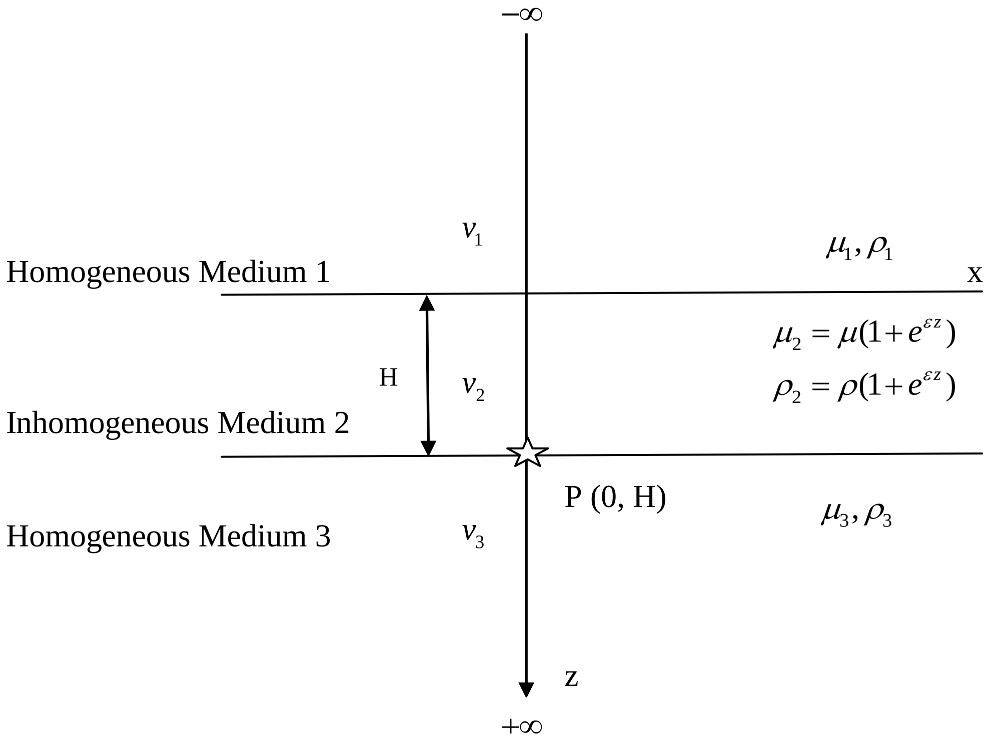

Fig. 1 Geometry of the problem

or

$$\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial z^2} + k_1^2 v_1 = 0, \tag{7}$$

where, $k_1^2 = \dfrac{\rho_1}{\mu_1}\omega^2$

Similarly, the equation of motion for lower homogeneous isotropic medium is $(-\infty < x, y < \infty, 0 \le z < \infty)$

$$\frac{\partial^2 v_3}{\partial x^2} + \frac{\partial^2 v_3}{\partial z^2} + \frac{\rho_3}{\mu_3}\omega^2 v_3 = 0, \tag{8}$$

or

$$\frac{\partial^2 v_3}{\partial x^2} + \frac{\partial^2 v_3}{\partial z^2} + k_3^2 v_3 = 0, \tag{9}$$

where, $k_3^2 = \dfrac{\rho_3}{\mu_3}\omega^2.$

The equation of motion for intermediate inhomogeneous isotropic medium is $(-\infty < x, y < \infty, 0 < z < H)$

$$\frac{\partial}{\partial x}\left(\mu_2 \frac{\partial v_2}{\partial x}\right) + \frac{\partial}{\partial z}\left(\mu_2 \frac{\partial v_2}{\partial z}\right) - \rho_2 \frac{\partial^2 v_2}{\partial t^2} = 4\pi\sigma(r), \tag{10}$$

$\sigma(r)$ is the disturbances produced by the impulsive force at P. In terms of Dirac-delta function, these disturbances can be written as

$$\sigma(r) = \delta(x)\delta(z-H), \tag{11}$$

Inserting Eq. (11) in Eq. (10), Hence equation of motion for the inhomogeneous layer in terms of point source is given by

$$\frac{\partial}{\partial x}\left(\mu_2 \frac{\partial v_2}{\partial x}\right) + \frac{\partial}{\partial z}\left(\mu_2 \frac{\partial v_2}{\partial z}\right) - \rho_2 \frac{\partial^2 v_2}{\partial t^2} = 4\pi\delta(x)\delta(z-H), \tag{12}$$

Put $v_2(x,z,t) = v_2(x,z)e^{i\omega t}$ in Eq. (12)

$$\frac{\partial}{\partial x}\left(\mu_2 \frac{\partial v_2}{\partial x}\right) + \frac{\partial}{\partial z}\left(\mu_2 \frac{\partial v_2}{\partial z}\right) + \rho_2\omega^2 v_2 = 4\pi\delta(x)\delta(z-H), \tag{13}$$

From Eq. (1) and Eq. (13), we get

$$\mu\frac{\partial^2 v_2}{\partial x^2}+\mu e^{\varepsilon z}\frac{\partial^2 v_2}{\partial x^2}+\mu\frac{\partial^2 v_2}{\partial z^2}+\mu e^{\varepsilon z}\frac{\partial^2 v_2}{\partial z^2}+\mu\varepsilon e^{\varepsilon z}\frac{\partial v_2}{\partial z}$$
$$+(\rho+\rho e^{\varepsilon z})\omega^2 v_2=4\pi\delta(x)\delta(z-H), \qquad (14)$$

Dividing Eq. (14) throughout by μ and rearranging, we get

$$\frac{\partial^2 v_2}{\partial x^2}+\frac{\partial^2 v_2}{\partial z^2}+k_2^2 v_2=\left(\frac{4\pi}{\mu}\right)\delta(x)\delta(z-H)-e^{\varepsilon z}\frac{\partial^2 v_2}{\partial z^2}-\varepsilon e^{\varepsilon z}\frac{\partial v_2}{\partial z}-e^{\varepsilon z}\frac{\partial^2 v_2}{\partial x^2}-\frac{\rho e^{\varepsilon z}\omega^2}{\mu}v_2. \qquad (15)$$

where, $k_2^2=\frac{\rho}{\mu}\omega^2$

3 Boundary conditions

The geometry of the problem leads to the following conditions

1. Continuity conditions:

$$v_1=v_2, \quad \mu_1\frac{\partial v_1}{\partial z}=\mu_2\frac{\partial v_2}{\partial z}. \qquad (at \quad z=0, \quad -\infty<x<\infty) \qquad (16a)$$

2. Continuity conditions:

$$v_2=v_3, \quad \left(\mu+\mu e^{\varepsilon H}\right)\frac{\partial v_2}{\partial z}=\mu_3\frac{\partial v_3}{\partial z}. \qquad (at \quad z=H, \quad -\infty<x<\infty) \qquad (16b)$$

3. Stability conditions:

$$v_1\to 0 \quad as \quad z\to-\infty. \qquad (16c)$$

4. Stability conditions:

$$v_3\to 0 \quad as \quad z\to+\infty. \qquad (16d)$$

4 Solution of the problem

To solve Eq. (7), Eq. (9) and Eq. (15), the following transforms are used.

$$V_r(\xi,z)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}v_r(x,z)e^{i\xi x}dx \qquad (17a)$$

The inverse Fourier transform is given as

$$v_r(x,z) = \int_{-\infty}^{+\infty} V_r(\xi, z) e^{-i\xi x} d\xi \tag{17b}$$

Now using the above defining Fourier transforms for Eq. (7) and Eq. (9), we get

$$\frac{d^2V_1}{dz^2} - \alpha^2 V_1 = 0, \tag{18}$$

where, $\alpha^2 = \xi^2 - k_1^2$

$$\frac{d^2V_3}{dz^2} - \gamma^2 V_3 = 0, \tag{19}$$

where, $\gamma^2 = \xi^2 - k_3^2$

In terms of Fourier transforms, Eq. (15) can be written as

$$\frac{d^2V_2}{dz^2} - \beta^2 V_2 = \left(\frac{2}{\mu}\right)\delta(z-H) - e^{\varepsilon z}\frac{d^2V_2}{dz^2} - \varepsilon e^{\varepsilon z}\frac{dV_2}{dz} + e^{\varepsilon z}\xi^2 V_2, \tag{20a}$$

or

$$\frac{d^2V_2}{dz^2} - \beta^2 V_2 = \sigma_2(z), \tag{20b}$$

where, $\beta^2 = \xi^2 - k_2^2$

and $\sigma_2(z) = \left(\frac{2}{\mu}\right)\delta(z-H) - e^{\varepsilon z}\frac{d^2V_2}{dz^2} - \varepsilon e^{\varepsilon z}\frac{dV_2}{dz} + e^{\varepsilon z}\xi^2 V_2 - \frac{\rho e^{\varepsilon z}}{\mu}\omega^2 V_2$

Eqs (18-20) are solved by Green's Function technique under the prescribed boundary conditions in Eqs (16a), (16b), (16c) and (16d). First of all take the middle layer and it is solved with the help of Green's function $G_2(z/z_0)$, Stakgold [20]. The Eq. (20) will satisfy $G_2(z/z_0)$ as

$$\frac{d^2G_2(z/z_0)}{dz^2} - \beta^2 G_2(z/z_0) = \delta(z-z_0), \tag{21}$$

The homogeneous boundary conditions are:

$$\frac{dG_2(z/z_0)}{dz} = 0. \qquad \text{at z=0, H} \tag{22}$$

where z_0 is arbitrary line in the medium 2. Multiplying Eq. (20b) by $G_2(z/z_0)$, Eq. (21) by $V_2(\xi, z)$, subtracting and integrating with respect to z from z=0 to z=H, we get

$$G_2(H/z_0)\left[\frac{dV_2}{dz}\right]_{z=H} - G_2(0/z_0)\left[\frac{dV_2}{dz}\right]_{z=0} = \int_0^H \sigma_2(z) G_2(z/z_0)dz - V_2(z_0). \tag{23}$$

Similarly, if $G_1(z/z_0)$ and $G_3(z/z_0)$ are Green's functions corresponding to upper and lower homogeneous media, then Eq. (18) and Eq. (19) will satisfy as

$$\frac{dG_1(z/z_0)}{dz} = 0 \ \text{ at } \ \text{z=0;} \ \ \frac{dG_1(z/z_0)}{dz} \to 0 \ \text{ as } \ z \to -\infty, \tag{24}$$

and

$$\frac{dG_3(z/z_0)}{dz} = 0 \ \text{ at } \ \text{z=H;} \ \ \frac{dG_3(z/z_0)}{dz} \to 0 \ \text{ as } \ z \to \infty \tag{25}$$

where z_0 is arbitrary point in the medium 1 and 3. Multiplying Eq. (18) by $G_1(z/z_0)$, Eq. (24) by $V_1(\xi, z)$, subtracting and integrating, we get

$$G_1(0/z_0)\left[\frac{dV_1}{dz}\right]_{z=0} = -V_1(z_0), \tag{26}$$

Multiplying Eq. (19) by $G_3(z/z_0)$, Eq. (25) by $V_3(\xi, z)$, subtracting and integrating, we get

$$G_3(H/z_0)\left[\frac{dV_3}{dz}\right]_{z=H} = V_3(z_0). \tag{27}$$

Replacing z by z_0 and using symmetry of Green's function, Eq. (23), Eq. (26) and Eq. (27) become

$$V_2(z) = G_2(z/0)\left[\frac{dV_2}{dz}\right]_{z=0} - G_2(z/H)\left[\frac{dV_2}{dz}\right]_{z=H} + \int_0^H \sigma_2(z_0)G_2(z/z_0)dz_0, \quad (28)$$

$$V_1(z) = -G_1(z/0)\left[\frac{dV_1}{dz}\right]_{z=0}, \quad (29)$$

$$V_3(z) = G_3(z/H)\left[\frac{dV_3}{dz}\right]_{z=H}. \quad (30)$$

Using boundary condition (16a) in Eq. (28), we get

$$\left[\frac{dV_2}{dz}\right]_{z=0} = \frac{1}{A}\left\{G_2(0/H)\left[\frac{dV_2}{dz}\right]_{z=H} - \int_0^H \sigma_2(z_0)G_2(0/z_0)dz_0\right\}, \quad (31)$$

where, $A = G_2(0/0) + \dfrac{2\mu}{\mu_1}G_1(0/0)$

Similarly, using boundary condition (16b) in Eq. (28), we get

$$\left[\frac{dV_2}{dz}\right]_{z=H} = \frac{1}{\left\{AB - G_2^2(H/0) + \dfrac{\mu e^{\varepsilon H}}{\mu_3}AG_3(H/H)\right\}}\left\{\begin{array}{l} -G_2(H/0)\int_0^H \sigma_2(z_0)G_2(0/z_0)dz_0 \\ +A\int_0^H \sigma_2(z_0)G_2(H/z_0)dz_0 \end{array}\right\}, \quad (32)$$

where, $B = G_2(H/H) + \dfrac{\mu}{\mu_3}G_3(H/H).$

Using Eq. (31) and Eq. (32) in Eq. (28), substituting the value of $\sigma_2(z_0)$ and using the property of delta function, we get

$$V_2(z) = \frac{2(2\mu + \mu e^{\varepsilon H})}{\mu\mu_3}\left\{\frac{G_2(z/H)C - G_2(z/0)D}{AB - G_2^2(H/0) + \dfrac{\mu e^{\varepsilon H}}{\mu_3}AG_3(H/H)}\right\}$$

$$-\left\{\frac{G_2(z/H)G_2(H/0)-\left\{B+\dfrac{\mu e^{\varepsilon H}}{\mu_3}G_3(H/H)\right\}G_2(z/0)}{AB-G_2^2(H/0)+\dfrac{\mu e^{\varepsilon H}}{\mu_3}AG_3(H/H)}\right\}$$

$$\times\int_0^H\left\{e^{\varepsilon z_0}\frac{d^2V_2(z_0)}{dz_0^2}+\varepsilon e^{\varepsilon z_0}\frac{dV_2(z_0)}{dz_0}-e^{\varepsilon z_0}(\xi^2-\frac{\rho}{\mu}\omega^2)V_2(z_0)\right\}G_2(0/z_0)dz_0$$

$$-\left\{\frac{G_2(z/0)G_2(H/0)-G_2(z/H)A}{AB-G_2^2(H/0)+\dfrac{\mu e^{\varepsilon H}}{\mu_3}AG_3(H/H)}\right\}\int_0^H\left\{\begin{array}{l}e^{\varepsilon z_0}\dfrac{d^2V_2(z_0)}{dz_0^2}\\+\varepsilon e^{\varepsilon z_0}\dfrac{dV_2(z_0)}{dz_0}\\-e^{\varepsilon z_0}(\xi^2-\dfrac{\rho}{\mu}\omega^2)V_2(z_0)\end{array}\right\}G_2(H/z_0)dz_0$$

$$-\int_0^H\left\{e^{\varepsilon z_0}\frac{d^2V_2(z_0)}{dz_0^2}+\varepsilon e^{\varepsilon z_0}\frac{dV_2(z_0)}{dz_0}-e^{\varepsilon z_0}(\xi^2-\frac{\rho}{\mu}\omega^2)V_2(z_0)\right\}G_2(z/z_0)dz_0, \quad (33)$$

where, $C=G_3(H/H)A, \qquad D=G_3(H/H)G_2(H/0).$

$V_2(z)$ can be found from Eq. (33) which is an integral equation . Also the value of $V_2(z)$ can be obtained by using successive approximations and this will give the displacement in the intermediate inhomogeneous layer. First of all we neglect the terms having ε , we get

$$V_2(z)=\frac{4}{\mu_3}\left\{\frac{G_2(z/H)C-G_2(z/0)D}{AB-G_2^2(H/0)}\right\}. \qquad (34)$$

Now put Eq. (34) back in the right hand side of Eq. (33), we get

$$V_2(z)=\frac{2(2\mu+\mu e^{\varepsilon H})}{\mu\mu_3}\left\{\frac{G_2(z/H)C-G_2(z/0)D}{AB-G_2^2(H/0)+\dfrac{\mu e^{\varepsilon H}}{\mu_3}AG_3(H/H)}\right\}$$

$$-\frac{4}{\mu_3}\left\{\frac{G_2(z/H)G_2(H/0)-\left\{B+\frac{\mu e^{\varepsilon H}}{\mu_3}G_3(H/H)\right\}G_2(z/0)}{AB-G_2^2(H/0)+\frac{\mu e^{\varepsilon H}}{\mu_3}AG_3(H/H)}\right\}$$

$$\times\int_0^H\left\{e^{\varepsilon z_0}\frac{d^2\Theta(z_0)}{dz_0^2}+\varepsilon e^{\varepsilon z_0}\frac{d\Theta(z_0)}{dz_0}-e^{\varepsilon z_0}(\xi^2-\frac{\rho}{\mu}\omega^2)\Theta(z_0)\right\}G_2(0/z_0)dz_0$$

$$-\frac{4}{\mu_3}\left\{\frac{G_2(z/0)G_2(H/0)-G_2(z/H)A}{AB-G_2^2(H/0)+\frac{\mu e^{\varepsilon H}}{\mu_3}AG_3(H/H)}\right\}\int_0^H\left\{\begin{matrix}e^{\varepsilon z_0}\frac{d^2\Theta(z_0)}{dz_0^2}\\+\varepsilon e^{\varepsilon z_0}\frac{d\Theta(z_0)}{dz_0}\\-e^{\varepsilon z_0}(\xi^2-\frac{\rho}{\mu}\omega^2)\Theta(z_0)\end{matrix}\right\}G_2(H/z_0)dz_0$$

$$-\frac{4}{\mu_3}\int_0^H\left\{e^{\varepsilon z_0}\frac{d^2\Theta(z_0)}{dz_0^2}+\varepsilon e^{\varepsilon z_0}\frac{d\Theta(z_0)}{dz_0}-e^{\varepsilon z_0}(\xi^2-\frac{\rho}{\mu}\omega^2)\Theta(z_0)\right\}G_2(z/z_0)dz_0, \quad (35)$$

where,

$$\Theta(z_0)=\left\{\frac{G_2(z_0/H)C-G_2(z_0/0)D}{AB-G_2^2(H/0)}\right\}$$

We see that Eq. (35) completely represents the elastic displacements which are due to a unit impulsive force in space and time. Here in Eq. (35); G_1, G_2 and G_3 are unknown. We adopt the following method to find the unknown Green's function so that the value of $V_2(z)$ can be determined from Eq. (35).

We have considered $G_1(z/z_0)$ as a solution of Eq. (18).

A solution of Eq. (18) can also be found as

$$\frac{d^2L}{dz^2}-\alpha^2L=0 \quad (36)$$

The two independent solutions of Eq. (36) will vanish at $z = -\infty$ and $z = \infty$ are

$$L_1(z) = e^{\alpha z} \qquad and \qquad L_2(z) = e^{-\alpha z} \tag{37}$$

Hence, the solution of Eq. (36) for an infinite medium is

$$\begin{aligned} &\frac{L_1(z)L_2(z_0)}{M} \qquad for \qquad z < z_0, \\ &\frac{L_1(z_0)L_2(z)}{M} \qquad for \qquad z > z_0. \end{aligned} \tag{38}$$

where, $M = L_1(z)L'_2(z) - L_2(z)L'_1(z) = -2\alpha$.

Hence, we can write the solution of Eq. (18) as

$$-\frac{e^{-\alpha|z-z_0|}}{2\alpha}. \tag{39}$$

Since $G_1(z/z_0)$ is to satisfy the condition (Eq. (24)

$$\frac{dG_1(z/z_0)}{dz} = 0 \quad \text{at} \quad z=0; \quad \frac{dG_1(z/z_0)}{dz} \to 0 \text{ as } \quad z \to -\infty, \tag{40}$$

Therefore, we assume that

$$G_1(z/z_0) = -\frac{e^{-\alpha|z-z_0|}}{2\alpha} + Pe^{\alpha z} + QBe^{-\alpha z}. \tag{41}$$

The conditions as mentioned in Eq. (40) give

$$G_1(z/z_0) = -\frac{1}{2\alpha}\left[e^{-\alpha|z-z_0|} + e^{\alpha(z+z_0)}\right], \tag{42}$$

Similarly

$$G_3(z/z_0) = -\frac{1}{2\gamma}\left[e^{-\gamma|z-z_0|} + e^{-\gamma(z+z_0-2H)}\right], \tag{43}$$

Green's function $G_2(z/z_0)$ for the intermediate inhomogeneous layer can be obtained in the similar manner as above by using the boundary conditions Eq. (16a) and Eq. (16b).

$$G_2(z/z_0) = -\frac{1}{2\beta}\left[e^{-\beta|z-z_0|} + e^{\beta z}\left\{\frac{e^{-\beta(z_0+H)} + e^{-\beta(H-z_0)}}{e^{\beta H} - e^{-\beta H}}\right\} + e^{-\beta z}\left\{\frac{e^{\beta(H-z_0)} + e^{-\beta(H-z_0)}}{e^{\beta H} - e^{-\beta H}}\right\}\right]. \tag{44}$$

Substitute the value of Eq. (42), Eq. (43) and Eq. (44) in Eq. (35), simplifying and neglecting square and higher powers of ε, we get

$$V_2(z) = \frac{-2(\mu\beta\cosh\beta z + \mu_1\alpha\sinh\beta z)}{\mu_1\mu_3\alpha\beta^2\gamma\left\{AB - G_2^2(H/0)\right\}\Phi(\varepsilon)\sinh\beta H}, \tag{45}$$

where,

$$\Phi(\varepsilon) = 1 + \frac{\varepsilon}{4\left\{AB - G_2^2(H/0)\right\}}\left[\begin{array}{l} \dfrac{(\mu_3\gamma - \mu_1\alpha)}{\mu_1\mu_3\alpha\beta^2\gamma} \\ -\dfrac{H\left\{(5\mu_1\mu_3\alpha\gamma + 3\mu^2\beta^2) + (\mu_1\mu_3\alpha\gamma + \mu^2\beta^2)\coth\beta H\right\}}{\mu_1\mu\mu_3\alpha\beta^2\gamma} \\ -\dfrac{H^2\left\{\mu\beta(\mu_1\alpha + \mu_3\gamma) + (\mu_1\mu_3\alpha\gamma + \mu^2\beta^2)\coth\beta H\right\}}{\mu_1\mu\mu_3\alpha\beta\gamma} \\ -\dfrac{H\xi^2\left\{(\mu_1\mu_3\alpha\gamma - \mu^2\beta^2) + \mu\beta(\mu_3\gamma - \mu_1\alpha)\coth\beta H\right\}}{\mu_1\mu\mu_3\alpha\beta^4\gamma} \\ +\dfrac{H^2\xi^2\left\{\mu\beta(\mu_1\alpha + \mu_3\gamma) + (\mu_1\mu_3\alpha\gamma + \mu^2\beta^2)\coth\beta H\right\}}{\mu_1\mu\mu_3\alpha\beta^3\gamma} \\ +\dfrac{\xi^2(\mu_3\gamma - \mu_1\alpha)}{\mu_1\mu_3\alpha\beta^4\gamma} \end{array}\right].$$

Taking inverse Fourier transform of Eq. (45), the displacement in the intermediate inhomogeneous layer is

$$v_2(x,z) = -\int_{ic-\infty}^{ic+\infty}\frac{2(\mu\beta\cosh\beta z + \mu_1\alpha\sinh\beta z)e^{-i\xi x}}{\mu_1\mu_3\alpha\beta^2\gamma\left\{AB - G_2^2(H/0)\right\}\Phi(\varepsilon)\sinh\beta H}d\xi, \tag{46}$$

Eq. (46) is obtained by performing contour integration. The dispersion equation of surface waves in non-homogeneous elastic media subjected to point source will be obtained by equating the denominator of the above integral with zero.

Replacing β by ik, the dispersion relation will become

$$\tan kH = \frac{\mu k(\mu\alpha_1 + \mu_3\gamma)}{\mu^2 k^2 - \mu_1\mu_3\alpha\gamma} + \frac{\varepsilon e^{\varepsilon z}}{4\left\{\mu^2k^2 - \mu_1\mu_3\alpha\gamma\right\}}\left[\begin{array}{l}(\mu_3\gamma - \mu_1\alpha)\tan kH \\ -\dfrac{H}{\mu}\left\{(5\mu_1\mu_3\alpha\gamma - 3\mu^2k^2)\tan kH + (3\mu_1\alpha + 5\mu_3\gamma)\right\} \\ -\dfrac{H^2k}{\mu}\left\{(\mu_1\mu_3\alpha\gamma - \mu^2k^2) + \mu k(\mu_1\alpha + \mu_3\gamma)\tan kH\right\} \\ +\dfrac{H\xi^2}{\mu k^2}\left\{(\mu_1\mu_3\alpha\gamma + \mu^2k^2)\tan kH + \mu k(\mu_3\gamma - \mu_1\alpha)\right\} \\ -\dfrac{H^2\xi^2}{\mu k^2}\left\{(\mu_1\mu_3\alpha\gamma - \mu^2k^2) - \mu k(\mu_1\alpha + \mu_3\gamma)\tan kH\right\} \\ -\dfrac{\xi^2}{k^2}\left\{(\mu_3\gamma - \mu_1\alpha)\tan kH\right\}\end{array}\right] \tag{47}$$

Special Case

In the absence of non-homogeneity i.e. ε =0, the relation reduces to

$$\tan kH = \frac{\mu k(\mu\alpha_1 + \mu_3\gamma)}{\mu^2 k^2 - \mu_1\mu_3\alpha\gamma} \tag{48}$$

The Eq. (48) is the dispersion relation for love waves in homogeneous media given by Ewing et al. [3].

5 Transmitted waves

The equation for the transmitted surface waves can be obtained from Eq. (46). We note that the poles of the integral are roots $P_{2,n}$ (n=1, 2, 3...) of

$$J(\xi, H) = \mu_1\mu_3\alpha\beta^2\gamma\left\{AB - G_2^2(H/0)\right\}\Phi(\varepsilon)\sinh\beta H$$

Simplifying, we get

$$v_2(x,z) = 2\pi \sum_{n=1}^{\infty} \frac{e^{-ip_{2,n}x} \left\{ \mu k_{2,n} \cos k_{2,n} z + \mu_1 \alpha_{2,n} \sin k_{2,n} z \right\}}{\left. \frac{dJ(\xi,H)}{d\xi} \right|_{\xi=p_{2,n}}} \qquad (49)$$

where, $k\big|_{\xi=p_{2,n}} = k_{2,n}$, $\alpha\big|_{\xi=p_{2,n}} = \alpha_{2,n}$.

Eq. (49) is the expression for surface waves travelling in the x-axis.

6 Numerical analysis

The effects of non-homogeneity in the intermediate layer are studied numerically by taking parameters in following table Gubbins, [21]. In fig. 2, the various curves are plotted between kH v/s α at various values of non-homogeneity parameter $\varepsilon' = \frac{\varepsilon}{4\left\{\mu^2 k^2\right\}}$ by taking values of ε' = 0.1 to 0.4. In fig. 3, we have shown the effect of another non-homogeneity factor $\varepsilon'' = \frac{\varepsilon H}{4\left\{\mu^2 k^2\right\}}$ by taking ε'' = 0.0 to 0.4. Here, the various curves are plotted between kH v/s α. The amplitude of the scattered waves falls off very rapidly as the kH increases slowly. The amplitude of the reflected SH-wave decreases rapidly with the kH and ultimately becomes saturated which shows that the reflected SH-wave take a very long time to dissipate making these the most destructive waves during the earthquake. It is clear from graphs that the phase velocity of SH-waves is affected by non-homogeneity parameters.

Table Material Parameters

Layer	**Rigidity**	**Density**
I	$\mu_1 = 6.54 \times 10^{10} N/m^2$	$\rho_1 = 3409 Kg/m^3$
II	$\mu = 11.77 \times 10^{10} N/m^2$	$\rho = 4148 Kg/m^3$
III	$\mu_3 = 8.84 \times 10^{10} N/m^2$	$\rho_3 = 3944 Kg/m^3$

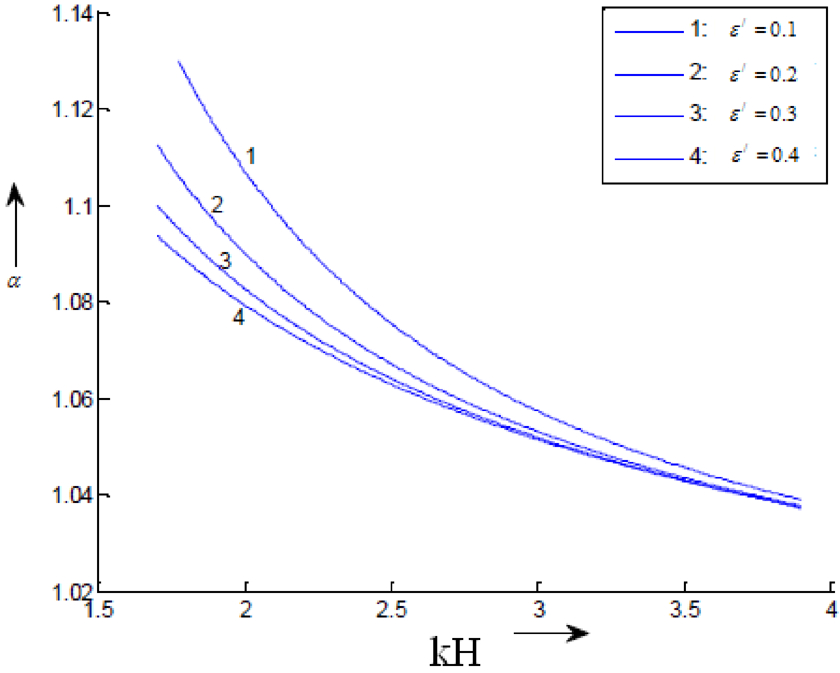

Fig. 2 Dispersion of SH-wave for $\varepsilon^{/}$

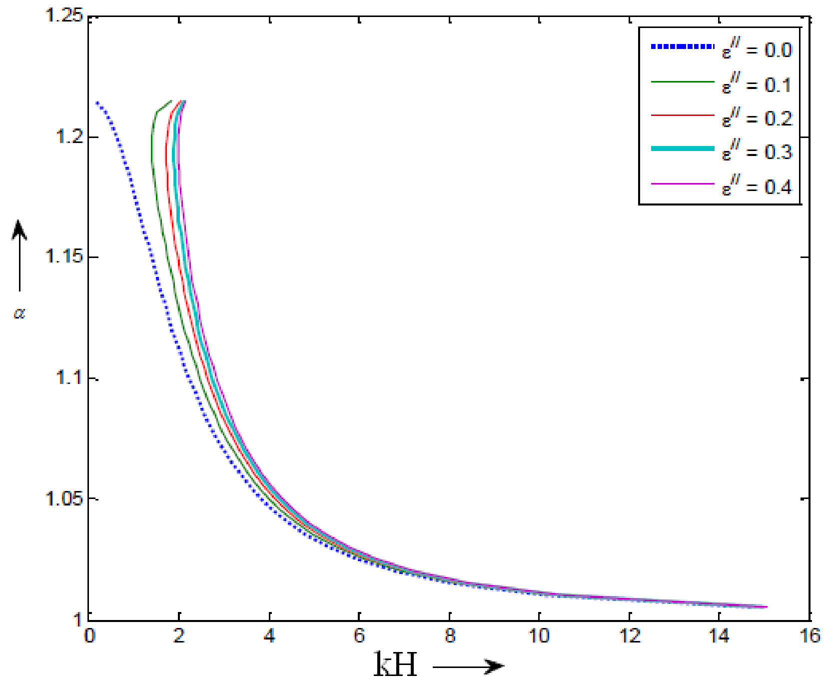

Fig. 3 Dispersion of SH-wave for $\varepsilon^{//}$

7 Conclusions

In this problem we assume the upper layer and lower layer to be homogeneous, isotropic and semi infinite, whereas the intermediate layer is taken non-homogeneous isotropic with exponential variation in rigidity and density. We have employed Green's function method to find the frequency equation due to a line source. Displacement in the intermediate layer is derived in closed form and the dispersion curves are drawn for various values of inhomogeneity parameters. Eq. (47) gives the dispersion of surface waves in non-homogeneous elastic media subjected to point source. We observe that

1. Dimensionless wave number kH and the inhomogeneity parameters ($\varepsilon^{/}$ and $\varepsilon^{//}$) affect the phase velocity of surface waves. In general, phase velocity decreases with increase in wave number kH but at a particular value of kH, phase velocity increases with increase in $\varepsilon^{/}$ and $\varepsilon^{//}$.

2. Effects of inhomogeneity parameters ($\varepsilon^{/}$ and $\varepsilon^{//}$) on phase velocity are negligible after certain range of dimensionless wave number kH.

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