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ON THE INTEGRALS OF MOTION FOR UNIFIED CHAOTIC SYSTEM

Jhinjhit Podder

Department of Mathematics
Nabagram Hiralal Paul College
Nabagram, Hooghly, Pin-712246, West Bengal, India.
E-mail: jhinjhitpodder@gmail.com

Nurul Islam

Department of Mathematics
Ramakrishna Mission Residential College (Autonomous)
Narendrapur, Kolkata-700103, West Bengal, India.
E-mail: dr.i.nurul@gmail.com

Abstract:
This paper introduces the dynamical behavior of unified nonlinear chaotic system which describes three chaotic systems containing Lorenz chaotic system, Lü chaotic system and Chen chaotic system. Here the integrals of motion for unified chaotic system are derived by considering their behavior in the neighbourhood of a singularity.

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1. Introduction:

The Lorenz chaotic system [5] is given by

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= x(b-z)-y \\ \dot{z} &= xy-cz. \end{aligned} \tag{1}$$

$$\text{for } a=10, b=\frac{8}{3}, c=28.$$

The Lü chaotic system [6] is given by

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= -xz+by \\ \dot{z} &= xy-cz \end{aligned} \tag{2}$$

$$\text{for } a=30, b=22.2, c=2.93.$$

The Chen chaotic system [1] is given by

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= (b-a)x-xz+by \\ \dot{z} &= xy-cz \end{aligned} \tag{3}$$

$$\text{for } a=35, b=28, c=3.$$

In unified chaotic system [2] the above three systems are combined together as follows:

$$\begin{aligned} \dot{x} &= (25a+10)(y-x) \\ \dot{y} &= (28-35a)x-xz+(29a-1)y \\ \dot{z} &= xy-\frac{8+a}{3}z \end{aligned} \tag{4}$$

where a is a parameter of the system. It represents Lorenz chaotic system for $a=0$,Lü chaotic system for $a=0.8$,Chen chaotic system for $a=1$ .In this paper our intension is to find integrals of motion of unified chaotic system (4) by considering its leading order behavior in the neighbourhood of a singularity at $t = t^*$ (say).

2. Integrability conditions:

We consider the integrals of motion [3, 4] in the form $I(x,y,z,t)=W(x,y,z)e^{\mu.t}$

Let the leading order behavior of the system in the neighbourhood of $t = t^*$ is given by:

$$x = \frac{A}{(t-t^*)^p},\ y = \frac{B}{(t-t^*)^q},\ z = \frac{C}{(t-t^*)^r}. \quad (5)$$

where A, B, C, p, q, r are constants, $ABC \neq 0, p > 0, q > 0, r > 0$.

Using (5) in system (4) one gets after simple mathematical calculations

$$\begin{aligned} &p - q + 1 = 0, -Ap = (25a + 10)B \\ &r = 2, -Bq = -AC \\ &2 - 2p = 0, -rC = AB. \end{aligned} \quad (6)$$

It yields p =1, q =2, r =2. Then we get, $x = \frac{A}{(t-t^*)}, y = \frac{B}{(t-t^*)^2}, z = \frac{C}{(t-t^*)^2}$. (7)

Here we choose all possible terms having singularity up to fifth order and arrange them in the following table:

Table: singular terms up to fifth order

Order	Terms with real coefficients
$(t-t^*)^{-1}$	x
$(t-t^*)^{-2}$	x^2, y, z
$(t-t^*)^{-3}$	x^3, xy, xz
$(t-t^*)^{-4}$	$x^4, y^2, z^2, x^2y, x^2z, yz$
$(t-t^*)^{-5}$	$x^5, xy^2, xz^2, x^3y, x^3z, xyz$

$W(x, y, z)$ can be considered in general form as,

$W(x, y, z) = A_1x + B_1x^2 + B_2y + B_3z + C_1x^3 + C_2xy + C_3xz + D_1x^4 + D_2y^2 + D_3z^2$
$D_4x^2y + D_5x^2z + D_6yz + E_1x^5 + E_2xy^2 + E_3xz^2 + E_4x^3y + E_5x^3z + E_6xyz.$

where $A_i (i = 1(1)3), B_i (i = 1(1)3), C_i (i = 1(1)3), D_i (i = 1(1)6), E_i (i = 1(1)6)$ are arbitrary constants.

Since an integral of motion takes a constant value when it is considered in the neighbourhood of a singularity, we must have $dI = 0$.

$$\text{i.e. } I_x\dot{x} + I_y\dot{y} + I_z\dot{z} + I_t = 0. \quad (8)$$

This gives, $(\mu A_1 + \beta B_2 - \alpha A_1)x + (\mu B_1 + \beta C_2 - 2\alpha B_1)x^2 + (\mu B_2 + \gamma B_2 + \alpha A_1)y +$

$$(\mu B_3 - \delta B_3)z + (\mu C_1 + \beta D_4 - 3\alpha C_1)x^3 + (\mu C_2 + B_3 + 2\beta D_2 + \gamma C_2 + 2\alpha B_1 - \alpha C_2) xy(\mu C_3 - \delta C_3 + \beta D_6 - B_2 - \alpha C_3)xz + (\mu D_1 + \beta E_4 - 4\alpha D_1)x^4 + (\mu D_2 + 2\gamma D_2 + \alpha C_2)y^2 + (\mu D_3 - 2\delta D_3)z^2 + (\mu D_4 + C_3 + 2\beta E_2 + \gamma D_4 + 3\alpha C_1 - 2\alpha D_4)x^2 y + (\mu D_5 - \delta D_5 - C_2 + \beta E_6 - 2\alpha D_5)x^2 z + (\mu D_6 - \delta D_6 + \gamma D_6 + \alpha C_3)yz + (\mu E_1 - 5\alpha E_1)x^5 + (\mu E_2 + D_6 + 2\gamma E_2 + 2\alpha D_4 - \alpha E_2)xy^2 + (\mu E_3 - 2\delta E_3 - D_6 - \alpha E_3)xz^2 + (\mu E_4 + D_5 + \gamma E_4 + 4\alpha D_1 - 3\alpha E_4)x^3 y + (\mu E_5 - \delta E_5 - D_4 - 3\alpha E_5)x^3 z + (\mu E_6 - \delta E_6 + 2D_3 + \gamma E_6 - 2D_2 + 2\alpha D_5 - \alpha E_6)xyz + (2E_3 - 2E_2 + 3\alpha E_5)x^2 yz + (E_5 + 5\alpha E_1)x^4 y + (E_6 + 3\alpha E_4)x^2 y^2 - E_4 x^4 z - E_6 x^2 z^2 + \alpha E_2 y^3 + \alpha E_3 yz^2 + \alpha E_6 y^2 z = 0.$$

where
$$\alpha = (25a + 10)$$
$$\beta = (28 - 35a)$$
$$\gamma = (29a - 1)$$
$$\delta = (\frac{8 + a}{3}).$$

Neglecting the terms having singularity of order greater than five and equating the coefficients of the terms with singularity of order less or equal to five, we get:

$$\begin{aligned}
&(\mu - \alpha)A_1 + \beta B_2 = 0 \\
&(\mu - 2\alpha)B_1 + \beta C_2 = 0 \\
&(\mu + \gamma)B_2 + \alpha A_1 = 0 \\
&(\mu - \delta)B_3 = 0 \\
&(\mu - 3\alpha)C_1 + \beta D_4 = 0 \\
&(\mu + \gamma - \alpha)C_2 + 2\alpha B_1 + 2\beta D_2 + B_3 = 0 \\
&(\mu - \delta - \alpha)C_3 - B_2 + \beta D_6 = 0 \\
&(\mu - 4\alpha)D_1 + \beta E_4 = 0 \\
&(\mu + 2\gamma)D_2 + \alpha C_2 = 0 \\
&(\mu - 2\delta)D_3 = 0 \\
&(\mu + \gamma - 2\alpha)D_4 + C_3 + 2\beta E_2 + 3\alpha C_1 = 0 \\
&(\mu - \delta - 2\alpha)D_5 - C_2 + \beta E_6 = 0 \\
&(\mu + \gamma - \delta)D_6 + \alpha C_3 = 0 \\
&(\mu - 5\alpha)E_1 = 0 \\
&(\mu + 2\gamma - \alpha)E_2 + D_6 + 2\alpha D_4 = 0
\end{aligned} \qquad (9)$$

$(\mu-2\delta-\alpha)E_3-D_6=0$
$(\mu+\gamma-3\alpha)E_4+D_5+4\alpha D_1=0$
$(\mu-\delta-3\alpha)E_5-D_4=0$
$(\mu-\delta+\gamma-\alpha)E_6+2D_3-2D_2+2\alpha D_5=0.$

3. Integrals of motion:

The integrals of motion are found for some particular values of μ as follows:

Case I: Let us consider $\mu=\delta, B_3\neq 0.$

In this case the integral of motion is:

$$I(x,y,z,t)=[-\frac{\beta}{\delta-2\alpha}x^2+(\frac{2\alpha\beta}{\delta-2\alpha}+\frac{2\alpha\beta}{\delta+2\gamma}+\alpha-\delta+\gamma)z+xy- \beta\frac{\frac{1}{2\alpha}-\frac{\beta}{2\alpha}\frac{\alpha+\delta+2\gamma}{(\delta+2\gamma)(\gamma-\alpha+\beta)}}{(\delta+\gamma-3\alpha)(\delta-4\alpha)-4\alpha\beta}x^4-\frac{\alpha}{\delta+2\gamma}y^2+\frac{\alpha+2\gamma+\delta}{(\gamma-\alpha+\beta)(\delta+2\gamma)} xyz+(\frac{\beta}{2\alpha}-\frac{1}{2\alpha}+\frac{\alpha+\delta+2\gamma}{(\delta+2\gamma)(\gamma-\alpha+\beta)})x^2z+ \frac{\frac{1}{2\alpha}-\frac{\beta}{2\alpha}\frac{\alpha+\delta+2\gamma}{(\delta+2\gamma)(\gamma-\alpha+\beta)}}{(\delta+\gamma-3\alpha)(\delta-4\alpha)-4\alpha\beta}x^3y]C_2e^{\mu.t} \tag{10}$$

where C_2 is non zero arbitrary constant.

Case II: We now take $\mu=2\delta+\alpha, E_3\neq 0.$

It yields integral of motion as:

$$I(x,y,z,t)=[(\frac{2\beta^2}{(2\delta+\gamma-\alpha)(2\delta-3\alpha)-3\alpha\beta}x^3-\frac{2\beta(2\delta-3\alpha)}{(2\delta+\gamma-\alpha)(2\delta-3\alpha)-3\alpha\beta}x^2y +xy^2-\frac{2\beta(2\delta-3\gamma)}{(2\delta+\gamma-\alpha)(2\delta-3\alpha)-3\alpha\beta}\frac{1}{\delta-2\alpha}x^3z)E_2+xz^2E_3+ (-\frac{\beta}{2\delta-3\alpha}x^4-(2\delta+\gamma-2\alpha-\frac{4\alpha\beta}{2\delta-3\alpha})x^2z+x^3y)E_4]e^{\mu.t}. \tag{11}$$

where E_2, E_3, E_4 are non zero arbitrary constant.

Case III: Let us now consider $\mu=2\delta, D_3\neq 0.$

In this case the integral of motion is of the form:

$$I(x,y,z,t)=[(\frac{2\alpha\beta}{\delta-4\alpha}-\delta-\gamma+\alpha)z^2-\frac{\beta}{\delta-4\alpha}x^2z+xyz]E_6e^{\mu.t} \quad (12)$$

where E_6 is non zero arbitrary constant.

Case IV: $\mu=\delta+\alpha, C_3\neq 0$ yields integral of motion as:

$$I(x,y,z,t)=[(-\frac{\beta}{\delta-2\alpha}x^3+x^2y-\frac{1}{2\alpha}x^3z-\frac{2\alpha}{\delta+2\gamma}xy^2)D_4+(-\frac{\alpha}{\alpha+\gamma}yz+xz+ \frac{\alpha}{\delta(\alpha+\gamma)}xz^2+\frac{\alpha}{(\alpha+\gamma)(\delta+2\alpha)}xy^2)C_3]e^{\mu.t} \quad (12)$$

where C_3, D_4 are non zero arbitrary constants.

4. Conclusion:

This paper introduces the dynamical behavior of unified chaotic system which describes a three family chaotic system for different values of parameter a and in tern the corresponding different values of $\alpha,\beta,\gamma,\delta$ .This paper gives a better understanding about the integrals of motion of Lorenz, Lü and Chen chaotic systems. We can expect that the findings would be of much interest while considering various other properties of the unified chaotic system.

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