

**KAHLERIAN SPACE WITH BOCHNER CURVATURE TENSOR**

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**Abstract**

In this paper, we have defined some important definitions. In the end, an important theorem on kahler space with Bochner Curvature Tensor has been established.

**Key Words:** Bochner Curvature tensor, Riemannian Curvature tensor, Ricci tensor, recurrence vector.

**1. INTRODUCTION:**

$n (=2m)$  dimensional kahlerian space is a Riemannian space which admits a tensor field  $F_i^h$  and satisfying the following relations [1]

(1.1)  $F_j^h F_h^i = -\delta_j^i$

(1.2)  $F_{ij} = F_i^a g_{aj}$

(1.3)  $F_{ij} = -F_{ji}$

(1.4)  $\nabla_j F_j^i = 0$

(1.5) Let  $R_{ijk}^h = \partial_i \{ \begin{smallmatrix} h \\ j \quad k \end{smallmatrix} \} - \partial_j \{ \begin{smallmatrix} h \\ i \quad k \end{smallmatrix} \} + \{ \begin{smallmatrix} h \\ i \quad l \end{smallmatrix} \} \{ \begin{smallmatrix} l \\ j \quad k \end{smallmatrix} \} - \{ \begin{smallmatrix} h \\ j \quad l \end{smallmatrix} \} \{ \begin{smallmatrix} l \\ i \quad k \end{smallmatrix} \}$

$R = R_{jk}^g$  and  $R_{jk} = R'_{ijk}$  are the Riemannian Curvature tensor, the scalar curvature and Ricci tensor respectively.

In the real coordinate system, the Bochner Curvature tensor may be defined as

$$(1.6) \quad K_{ijk}^h = R_{ijk}^h + \{1/(n+4)\}(R_{ik}\delta_j^h - R_{jk}\delta_i^h + g_{ik}R_j^h - g_{jk}R_i^h + S_{ik}F_j^h - S_{jk}F_i^h + F_{ik}S_j^h - F_{jk}F_i^h + 2S_{ij}F_k^h) - \{1/(n+2)(n+4)\}R(g_{ik}\delta_j^h - g_{jk}\delta_i^h + F_{ik}F_j^h - F_{jk}F_i^h + 2F_{ij}F_k^h)$$

Wherein,

$$S_{ij} = F_i^a R_{aj}$$

In the Kahlerian space, the Kahlerian Conharmonic Curvature tensor is defined as

$$(1.7) \quad T_{ijk}^h = R_{ijk}^h + \{1/(n+4)\}(R_{ik}\delta_j^h - R_{jk}\delta_i^h + g_{ik}R_j^h - g_{jk}R_i^h + S_{ik}F_j^h - S_{jk}F_i^h + F_{ik}S_j^h - F_{jk}S_i^h + 2S_{ij}F_k^h + 2F_{ij}S_k^h)$$

**Remark 1.1 :**

It is to be noted that if a Kahlerian space is an Einstein one, then the Ricci tensor satisfies [2] [3]

$$(1.8) \quad R_{ij} = (1/n)Rg_{ij}$$

$$(1.9) \quad \nabla_a R = 0$$

Yields

$$(1.10) \quad \nabla_a R_{ji} = 0$$

$$(1.11) \quad \nabla_a S_{ij} = 0$$

$$(1.12) \quad S_{ij} = (1/n)RF_{ij}$$

**2. KAHLERIAN CONHARMONIC CURVATURE TENSOR :**

If a Kahlerian space is an Einstein one, then the Bochner Curvature tensor becomes

$$(2.1) \quad U_{ijk}^h = R_{ijk}^h + \{1/n(n+2)\}R(g_{ik}\delta_j^h - g_{jk}\delta_i^h + F_{ik}F_j^h - F_{jk}F_i^h + 2F_{ij}F_k^h)$$

If a Kahlerian space is an Einstein one then the Kahlerian Conharmonic Curvature tensor reduces in the form

$$(2.2) \quad E_{ijk}^h = R_{ijk}^h + \{2/n(n+4)\}R(g_{ik}\delta_j^h - g_{jk}\delta_i^h + F_{ik}F_j^h - F_{jk}F_i^h + 2F_{ij}F_k^h)$$

By virtue of equations (2.1) and (2.2), we obtain

$$(2.3) \quad U_{ijk}^h = E_{ijk}^h - \{1/(n+2)(n+4)\}R(g_{ik}\delta_j^h - g_{jk}\delta_i^h + F_{ik}F_j^h - F_{jk}F_i^h + 2F_{ij}F_k^h)$$

In this regard, we have the following definitions:

**Definition 2.1 :**

A Kahler space is termed recurrent if it satisfies the relation

$$(2.4) \quad \nabla_a R_{ijk}^h = \lambda_a R_{ijk}^h$$

for some non-zero recurrence vector  $\lambda_a$ .

**Definition 2.2 :**

A Kahlerian space is said to be Ricci recurrent if it satisfies the relation

$$(2.5) \quad \nabla_a R_{ij} = \lambda_a R_{ij}$$

Contracting equation (2.5) with  $g^{ij}$  yields

$$(2.6) \quad \nabla_a R = \lambda_a R$$

**Definition 2.3 :**

A Kahler space satisfying the relation

$$(2.7) \quad \nabla_a E^h_{ijk} = \lambda_a E^h_{ijk}$$

Where  $\lambda_a$  is a non-zero recurrence vector, is termed Einstein kahlerian Conharmonic recurrent space.

**Definition 2.4 :**

A kahlerian space satisfying the relation

$$(2.8) \quad \nabla_a U^h_{ijk} = \lambda_a U^h_{ijk}$$

Where  $\lambda_a$  is a non-zero recurrence vector, is called, an Einstein kahlerian space with recurrent Bochner Curvature tensor.

**Theorem 2.1 :**

Kahlerian Conharmonic Curvature tensor satisfies the following relation [4]

$$(2.9) \quad \nabla_a E^h_{ijk} + \nabla_i E^h_{jak} + \nabla_j E^h_{aik} = 0$$

**Theorem 2.2 :**

In a kahlerian space, the Bochner Curvature tensor satisfies the following relation

$$(2.10) \quad \nabla_a U^h_{ijk} + \nabla_i U^h_{jak} + \nabla_j U^h_{aik} = 0$$

**Proof:**

Inserting equation (2.8) in the L.H.S. of equation (2.10), we get

$$(2.11) \quad \nabla_a U^h_{ijk} + \nabla_i U^h_{jak} + \nabla_j U^h_{aik} = \lambda_a U^h_{ijk} + \lambda_i U^h_{jak} + \lambda_j U^h_{aik}$$

By virtue of equations (2.11) and (2.3), we obtain

$$(2.12) \quad \nabla_a U^h_{ijk} + \nabla_i U^h_{jak} + \nabla_j U^h_{aik} = \lambda_a [E^h_{ijk} - \{R/(n+2)(n+4)\}(g_{ik}\delta^h_j - g_{jk}\delta^h_i + F_{ik}F^h_j - F_{jk}F^h_i + 2F_{ij}F^h_k)] + \lambda_i [E^h_{jak} - \{R/(n+2)(n+4)\}(g_{ik}\delta^h_a - g_{ak}\delta^h_j + F_{jk}F^h_a - F_{ak}\delta^h_j + 2F_{ja}F^h_k)] + \lambda_j [E^h_{aik} - \{R/(n+2)(n+4)\}(g_{ak}\delta^h_i - g_{ik}\delta^h_a + F_{ak}F^h_i - F_{ik}F^h_a + 2F_{ai}F^h_k)]$$

As a consequence of equations (2.12),(2.6) and (2.7), we get

$$(2.13) \quad \nabla_a U^h_{ijk} + \nabla_i U^h_{jak} + \nabla_j U^h_{aik} = \nabla_a E^h_{ijk} - \{\nabla_a R/(n+2)(n+4)\}(g_{ik}\delta^h_j - g_{jk}\delta^h_i + F_{ik}F^h_j - F_{jk}F^h_i + 2F_{ij}F^h_k) + \nabla_i E^h_{jak} - \{\nabla_i R/(n+2)(n+4)\}(g_{ik}\delta^h_a - g_{ak}\delta^h_j + F_{jk}F^h_a - F_{ak}\delta^h_j + 2F_{ja}F^h_k) + \nabla_j E^h_{aik} - \{\nabla_j R/(n+2)(n+4)\}(g_{ak}\delta^h_i - g_{ik}\delta^h_a + F_{ak}F^h_i - F_{ik}F^h_a + 2F_{ai}F^h_k)]$$

In view of equation (1.9) then equation (2.13) reduces in the form

$$(2.14) \quad \nabla_a U^h_{ijk} + \nabla_i U^h_{jak} + \nabla_j U^h_{aik} = \nabla_a E^h_{ijk} + \nabla_i E^h_{jak} + \nabla_j E^h_{aik}$$

By virtue of equations (2.14) and (2.9), we obtain

$$\nabla_a U^h_{ijk} + \nabla_i U^h_{jak} + \nabla_j U^h_{aik} = 0$$

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