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Approximation Solution to a Three Dimensional Free Convective Flow of Hydromagnetic Porous Plate

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Abstract

An analytical solution to the problem of the three-dimensional free convection flow of an incompressible, viscous, electrically conducting fluid past an infinite, vertical porous plate with transverse sinusoidal suction velocity is presented. A uniform magnetic field is assumed to be applied transversely to the direction normal to the plate. The velocity components and temperature components in the case of noble gases (prandtl number $(p_r) = 0.5$ aprrox.) are presented in graphs. The effects of different physical parameters like Hartmann number(M), Prandtl number(p_r), and Grashof number (G) are discussed and the results obtained are physically interpreted.

Key Words: Three Dimensional Free Convective Flow**,** Hydromagnetic Porous Plate

Introduction:

Many natural phenomena and technological problem are easily affected to MHD analysis. Forced, mixed and free convection flows and heat transfer in fluid saturated porous media are encountered in many geophysical and engineering applications. Geophysical applications are thermally enhanced oil recovery energy storage. Pore water convection near salt domes (for the storage of nuclear wastes) and moment of contaminants in ground water. From technological point of view, MHD convection flow problems are very significant in the fields of stellar and planetary magnetospheres, aeronautics, chemical engineering, electronics and geophysics. Engineers employ MHD principle, in the design of heat exchangers pumps and flow meters, in space vehicle propulsion thermal protection, braking, control and re-entry, in creating novel power generating system etc. Many investigators have studied the phenomena of MHD convection$[1-4]$, MHD free convection, mass transfer flow and radiation $[5-10 \; \& \; 18\,]$ are worth mentioning.

An investigation of the problems of laminar flow control has become very important in field of Aeronautical Engineering in view of its applications to reduce drag and hence the vehicle power requirement by a substantial amount. The effect of the flow of an electrically conducting and radiating fluid over a moving heated porous plate in the presence of induced magnetic field has been investigated by C.Israll- cookey and C.Nevaigeve$[11]$ (2010)

Singh et.al (1978)$[12]$ has been investigated the effect of the flow caused by the periodic suction velocity perpendicular to the main flow when the difference between the wall temperature and free stream temperature gives rise to buoyancy force in the direction of the free stream on heat transfer characteristics. Ahmed and Sarma (1997) $[13]$ have extended the work of Singh et.al (1978) to the case when the medium is porous. Gupta and Johari (2001) $[14]$ have analyzed the effect of magnetic field on the three-dimensional forced flow of an incompressible viscous fluid past a porous plate. Singh and Sharma (2001)$[15]$ have studied the effect of the periodic permeability on the free convective flow of a viscous incompressible fluid through a highly porous medium. M.C. Gorla and K.D. Singh (2005)$[17]$ have discussed free convection flow of a viscous; incompressible fluid past an impulsively starred infinite, vertical porous limiting surface with transverse sinusoidal suction when the free stream velocity oscillates in time about a constant mean.

In this paper, we have studied the free convection flow of an electrically conducting viscous incompressible fluid past an infinite vertical porous plate with a slightly sinusoidal transverse suction velocity distribution in presence of uniform magnetic field.

Mathematical Analysis:

We consider the flow of a viscous, incompressible and electrically conducting fluid past an infinite, vertical porous plate subjected to a slightly sinusoidal transverse suction velocity in the presence of a uniform magnetic field is considered. The sinusoidal suction velocity distribution at the plate is consider ed to be of the form

$v(z) = -v_0 \left(1 + \varepsilon \, Cos \, \pi \frac{z}{v} \right)$

Which consists of a basic steady distribution $v_0 > 0$ superimposed with a very weak distribution $\epsilon \, v_0 \, Cos \, \pi \, \frac{z}{v}$. So the amplitude ε of the suction velocity variation is assumed to be small and negative sign indicates that the suction is towards the plate. We choose the $x' - z'$ plane such that x axis is taken along the plate in the direction of flow and y'-axis is perpendicular to the plane of plate and directed into the fluid which is flowing with the free stream velocity U_0. Denoting velocity components u, v, w in the direction x, y, z and temperature by$T$.
Then under the usual Boussineque's approximation the non dimensional equation governing the problem are given by:-

Equation of continuity $\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ (1)

x Component of momentum equation

$$v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = G\theta + \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - M \left(u - 1 \right) \quad (2)$$

Y component of momentum equation

$$v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = - \frac{\partial p}{\partial y} + \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (3)$$

Z component of momentum equation

$$v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = - \frac{\partial p}{\partial z} + \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - Mw \quad (4)$$

Energy equation

$$v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} = \frac{1}{p} \left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) \quad (5)$$

With relevant boundary conditions

$$y = 0 : u = 0, v = -\left(1 + \epsilon \, Cos \, \pi \, z \right), w = 0, \theta = 1 \quad (6)$$

$$y \to \infty : u = 1, v = -1, w = 0, p = p_\infty, \theta = 0 \quad (7)$$

Method of solutions:

We assume the solutions of the equations (1) to (5) to be of the form:

$$u = u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \cdots \quad (8)$$
$$v = v_0 + \varepsilon v_1 + \varepsilon^2 v_2 + \cdots \quad (9)$$
$$w = w_0 + \varepsilon w_1 + \varepsilon^2 w_2 + \cdots \quad (10)$$
$$p = p_0 + \varepsilon p_1 + \varepsilon^2 p_2 + \cdots \quad (11)$$
$$\theta = \theta_0 + \varepsilon \theta_1 + \varepsilon^2 \theta_2 + \cdots \quad (12)$$

Substituting these in the equations (1) to (5) and equating the co-efficient of same degree terms and neglecting ε^2 we get the following sets of the differential equations:

Zeroth-order equations:

$$\frac{\partial v_0}{\partial y} = 0 \quad (13)$$
$$\frac{\partial^2 u_0}{\partial y^2} - v_0 \frac{\partial u}{\partial y} - M u_0 = -G\theta_0 - M \quad (14)$$
$$\frac{\partial^2 v_0}{\partial y^2} - v_0 \frac{\partial v_0}{\partial y} = \frac{\partial p_0}{\partial y} \quad (15)$$
$$\frac{\partial^2 w}{\partial y^2} - v_0 \frac{\partial w_0}{\partial y} - M w = 0 \quad (16)$$
$$\frac{\partial^2 \theta_0}{\partial y^2} - v_0 p_r \frac{\partial \theta_0}{\partial y} - M u_0 = 0 \quad (17)$$

First order equations:

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \quad (18)$$

$$v_1 \frac{\partial u_0}{\partial y} - \frac{\partial u_1}{\partial y} = G\theta_1 + \left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) - M^2 u_1 \quad (19)$$

$$-\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right) \quad (20)$$

$$-\frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) - M w_1 \quad (21)$$

$$v_1 \frac{\partial \theta_0}{\partial y} - \frac{\partial \theta_1}{\partial z} = \frac{1}{p_r}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) \quad (22)$$

With boundary conditions

$$y = 0 : u_0 = 0, v_0 = -1, w_0 = 0, \theta_0 = 1, u_1 = 0, v_1 = -Cos\pi z, w_1 = 0, \theta_1 = 0 \quad (23)$$

$$y \to \infty : u_0 = 1, v_0 = -1, w_0 = 0, p_0 = p_\infty, \theta_0 = 0, u_1 = 0, v_1 = -1, w_1 = 0, p_1 = 0, \theta_1 = 0 \quad (24)$$

The solutions of the equations (13) to (17) under the boundary conditions (23) and (24) respectively

$$u_0 = 1 - e^{-r_1 y} + G_1 \left(e^{-r_1 y} - e^{-p_r y} \right) \qquad (25)$$
$$\theta_0 = e^{-p_r y} \qquad (26)$$

With $v_0 = -1, w_0 = 0, p_0 = p_\infty$ (27)

Where $r_1 = \frac{1}{2}\left[1 + (1 + 4M)^{1/2}\right]$

$$G_1 = \frac{G}{(p_r^2 - p_r) - M}$$

This is the solution of the problem of two dimensional free convective flow past a porous vertical plate with constant suction and transversely applied uniform magnetic field.

Cross flow solution:

We shall first consider the equation (18), (20) and (21) for $v_1(y,z), w_1(y,z)$ and $p_1(y,z)$ which are independent of main flow component u_1 and temperature field θ_1. The suction velocity consists of a basic uniform distribution with superimposed weak sinusoidal distribution.
Hence the velocity components v, w and p are also separated into mean and small sinusoidal components v_1, w_1 and p_1. We assume v_1, w_1 and p_1 to be of the following form.

$$v_1(y,z) = v_{11}(y) \,Cos\, \pi z \qquad (29)$$

$$w_1(y,z) = w_{11}(y) \,Sin\, \pi z \qquad (30)$$

$$p_1(y,z) = p_{11}(y) \,Cos\, \pi z \qquad (31)$$

On substitution of (29), (30) and (31), the equation (18) is satisfied and the equation (20) and (21) reduce to the following differential equations:-

$$v_{11}'' + v_{11}' - \pi^2 v_{11} = p_{11}' \qquad (32)$$

$$v_{11}''' + v_{11}'' - (\pi^2 + M)\, v_{11}' = \pi^2 p_{11} \qquad (33)$$

The relevant boundary conditions for these equations are:-

$$y = 0 : \; v_{11} = -1 \; v_{11}' = 0 \qquad (34)$$

$$y \to \infty : \; v_{11} = 0 \; p_{11} = 0 \qquad (35)$$

Solutions for flow and temperature field:

We shall now consider the equation (19), and (22). The solutions for the velocity component u and temperature field θ are also separated into mean and sinusoidal components u_1, θ_1.To reduce the partial differential equations (19) and (22) into ordinary differential equations, we consider for the following forms for u_1 and θ_1.

$$u_1\,(y,z) = u_{11}\,(y)\,Cos\,\pi z \tag{36}$$

$$\theta_1\,(y,z) = \theta_{11}\,(y)\,Cos\,\pi z \tag{37}$$

Using the expressions for u_1, v_1 and θ_1 in (19) and (22), we get the following ordinary differential equations:

$$u_{11}'' + u_{11}' - (\pi^2 + M)\,u_{11} = v_{11}\,u_0' - G\,\theta_{11} \tag{38}$$

$$\theta_{11}'' + p_r\,\theta_{11}' - \pi^2\,\theta_{11} = p_r\,v_{11}\,\theta_0' \tag{39}$$

With boundary conditions

$$y = 0 : \quad u_{11} = 0, \theta_{11} = 0 \tag{40}$$

$$y \to \infty: u_{11} = 0, \theta_{11} = 0 \tag{41}$$

Solving these equations and using (40) and (41) we get

$$u_1 = \frac{1}{r_2 - r_2'}\left[\begin{array}{c} (1 - G_1)\left\{\begin{array}{c} C_1\left(e^{-ny} - e^{-(r_1 + r_2')y}\right) - \\ C_2\left(e^{-ny} - e^{-(r_1 + r_2)y}\right) \end{array}\right\} \\ + C_3\left(e^{-ny} - e^{-my}\right) - C_4\left(e^{-ny} - e^{-(r_2 + 1 + p_r)y}\right) \\ + C_5\left(e^{-ny} - e^{-(r_2 + p_r)y}\right) \end{array}\right] Cos\,\pi z \tag{42}$$

$$\theta_1 = \frac{p_r^2}{r_2 - r_2'}\left[\frac{r_2'\left(e^{-my} - e^{-(r_2 + p_r)y}\right)}{r_2\,(r_1 + p_r)} - \frac{r_2\left(e^{-my} - e^{-(r_2' + p_r)y}\right)}{r_2'\,(r_1' + p_r)}\right] Cos\,\pi z \tag{43}$$

Where

$$r_1' = \frac{1}{2}\left(1 - (1 + 4M)^{1/2}\right)$$

$$r_2 = \frac{1}{2}\left(r_1 - (r_1^2 + 4\pi^2)^{1/2}\right)$$

$$r_2' = \frac{1}{2}\left(r_1' - \left(r_1'^2 + 4\pi^2\right)^{1/2}\right)$$
$$n = \frac{1}{2}\left(1 + \{1 + 4(\pi^2 + M)\}^{1/2}\right)$$
$$m = \frac{1}{2}\left[p_r + (p_r^2 + 4\pi^2)^{1/2}\right]$$
$$C_1 = \frac{r_2}{r_2'}$$

$$C_2 = \frac{r_1 r_2'}{r_2(3r_1 - 1)}$$

$$C_3 = \frac{G p_r^2}{m(p_r - 1) - M}\left[\frac{r_2'}{r_2(r_1 + p_r)} - \frac{r_2}{r_2'(r_1' + p_r)}\right]$$

$$C_4 = \frac{p_r r_2'}{A}\left[G_1 - \frac{G p_r}{r_2(r_1 + p_r)}\right]$$

$$C_5 = \frac{p_r r_2}{B}\left[G_1 - \frac{G p_r}{r_2'(r_1' + p_r)}\right]$$

$$A = p_r[(p_r - 1) + 2r_2] + [r_2(r_1 - 1) - M]$$

$$B = p_r\left[(p_r - 1) + 2r_2'\right] + \left[r_2'\left(r_1' - 1\right) - M\right]$$

Skin friction and heat transfer:

The non dimensional skin- friction in the direction of the free stream at the wall $y = 0$ is given by

$$\tau = \frac{\tau'}{\rho' u_0^2} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = u_0'(0) + \varepsilon u_1'(0)$$

$$\tau = r_1 + G_1(p_r - r_1) + \frac{\varepsilon}{r_2 - r_2'}\left[\begin{array}{c} (1 - G_1) \\ \left\{\begin{array}{l} C_1(r_1 + r_2' - n) \\ -C_2(r_1 + r_2 - n) \end{array}\right\} \\ + C_3(m - n) \\ - C_4(p_r + r_2 - n) \\ + C_5(p_r - r_2' - n) \end{array}\right] \mathrm{Cos}\,\pi z \qquad (44)$$

The rate of heat transfer q is given by

$$q = \frac{q' v}{u_0 k}(T_w' - T_\infty') = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = -[\theta_0'(0) + \varepsilon\,\theta_1'(0)] = N_u$$

$$= p_r - \frac{\varepsilon p_r^2}{r_2 - r_2'} \left[\frac{r_2'(p_r + r_2 - m)}{r_2(p_r + r_1)} - \frac{r_2(p_r + r_2' - m)}{r_2'(p_r + r_1')} \right] Cos\pi z \qquad (45)$$

Discussion:

The effect of velocity profile has been shown in fig.1. It is observed that velocity increases with the increases of Grashof number G in the case of mix of noble gases ($p_r = 0.5$).

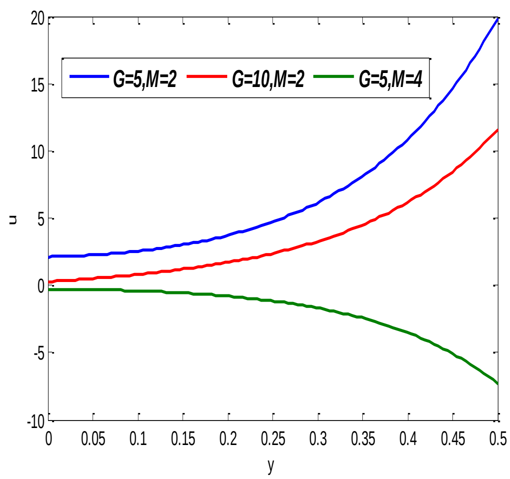

Fig:1 Velocity Profile for Z=0

In the fig.2 temperature_decreases with increasing Hartmann number M.

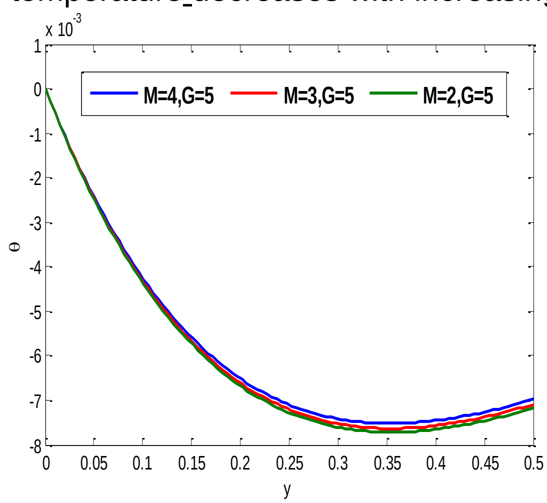

Fig:2 Temperature Profile for Z = 0

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