Mathematica Aeterna, Vol. 3, 2013, no. 6, 489 - 496

PLANE GRAVITATIONAL WAVES WITH COSMIC STRINGS IN BIMETRIC RELATIVITY

S.D. Deo
Head,Department of Mathematics
N.S.Science and Arts college,
Bhadrawati,Dist-Chandrapur,(M.S.)
442902, India.
E-mail: s_deo01@yahoo.in

Sulbha R. Suple
159, Parvati Nagar
Rameshwari Road,
Nagpur (M.S.)
440027, India.
E-mail: rajsulu4762@rediffmail.com

ABSTRACT

In this paper, we will study $Z = \left(\frac{t}{\sqrt{x^2 + y^2 + z^2}} \right)$ type higher five dimensional and n-dimensional plane gravitational wave with the matter cosmic strings in the context of Rosen's Bimetric theory of relativity and observed that the cosmic strings does not exist in this theory.

AMS: 83C05 (General relativity)

Keywords: - Plane gravitational waves, cosmic strings, Bimetric Relativity

1. INTRODUCTION

The general theory of relativity is one of the most beautiful structures in all theoretical physics. In an attempt to get rid of the singularities appear in the Einstein's General Theory of Relativity

(GR), Rosen [1] [2] (1940, 1973) has proposed a modified theory of gravitation within the frame work of general relativity which is called Bimetric Theory of Relativity (BR).

In this theory he has proposed a new formulation of the general relativity by introducing a background Euclidean metric tensor γ_{ij} in addition to the usual Riemannian metric tensor g_{ij} at each point of the four dimensional space-time. With the flat background metric γ_{ij} the physical content of the theory is the same as that of the general relativity.

Thus, now the corresponding two line elements in a coordinate system x^i are –

$$ds^2 = g_{ij} dx^i dx^j \tag{1.1}$$

And
$$d\sigma^2 = \gamma_{ij} dx^i dx^j \tag{1.2}$$

Where ds is the interval between two neighboring events as measured by means of a clock and a measuring rod. The interval $d\sigma$ is an abstract or geometrical quantity not directly measurable. One can regard it as describing the geometry that would exist if no matter were present.

H Takeno (1961) [3] propounded a rigorous discussion of plane gravitational waves, defined various terms by formulating a meaningful mathematical version and obtained numerous results.

A fairly general case of "plane" gravitational wave is represented by the metric

$$ds^2 = -Adx^2 - 2Ddxdy - Bdy^2 - dz^2 + dt^2 \tag{1.3}$$

both for weak field approximation and for exact solutions of Einstein field equations.

Reformulating Takeno's (1961) [3] definition of plane wave, Z = $\left(\frac{t}{\sqrt{x^2 + y^2 + z^2}}\right)$ type higher five dimensional and n-dimensional gravitational waves are defined by using the line elements,

$$ds^2 = \frac{-3At^2}{(x^2 + y^2 + z^2)^2}\left(x^2 dx^2 + y^2 dy^2 + z^2 dz^2\right) - Bdu^2 + Adt^2 \tag{1.4}$$

$$ds^2 = \frac{-3At^2}{(x^2 + y^2 + z^2)^2}\left(x^2 dx^2 + y^2 dy^2 + z^2 dz^2\right) - \sum_{i=4}^{n-1} Bdu^{i^2} + Adt^2 \tag{1.5}$$

Mohseni, Tucker and Wang [4] have studied the motion of spinning test particles in plane gravitational waves. S Kessari, D Singh et al [5], analyzed the motion of electrically

neutral massive spinning test particle in the plane gravitational and electromagnetic wave background.
The theory of plane gravitational waves have been studied by many investigators , H Takeno [6]; Pandey [7]; Lal and Shafiullah [8];Lu Hui qing [9];Bondi, H. et.al.[10], Torre,C.G.[11]; Hogan, P.A.[12];Deo and Ronghe[13],[14] and they obtained the solutions .

In this paper,we will study $Z = \left(\frac{t}{\sqrt{x^2 + y^2 + z^2}} \right)$ type higher five dimensional and n-dimensional plane gravitational wave with the matter cosmic strings and will observe the result in the context of Bimetric theory of relativity.

2. FIELD EQUATIONS IN BIMETRIC RELATIVITY

Rosen N. has proposed the field equations of Bimetric Relativity from variation principle as

$$K_i^j = N_i^j - \tfrac{1}{2} N g_i^j = -8\pi\kappa T_i^j \tag{2.1}$$

where $$N_i^j = \tfrac{1}{2} \gamma^{\alpha\beta} \left[g^{hj} g_{hi}|_{\alpha} \right]|_{\beta} \tag{2.2}$$

$$N = N_\alpha^\alpha \qquad \kappa = \sqrt{\frac{g}{\gamma}} \tag{2.3}$$

and $$g = |g_{ij}| \qquad , \gamma = |\gamma_{ij}| \tag{2.4}$$

Where a vertical bar (|) denotes a covariant differentiation with respect to γ_{ij}.

And , T_i^j the energy momentum tensor for cosmic cloud strings is given by

$$T_i^j = T_i^j{}_{\text{strings}} = \rho v_i v^j - \lambda x^i x^j \tag{2.5}$$

Here ρ is the rest energy density for a cloud with particle attached along the extension, thus $\rho = \rho_p + \lambda$ where ρ_p is the particle energy density, λ is the tension density of the strings. As pointed out by Letelier[15] λ may be positive or negative and v_i is the flow vector of matter.

The flow of the matter is taken orthogonal to the hyper-surface of homogeneity so that $v_5 v^5 = -1$, $v_n v^n = -1$ for five and n-dimensional space and x^i representing the direction vector of anisotropy ie $x_3 x^3 = 1$ and $v_i x^i = 0$

3. Z = $\left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)$ type higher five dimensional plane gravitational wave with cosmic strings

For Z = $\left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)$ plane gravitational wave in higher five dimensions, we have the line element as

$$ds^2 = \frac{-3At^2}{(x^2+y^2+z^2)^2}\left(x^2dx^2 + y^2dy^2 + z^2dz^2\right) - Bdu^2 + Adt^2 \tag{3.1}$$

Where $A = A(Z)$, B = B (Z) and Z= $\left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)$

Corresponding to the equation (3.1),we consider the line element for background metric γ_{ij} as

$$d\sigma^2 = -(dx^2 + dy^2 + dz^2 + du^2) + dt^2 \tag{3.2}$$

Since γ_{ij} is the Lorentz metric i.e. (-1,-1,-1,-1, 1),therefore γ- covariant derivative becomes the ordinary partial derivative.

Using equations (2.1) to (2.5) with (3.1) and (3.2), we get,

$$\left\{\frac{1}{2}\left(\frac{\bar{A}^2}{A^2} - \frac{\bar{\bar{A}}}{A}\right) + \frac{1}{4}\left(\frac{\bar{B}^2}{B^2} - \frac{\bar{\bar{B}}}{B}\right)\right\} D = 0 \tag{3.3}$$

$$\left\{\frac{1}{2}\left(\frac{\overline{A}^{2}}{A^{2}}-\frac{\overline{\overline{A}}}{A}\right)+\frac{1}{4}\left(\frac{\overline{B}^{2}}{B^{2}}-\frac{\overline{\overline{B}}}{B}\right)\right\}D=-8\pi\kappa\lambda \tag{3.4}$$

$$\left\{\left(\frac{\overline{A}^{2}}{A^{2}}-\frac{\overline{\overline{A}}}{A}\right)-\frac{1}{4}\left(\frac{\overline{B}^{2}}{B^{2}}-\frac{\overline{\overline{B}}}{B}\right)\right\}D=0 \tag{3.5}$$

$$\left\{\frac{1}{2}\left(\frac{\overline{A}^{2}}{A^{2}}-\frac{\overline{\overline{A}}}{A}\right)+\frac{1}{4}\left(\frac{\overline{B}^{2}}{B^{2}}-\frac{\overline{\overline{B}}}{B}\right)\right\}D=-8\pi\kappa\rho \tag{3.6}$$

where $D=\left[\frac{t^{2}-(x^{2}+y^{2}+z^{2})}{(x^{2}+y^{2}+z^{2})^{2}}\right]$

and $\overline{A}=\frac{\partial A}{\partial Z}$, $\overline{\overline{A}}=\frac{\partial^{2}A}{\partial Z^{2}}$

Using equation (3.3) to (3.6) we have

$$\lambda=0=\rho \tag{3.7}$$

It gives us nil contribution of cosmic cloud string in Z = $\left(\frac{t}{\sqrt{x^{2}+y^{2}+z^{2}}}\right)$ type higher five dimensional plane gravitational wave in the theory of Bimetric relativity.

Hence for vacuum case $\lambda=0=\rho$, the field equation reduced to

$$\left(\frac{\overline{A}^{2}}{A^{2}}-\frac{\overline{\overline{A}}}{A}\right)D=0$$

ie

$$\left(\frac{\overline{A}^{2}}{A^{2}}-\frac{\overline{\overline{A}}}{A}\right)=0 \tag{3.8}$$

Solving equations (3.8), we have

$$A=R_{1}\,e^{S_{1}Z} \tag{3.9}$$

Now using (3.8) in (3.3), we have

$$B = R_2\, e^{S_2 Z} \tag{3.10}$$

where R_1 , S_1 and R_2, S_2 are the constants of integration.

Thus substituting the value of (3.9) and (3.10) in (3.1), we get the vacuum line element as

$$ds^2 = \frac{-3R_1 e^{S_1 Z} t^2}{(x^2+y^2+z^2)^2}\left(x^2dx^2 + y^2dy^2 + z^2dz^2\right) - R_2 e^{S_2 Z} du^2 + R_1\, e^{S_1 Z} dt^2 \tag{3.11}$$

4. Z = $\left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)$ type n-dimensional plane gravitational wave with Cosmic strings

For Z = $\left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)$ plane gravitational waves in n-dimensions, we have the line element as

$$ds^2 = \frac{-3At^2}{(x^2+y^2+z^2)^2}\left(x^2dx^2 + y^2dy^2 + z^2dz^2\right) - \sum_{i=4}^{n-1} Bdu^{i^2} + Adt^2 \tag{4.1}$$

Where $A = A(Z)$, B=B (Z) and Z = $\left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)$

Corresponding to the equation (4.1), we consider the line element for background metric γ_{ij} as

$$d\sigma^2 = -\left(dx^2 + dy^2 + dz^2 + \text{............}(n-1) terms\right) + dt^2 \tag{4.2}$$

Using equations (2.1) to (2.5) with (4.1) and (4.2),

We get the field equations as

$$\left\{\frac{1}{2}\left(\frac{\overline{A}^2}{A^2} - \frac{\overline{\overline{A}}}{A}\right) + \left(\frac{n-4}{4}\right)\left(\frac{\overline{B}^2}{B^2} - \frac{\overline{\overline{B}}}{B}\right)\right\} D = 0 \tag{4.3}$$

$$\left\{\frac{1}{2}\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\left(\frac{n-4}{4}\right)\left(\frac{\bar{B}^2}{B^2}-\frac{\bar{\bar{B}}}{B}\right)\right\}D=-8\pi\kappa\lambda \tag{4.4}$$

$$\left\{\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\left(\frac{n-6}{4}\right)\left(\frac{\bar{B}^2}{B^2}-\frac{\bar{\bar{B}}}{B}\right)\right\}D=0 \tag{4.5}$$

$$\left\{\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\left(\frac{n-6}{4}\right)\left(\frac{\bar{B}^2}{B^2}-\frac{\bar{\bar{B}}}{B}\right)\right\}D=0 \tag{4.6}$$

$$\left\{\frac{1}{2}\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\left(\frac{n-4}{4}\right)\left(\frac{\bar{B}^2}{B^2}-\frac{\bar{\bar{B}}}{B}\right)\right\}D=-8\pi\kappa\rho \tag{4.7}$$

where $D=\left[\frac{t^2-(x^2+y^2+z^2)}{(x^2+y^2+z^2)^2}\right]$ and $\bar{A}=\frac{\partial A}{\partial Z}$, $\bar{\bar{A}}=\frac{\partial^2 A}{\partial Z^2}$

Using equation (4.3) to (4.7), we get

$$\lambda=0=\rho \tag{4.8}$$

that is cosmic cloud string does not exist in $Z=\left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)$ plane gravitational waves in Rosen's Bimetric theory of relativity.

Hence for vacuum case $\lambda=0=\rho$, we obtained the same result as defined in (3.9) and (3.10).

Thus substituting the value of (3.9) and (3.10) in (4.1), we get the vacuum line element as

$$ds^2=\frac{-3R_1 e^{S_1Z}t^2}{(x^2+y^2+z^2)^2}\left(x^2dx^2+y^2dy^2+z^2dz^2\right)-\sum_{i=4}^{n-1}R_2e^{S_2Z}du^{i^2}+R_1 e^{S_1Z}dt^2 \tag{4.9}$$

5. CONCLUSION

In the study of $\mathbf{Z} = \left(\frac{t}{\sqrt{x^2 + y^2 + z^2}}\right)$ type higher five dimensional and n-dimensional plane gravitational waves, there is no contribution of cosmic cloud strings in Bimetric theory of relativity respectively.

ACKNOWLEDMENT:-The authors are thankful to Dr. R. D. Giri, Prof. Emirates, P.G.T.D. (Mathematics), R. T. M. N. U., Nagpur, India for his constant inspiration.

REFERENCES:

[1] Rosen N General Relativity and Flat space I. Phys. Rev. (1940) 57, 147.

[2] Rosen N. A bimetric theory of gravitation I Gen. Rela. Grav. (1973) 04, 435-47.

[3] Takeno H. The mathematical theory of plane gravitational waves in General Relativity. Scientific report of Research Institute for theoretical Physics, Hiroshima University, Hiroshima, Ken, Japan (1961).

[4] Mohseni, M .; Tucker, R. W. ; Wang, C. On the motion of spinning test particles in plane gravitational waves. Class. Quantum Grav. (2001), 18 3007-3017

[5] Kessari, S ; Singh, D. et al. Scattering of spinning test particles by plane gravitational and Electromagnetic waves. gr-qc/0203038, Class.Quant.Grav.,(2002). 19 4943-4952

[6] Takeno, H A comparison of plane wave solutions in general relativity with those in non-symmetric theory.Prog.Theor.Phys. (1958).20, 267-276

[7] Pandey, S. N. Plane wave solutions in Finzi's non-symmetric unified field theory. Theo. Math. Phys. (1979).39, 371-375

[8] Lal, K. B. ; Shafiullah, On plane wave solutions of non – symmetric field equations of unified theories of Einstein Bonner and Schrödinger . Annali de Mathematica ed pura Applicata., (1980). 126, 285-298.

[9] Lu Hui quing. Plane gravitational waves under a non - zero cosmological constant. Chin. Astronomy Astrophys. (1988) 12, 186-190.

[10] Bondi, H. ; Pirani, F.A.E. and Robinson, I Gravitational waves in general relativity III. Exact plane waves. Proc. Roy.Soc.Lond. .(1959). A23, 25, 519-533

[11] Torre, C.G. Gravitational waves – Just plane symmetry. Gen. Rela. Grav. (2006) 38, 653-662

[12] Hogan, P.A Gravitational waves and Bertotti-Robinson space- time. Math. Proc. Roy. Irish Acad. .(1999), 99A, 51-55.

[13] Ronghe A.K. and Deo S.D. - Plane Gravitational Waves In Bimetric Relativity. JVR(2011) 6,1-11

[14] Ronghe A.K. and Deo S.D. Plane gravitational waves with wet dark energy. International Journal of Mathematical Archive-(2011) 2(3) Mar.-391-392

[15]Letelier P.S. Phys. Rev.D 28(1983)2414