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Lacunary series in some weighted meromorphic function spaces

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Abstract

In this paper, we study meromorphic functions $f^{\#}$ on the unit disc $\mathbb{D}$ with Hadamard gaps, under some conditions posed on the funtions.

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1 Introduction

Let $\Delta = \{z \in \mathbb{C} : |z| < 1\}$ be the unit disk and $T = \{z_1 \in \mathbb{C} : |z_1| = 1\}$ be the unit circle in the complex plane $\mathbb{C}$; $\partial\Delta$ it's boundary, $dA(z)$ be the normalized area measure on the unit disk Δ, so that $A(\Delta) = 1$ and $d\theta$ be the Lebesgue measure on the unit circle T. Let $H(\Delta)$ and $M(\Delta)$ denote the classes of functions holomorphic and meromorphic in Δ, respectively. The Green's function of Δ with logarithmic singularity at $a \in \Delta$ is denoted by $g(z,a) = \log|\frac{1-\bar{a}z}{a-z}|$. Let $Aut(\Delta)$ be the group of all conformal mapping from Δ onto itself (also called disk automorphisms of Δ). It is well known that $Aut(\Delta)$ coincides with the set of all Möbius transformations of Δ onto itself:

$$Aut(\Delta) = \{\lambda\varphi_a : |\lambda| = 1, a \in \Delta\}.$$

Recall that the well known Bloch space (cf.[2]) is defined as follows:

$$\mathcal{B} = \{f : f\text{analytic in}\Delta\text{and } \sup_{z\in\Delta}(1-|z|^2)|f'(z)| < \infty\}$$

and the little Bloch space $\mathcal{B}_0$ (cf. [2]) is given as follows

$$\lim_{|z|\to 1^-}(1-|z|^2)|f'(z)| = 0.$$

Let $0 < p < \infty$. Then the Q_p-type spaces consist of analytic functions on Δ such that

$$\mathbf{Q_p} = \Big\{ f : f \text{ analytic in } \Delta \text{ and } \sup_{a\in\Delta} \int_{\Delta} |f'(z)|^2 g^p(z,a)\, dA(z) < \infty \Big\},$$

this definition is equivalent to

$$\sup_{a\in\Delta} \int_{\Delta} |f'(z)|^2 (1 - |\varphi_a(z)|^2)^p\, dA(z) < \infty$$

these classes are introduced and studied intensively in ([2, 3]).
Now, we give some definitions of different classes of meromorphic functions which recently have been studied intensively in the theory of complex function spaces, while the theory of such spaces like the class of normal functions $\mathcal{N}$, $\mathcal{B}^{\#}, Q_p^{\#}$ and $Q_K^{\#}$ spaces. For a meromorphic function f, a natural counterpart of the derivative $|f'(z)|$ of analytic case is *the spherical derivative* $f^{\#}(z)$ defined by

$$f^{\#}(z) = \frac{|f'(z)|}{1 + |f(z)|^2}.$$

The meromorphic counterpart of the Bloch space is the class of normal functions $\mathcal{N}$, which is defined as follows:

Definition 1.1 *(see [1, 31]) The class of normal functions is defined by*

$$\mathcal{N} = \{f \in M(\Delta) : \|f\|_{\mathcal{N}} = \sup_{a\in\Delta} (1 - |z|^2) f^{\#}(z) < \infty\}.$$

Moreover, the class of little normal functions is defined by

$$\mathcal{N}_0 = \{f \in M(\Delta) : \|f\|_{\mathcal{N}_0} = \lim_{|a|\to 1} (1 - |z|^2) f^{\#}(z) = 0\}.$$

For a point $a \in \Delta$ and $0 < r < 1$, the pseudo-hyperbolic disk $\Delta(a, r)$ with pseudo-hyperbolic center a and pseudo-hyperbolic radius r is defined by $\Delta(a, r) = \varphi_a(r\Delta)$.
The pseudo-hyperbolic disk $\Delta(a, r)$ is also an Euclidean disk: its Euclidean center and Euclidean radius are $\frac{(1-r^2)a}{1-r^2|a|^2}$ and $\frac{(1-|a|^2)r}{1-r^2|a|^2}$, respectively (see [32]).

Definition 1.2 *(see [22]) For some $r \in (0, 1)$ the class of spherical Bloch functions $\mathcal{B}^{\#}$ is defined by*

$$\mathcal{B}^{\#} = \{f \in M(\Delta) : \sup_{a\in\Delta} \int_{\Delta(a,r)} (f^{\#}(z))^2 dA(z) < \infty\}.$$

We clearly have $\mathcal{N} \subset \mathcal{B}^{\#}$. Yamashita in [35] has proved that there is an essential difference between $\mathcal{N}$ and $\mathcal{B}^{\#}$.

Definition 1.3 *(see [33]) Let $0 < p < \infty$. Then the meromorphic $Q_p^{\#}$ define by*

$$\mathbf{Q_p^{\#}} \;=\; \Big\{f : f \text{ meromorphic in } \Delta \text{ and } \sup_{a\in\Delta}\int_{\Delta} \big(f^{\#}(z)\big)^2 g^p(z,a)\, dA(z) < \infty\Big\},$$

Now, we give the following definition:

Definition 1.4 *Let $0 < p < \infty$, for a function $\omega : (0,1] \to (0,\infty)$, The meromorphic $Q_{p,\omega}^{\#}$ is define by*

$$\mathbf{Q_{p,\omega}^{\#}} \;=\; \Big\{f : f \text{ meromorphic in } \Delta \text{ and } \sup_{a\in\Delta}\int_{\Delta} \big(f^{\#}(z)\big)^2 \frac{(1-|\varphi_a(z)|)^p}{\omega(1-|z|)}\, dA(z) < \infty\Big\},$$

Thought this paper $\omega : (0,1] \to (0,\infty)$, stands for a nondecreasing right continuous function.

It should be remarked that there are some papers used the weight function ω to study some classes of function spaces, for more details, we refer to [10, 12, 16, 17, 18, 19, 30, 31].

Two quantities A_f and B_f, both depending on an analytic function f on Δ, are said to be equivalent, written as $A_f \approx B_f$, if there exists a finite positive constant C not depending on f such that for every analytic function f on Δ we have:

$$\frac{1}{C}B_f \le A_f \le CB_f.$$

If the quantities A_f and B_f, are equivalent, then in particular we have $A_f < \infty$ if and only if $B_f < \infty$.

Let $f(z)$, $g(z) \in H(\Delta)$ with Taylor series expansions given by

$$f(z) = \sum_{n=0}^{\infty} a_n z^n \text{ and } g(z) = \sum_{n=0}^{\infty} b_n z^n,$$

then, the Hadamard product of $f(z)$ and $g(z)$ is defined by

$$(f * g)(z) = \sum_{n=0}^{\infty} a_n b_n z^n.$$

We will need the following two lemmas in the sequel:

Lemma 1.5 *Let $0 < p < \infty$. If $\{n_k\}$ is an increasing sequence of positive integers satisfying $\frac{n_{k+1}}{n_k} \ge \lambda > 1$ for all k, then there is a constant A depending only on p and λ such that*

$$A^{-1}\Big(\sum_{k=1}^{\infty}|a_k|^2\Big)^{\frac{1}{2}} \le \Big(\frac{1}{2\pi}\int_0^{2\pi}\Big|\sum_{k=1}^{\infty} a_k e^{in_k\theta}|^p d\theta\Big)^{\frac{1}{p}} \le A\Big(\sum_{k=1}^{\infty}|a_k|^2\Big)^{\frac{1}{2}}$$

for any number $a_k (k = 1, 2, ...)$.

The above lemma is due to Zygmund [36].

Lemma 1.6 *Let* $\alpha > 0$, $p > 0$, $n \geq 0$, $a_n \geq 0$, $I_n = \{k : 2^n \leq k < 2^{n+1}, k \in \mathbb{N}\}$, $t_n = \sum\limits_{k \in I_n} a_k$ *and* $f(r) = \sum\limits_{n=1}^{\infty} a_n r^n$. *Then there exists a constant* K *depending only on* p *and* α *such that*

$$\frac{1}{K}\sum_{n=0}^{\infty} 2^{-n\alpha} t_n^p \leq \int_0^1 (1-r)^{\alpha-1} f(r)^p \, dr \leq K \sum_{n=0}^{\infty} 2^{-n\alpha} t_n^p.$$

For the proof of Lemma 1.6, we refer to [28].
Remark 1.1 It should be remarked that using simple computations will allow that Lemma 1.6 is still satisfied for the function $f(r) = \sum\limits_{n=1}^{\infty} a_n r^{n-1}$.

Lemma 1.7 *(see [3]) For* $0 < p \leq 1$, $a \in \Delta$ *and* $z = re^{i\theta}$,

$$I_{a,\theta} = \int_0^{2\pi} \frac{d\theta}{|1 - \bar{a} r e^{\theta}|^{2p}} \leq \frac{C}{(1-|a|r)^p}.$$

For our purpose we will use the following inequalities, which follow immediately from Holder's inequality. Let $a_n \geq 0$ and let N be a positive integer. Then for $0 < p \leq 1$,

$$\frac{1}{N^{1-p}}\Big(\sum_{n=1}^{N} a_n^p\Big) \leq \Big(\sum_{n=1}^{N} a_n\Big)^p \leq \Big(\sum_{n=1}^{N} a_n^p\Big); \tag{1}$$

for $1 \leq p < \infty$,

$$\Big(\sum_{n=1}^{N} a_n^p\Big) \leq \Big(\sum_{n=1}^{N} a_n\Big)^p \leq N^{p-1}\Big(\sum_{n=1}^{N} a_n^p\Big). \tag{2}$$

The following lemma is useful in our study

Lemma 1.8 *[34] Let* $\alpha \in (0, \infty)$ *and suppose that* $f(z) = \sum\limits_{j=1}^{\infty} a_j z^{n_j}$ *belongs to Hadamard gap class. Then* $f \in \mathcal{B}^\alpha$ *if and only if*

$$\sup_{j \in N} |a_j| n_j^{1-\alpha} < \infty \ , \quad where \ \ N = \{1, 2, 3, \ldots\}\, .$$

Hadamard gaps are known to study some classes and spaces of holomorphic and hyperholomorphic functions. A wide variety of characterization not only in the type of function spaces, where functions are holomorphic and hyperholomorphic, but also in the coefficients which extend over Taylor or Fourier series expansions. It is one of the important tasks in the study of function spaces to seek for characterizations of functions by the help of their Taylor or Fourier series expansions. In the past few decades both Taylor and Fourier series expansions were studied by the help of Hadamard gap class also called Hadamard's lacunarity condition (see e.g., [4, 7, 8, 11, 13, 25, 27, 29] and others). In this paper some characterizations of the lacunary series that belong to Q_p spaces of meromorphic are obtained under some mild assumptions. One key to the proofs of the main theorems is the paper by [29]. Another key is the recent paper [26, 27].

2 Series expansions

In this section, we obtain characterizations of the meromorphic $Q^{\#}_{p,\omega}$ functions by the coefficients of certain lacunary series expansions in the unit disc. Now, we give the following theorem:

Theorem 2.1 *Let* $0 < p \leq 1$, $I_n = \{k : 2^n \leq k < 2^{n+1}, k \in \mathbb{N}\}$ *and let* $f(z) = \sum\limits_{n=0}^{\infty} a_n z^n$ *be analytic function on* Δ. *Let*

$$\Big(\sum_{k=0}^{\infty} |a_k| r^{n_k}\Big)^2 \approx \sum_{k=0}^{\infty} |a_k| r^{n_k} \tag{3}$$

and

$$\Big(\sum_{n=0}^{\infty} a_n z^n\Big)\Big(\sum_{n=0}^{\infty} b_n z^n\Big) = \sum_{k=0}^{\infty} a_n b_n \sqrt{\omega(1-|z|)}\; z^n. \tag{4}$$

If

$$\sum_{n=0}^{\infty} 2^{n(1-p)} \sum_{k \in I_n} (-1)^{2k} |a_k|^4 < \infty,$$

then $f \in Q^{\#}_{p,0}$, *where*

$$Q^{\#}_{p,0} = \lim_{|a|\to 1} \int_{\Delta} (f^{\#}(z))^2 \frac{(1-|\varphi_a(z)|^2)^p}{\omega(1-|z|)}\; dA(z) = 0.$$

Proof: We have

$$\begin{aligned}
&\int_{\Delta} (f^{\#}(z))^2 \frac{(1-|\varphi_a(z)|^2)^p}{\omega(1-|z|)} dA(z) \\
&\leq \int_{\Delta} \Big(\sum_{n=1}^{\infty} n|a_n||z|^{n-1} \sum_{n=0}^{\infty} (-1)^n \big(\sum_{n=0}^{\infty} |a_n||z|^n\big)^{2n}\Big)^2 \frac{(1-|z|^2)^p (1-|a|^2)^p}{|1-\bar{a}z|^{2p} \omega(1-|z|)} dA(z) \\
&\leq \int_{\Delta} \Big(\sum_{n=1}^{\infty} n|a_n||z|^{n-1} \sum_{n=0}^{\infty} (-1)^n \big(\sum_{n=0}^{\infty} |a_n||z|^n\big)^{2n}\Big)^2 \frac{(1-|z|^2)^p}{|1-\bar{a}z|^p \omega(1-|z|)} dA(z)
\end{aligned}$$

$$\leq \int_0^1 \Big(\sum_{n=1}^{\infty} n|a_n|r^{n-1} \sum_{n=0}^{\infty} (-1)^n (\sum_{n=0}^{\infty} |a_n|r^n)^{2n} \Big)^2 \frac{1-|a|^2)^p(1-r^2)^p}{|1-\bar{a}z|^{2p}} r dr$$
$$\leq \int_0^1 \Big(\sum_{n=1}^{\infty} n|a_n|r^{n-1} \sum_{n=0}^{\infty} (-1)^n (\sum_{n=0}^{\infty} |a_n|r^n)^{2n} \Big)^2 (1-|a|^2)^p(1-r^2)^p (I_{a,\theta})\, dr$$

Using Lemma 1.7, we obtain

$$\int_{\Delta} (f^{\#}(z))^2 \frac{(1-|\varphi_a(z)|^2)^p}{\omega(1-|z|)} dA(z)$$
$$\leq 2^{p+1} C\pi \int_0^1 \Big(\sum_{n=1}^{\infty} n|a_n|r^{n-1} \sum_{n=0}^{\infty} (-1)^n (\sum_{n=0}^{\infty} |a_n|r^n)^{2n} \Big)^2 (1-r)^p r dr,$$

Using (1) and (2), we obtain

$$\int_{\Delta} (f^{\#}(z))^2 (1-|\varphi_a(z)|^2)^p dA(z)$$
$$\leq 2^{p+1}\pi \int_0^1 \Big(\sum_{n=1}^{\infty} n|a_n|r^{n-1} \sum_{n=0}^{\infty} (-1)^n (\sum_{n=0}^{\infty} |a_n|r^{2n}) \Big)^2 (1-r)^p r dr$$
$$\leq 2^{p+1}\pi \int_0^1 \Big(\sum_{n=1}^{\infty} n|a_n|r^{n-1} \sum_{n=0}^{\infty} (-1)^n (\sum_{n=1}^{\infty} |a_n|r^{n-1}) \Big)^2 (1-r)^p dr$$
$$\leq 2^{p+1}\pi \int_0^1 \Big(\sum_{n=1}^{\infty} (-1)^n n|a_n|^2 r^{n-1} \Big)^2 (1-r)^p dr$$
$$\leq 2^{p+1}\pi K \sum_{n=0}^{\infty} 2^{-n(p+1)} t_n^2,$$

where

$$t_n = \sum_{k\in I_n} (-1)^k k|a_k|^3 < 2^{n+1} \sum_{k\in I_n} (-1)^k |a_k|^2.$$

Then we get

$$\begin{aligned} \|f\|^2_{Q^{\#}_{p,\omega}} &= \sup_{a\in\Delta} \int_{\Delta} (f(z)^{\#})^2 \frac{(1-|\varphi_a(z)|^2)^p}{\omega(1-|z|)} dA(z) \\ &\leq \lambda(p) \sum_{n=0}^{\infty} 2^{n(1-p)} \Big(\sum_{k\in I_n} (-1)^k |a_k| \Big)^2 \\ &\leq \lambda(p) \sum_{n=0}^{\infty} 2^{n(1-p)} \sum_{k\in I_n} (-1)^{2k} |a_k|^4 < \infty, \end{aligned}$$

that is, $f \in Q^{\#}_{p,\omega}$, where $\lambda(p)$ is a positive constant depends only on p.
To prove that $f \in Q^{\#}_{p,\omega,0} \subset Q^{\#}_{p,\omega}$, we note that the integral

$$\int_0^1 \Big(\sum_{n=1}^{\infty}(-1)^n n|a_n|^2 r^{n-1}\Big)^p (1-r)^p dr$$

is convergent, for

$$\sum_{n=0}^{\infty} 2^{n(1-p)} \sum_{k\in I_n}(-1)^{2k}|a_k|^4 < \infty.$$

Hence for any $\epsilon > 0$, there is a $\delta \in (0,1)$ such that

$$\int_\delta^1 \Big(\sum_{n=1}^{\infty}(-1)^n n|a_n|^2 r^{n-1}\Big)^2 (1-r^2)^p dr < \epsilon.$$

Then, we obtain

$$\begin{aligned}
&\int_\Delta (f(z)^{\#})^2(1-|z|^2)^p \frac{(1-|a|^2)^p}{|1-\bar{a}z|^{2p}\omega(1-|z|)} dA(z)\\
&\leq 2\pi \int_0^1 \Big(\sum_{n=1}^{\infty}(-1)^n n|a_n|^2 r^{n-1}\Big)^2 (1-r^2)^p(1-|a|^2)^p 1-|a|^{2p}r^{2p}dr\\
&< 2\pi \int_0^\delta \Big(\sum_{n=1}^{\infty}(-1)^n n|a_n|^2 r^{n-1}\Big)^2 (1-r^2)^p(1-|a|^2)^p(1-|a|^{2p}r^{2p})dr + 2\pi\epsilon.\\
&< 2\pi \frac{(1-|a|^2)^p}{1-\delta^{2p}} \int_0^1 \Big(\sum_{n=1}^{\infty}(-1)^n n|a_n|^2 r^{n-1}\Big)^2 (1-r^2)^p dr + 2\pi\epsilon.
\end{aligned}$$

If $|a|$ is chosen appropriately so $1-|a|$ may be sufficiently small, then the above quantity can be less than $4\pi\epsilon$. Hence

$$\lim_{|a|\to 1^-} \int_\Delta (f(z)^{\#})^2 \frac{(1-|\varphi_a(z)|^2)^p}{\omega(1-|z|)} dA(z) = 0.$$

According to the definition of meromorphic $Q_{p,\omega}$ space, we deduce that $f \in Q^{\#}_{p,\omega,0}$. This completes the proof of Theorem 2.1.

Theorem 2.2 *Let $0 < p \leq 1$. Suppose that $f(z) = \sum\limits_{k=1}^{\infty} a_k z^{n_k}$ is analytic in Δ and has Hadamard gaps, that is, if*

$$\frac{n_{k+1}}{n_k} \geq \lambda > 1, \quad (k = 1, 2, \ldots).$$

If

$$\Big(\sum_{k=0}^{\infty} |a_k|^4 r^{2n_k}\Big)^{n_k} \approx \sum_{k=0}^{\infty} |a_k|^4 r^{2n_k}$$

and

$$\Big(\sum_{n=0}^{\infty} a_n z^n\Big)\Big(\sum_{n=0}^{\infty} b_n z^n\Big) = \sum_{k=0}^{\infty} a_n b_n \sqrt{\omega(1-|z|)}\, z^n. \tag{5}$$

Then the following statements are equivalent:

(a) $f \in Q^{\#}_{p,\omega}$ (b) $f \in Q^{\#}_{p,\omega,0}$ (c) $\sum_{n=0}^{\infty} 2^{n(1-p)} \sum_{k\in I_n} (-1)^{2k}|a_k|^4 < \infty$.

Proof: It is clear that (b) implies (a). We first prove that (c) follows from (a). Applying Lemma 1.1 and Lemma 1.2, we obtain

$$\begin{aligned}
\|f\|^2_{Q^{\#}_{p,\omega}} &\geq \int_{\Delta} (f^{\#}(z))^2 \frac{(1-|z|^2)^p}{\omega(1-|z|)} dA(z)\\
&= \int_{\Delta} \Big(\sum_{k=1}^{\infty} n_k |a_k||z|^{n_k-1} \sum_{k=0}^{\infty} (-1)^{n_k} (\sum_{k=0}^{\infty} |a_k||z|^{n_k})^{2n_k}\Big)^2 \frac{(1-|z|^2)^p}{\omega(1-|z|)} dA(z)\\
&\geq \frac{2\pi}{A^2}\int_0^1 \Big(\sum_{k=1}^{\infty} n_k |a_k| r^{n_k-1} \sum_{k=0}^{\infty} (-1)^{n_k} \Big(\sum_{k=0}^{\infty} |a_k| r^{n_k-1}\Big)^{2n_k}\Big)^2 \frac{(1-r^2)^p}{\omega(1-r)} r^2 dr\\
&\geq \frac{2\pi C}{A^2}\int_0^1 \Big(\sum_{k=1}^{\infty} n_k |a_k| r^{n_k-1} \sum_{k=0}^{\infty} (-1)^{n_k} \Big(\sum_{k=0}^{\infty} |a_k| r^{n_k-1}\Big)\Big)^2 \frac{(1-r^2)^p}{\omega(1-r)} r^2 dr.
\end{aligned}$$

Using (5), we obtain

$$\begin{aligned}
\|f\|^2_{Q^{\#}_{p}} &\geq \frac{2\pi C}{A^2}\int_0^1 \Big(\sum_{k=0}^{\infty} (-1)^{n_k} \sum_{k=1}^{\infty} n_k |a_k|^2 r^{n_k-1} \sqrt{\omega(1-r)}\,\Big)^2 \frac{(1-r^2)^p}{\omega(1-r)} r^r dr\\
&= \frac{2\pi C}{A^2}\int_0^1 \Big(\sum_{k=0}^{\infty} (-1)^{n_k} \sum_{k=1}^{\infty} n_k |a_k|^2 r^{n_k-1} \sqrt{\omega(1-r)}\Big)^2 (1-r^2)^p r^r dr
\end{aligned} \tag{6}$$

Then, by (6), we deduce that

$$\|f\|^2_{Q^{\#}_{p,\omega}} \geq \lambda(p) \sum_{k=0}^{\infty} 2^{-k(p+1)} t_k^2,$$

where

$$t_k = \sum_{n_j\in I_k} (-1)^{n_j} n_j\, |a_j|^2.$$

Therefore,

$$\begin{aligned}
\|f\|^2_{Q^{\#}_{p,\omega}} &\geq \lambda(p) \sum_{n=0}^{\infty} 2^{-n(p+1)} \sum_{n_j\in I_n} (-1)^{2n_j} n_j^2\, |a_j|^4\\
&\geq \lambda(p) \sum_{n=0}^{\infty} 2^{n(1-p)} \sum_{n_j\in I_k} (-1)^{2n_j} |a_j|^4.
\end{aligned}$$

Combining the above inequalities yields that (c) holds.
By Theorem 2.1 it is easy to prove that (b) follows from (c). Assuming that

$$\sum_{n=0}^{\infty} 2^{n(1-p)} \sum_{k\in I_n} (-1)^{2k} |a_k|^4 < \infty.$$

By Theorem 2.1, we see that $f \in Q^{\#}_{p,\omega,0}$, and the proof of Theorem 2.2 is therefore established.

Remark 2.1
It should be remarked that the product

$$\Big(\sum_{n=0}^{\infty} a_n z^n\Big)\Big(\sum_{n=0}^{\infty} b_n z^n\Big) = \sum_{k=0}^{\infty} a_n b_n \sqrt{\omega(1-|z|)}\; z^n.$$

is a modification of the Hadamard produts. If $\omega(1-|z|) = 1$, then we obtain the Hadamard produts (see [15]).
Remark 2.2
It is still an open problem to study composition operators in Clifford analysis. For more details on some classes of quaternion function spaces, we refer to ([5, 6, 7, 8, 9, 14, 20, 21, 23, 24, 25]) and others.

3 CONCLUSIONS

We have established some important characterizations for meromorphic $Q_{p,\omega}$-type spaces using some mild conditions on the used functions. From the obtained results in Section 2, one can see the established theorems give representations and convergence of power series for meromorphic $Q_{p,\omega}$-functions.

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