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Common due-window assignment and scheduling on a single machine with past-sequence-dependent set up times and a maintenance activity

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Abstract

This study investigates the single-machine scheduling problem with common due-win -dow assignment, past-sequence-dependent (p-s-d) setup times and a deteriorating maintenance activity. By past-sequence-dependent setup times, we mean that the setup time of a job is proportional to the sum of the processing times of the jobs already processed. The objective is to minimize a cost function based on the earliness, tardiness, due-window starting time, and due-window size. It is shown that the problem is polynomial solvable.

Mathematics Subject Classification: 90B35

Keywords: Scheduling, Common due-window assignment,Past-sequence-dependent setup times,Deteriorating maintenance activity

1 Introduction

Scheduling problems with due date assignment have received considerable attention in the last two decades due to the introduction of new operations management concepts and methods such as just-in-time production and supply chain management. In traditional scheduling models due dates are considered as exogenously given. However, in many practical situations due dates are endogenously determined that takes into account the production system's ability to meet the quoted due dates. Many studies consider due date assignment as part of the scheduling process and show that how firms' ability to control due dates can be a major factor for improving their performance. In order to avoid tardiness penalties, including the possibility of losing customers, companies are under increasing pressure to quote attainable due dates. At the same time, promising due dates too far into the future may not be acceptable to customers or may compel the firm to offer price discounts in order to retain customers. Thus, there is an important trade-off between assigning relatively short due dates to customer orders and avoiding tardiness penalties. The due date assignment methods often used in manufacturing include the slack due date assignment method, and the unrestricted due date assignment method etc. For reviews of research results on scheduling models considering due-date assignment and their practical applications, the reader may refer to Cheng and Gupta [2], Gordon et al. [3,4], and Lauff and Werner [7].

Researchers have studied a variety of problems of scheduling against due dates where jobs incur no penalty when they are completed at a certain point in time. In practice, however, there are usually no penalties for jobs completed within a time interval, which we call a due window. The due window could be interpreted as the length of a time bucket in a material requirements planning (MRP) system. In real practice, due windows are very common. For instance, to achieve JIT production, companies adopt the make-toorder strategy, i.e., products are not manufactured for inventory but according to specified customer orders. The customer, in general, allows a time interval within which its order should be delivered ([9]). Cheng [1] pioneers research on scheduling with due windows. The left end and the right end of the due window are called the starting time and finishing time of the window, respectively. A job completed on or before (after) the starting (finishing) time of the due window is considered to be early (tardy). Apparently, a late and wide due window increases the manufacturer's production and delivery flexibility. On the other hand, a large due window and delaying job completion reduce the manufacturer's competitiveness and its level of customer service.

In a recent paper, Kubzin and Strusevich [6] considered a setting where the maintenance activity is deteriorating, that is delaying the maintenance activity increases the time required to perform it. The longer the maintenance activity is delayed, the worse the system conditions become, so that the maintenance requires more effort and time. Kubzin and Strusevich [6] study makespan minimization on a two-machine flowshop and a two-machine open shop. In this paper, in order to more realistically model a production system, we investigate the problem of common due-window assignment and past-sequence-dependent (p-s-d) setup times and a deteriorating maintenance activity simultaneously. The objective is to find jointly the optimal maintenance location, the optimal location and size of the due-window, and the optimal job sequence to minimize the total earliness, tardiness, and due-window starting time and size costs. We provide a polynomial time solution for the studied problem and show that the maintenance activity is optimally scheduled immediately after any job processed before the due-window, within the due-window, or after the due-window, or no maintenance activity is performed at all.

The rest of this paper is organized as follows: In Section 2 we describe and formulate the problem. In Section 3 we introduce several important lemmas. We provide a polynomial time solution for the problem in Section 4. We conclude the study and suggest some topics for future research in Section 5.

2 Problem description and formulation

We consider the problem of scheduling a set of n independent jobs on a single-machine. All the jobs are non-preemptive and available for processing at time zero. Each job J_j, has a processing time p_j and a due date d_j. For each given schedule, denote $[j]$ by the job in the jth position in this schedule. We assume that the setup time of a job is past-sequence-dependent, i.e., if it is scheduled in position j then its setup time is given by

$$S_{[1]} = 0, S_{[j]} = \sum_{k=1}^{j-1} bp_{[k]}, j = 2, \dots, n. \tag{1}$$

Where $b \geq 0$ is a normalizing constant,and we denote the p-s-d setup given in Eq. (1) by s_{psd} .

We assume that the post-processing operation of any job $J_{[j]}$ modelled by its set-up time $S_{[j]}$ is performed off-line; consequently, it is not affected by the availability of the machine and it can commence immediately upon completion of the main operation, resulting in

$$C_{[j]} = C_{[j-1]} + S_{[j]} + P_{[j]}^A$$

$$= \sum_{k=1}^{j-1} P_{[j]}^A + \sum_{k=1}^{j-1} S_{[j]} + S_{[j]} + P_{[j]}^A = \sum_{k=1}^{j} P_{[k]}^A + \sum_{k=1}^{j} S_{[k]} = \sum_{k=1}^{j} [b(j-k)+1] P_{[k]}^A$$

$j = 2, \cdots, n$,if all the jobs are processed consecutively without idle time and the first job starts at time 0.

Assume that all the jobs share a common due-window. Let $d_1\ (\geq 0)$ and $d_2\ (\geq 0)$ denote the due-window starting time and finishing time, respectively, and $D = d_2 - d_1$ denote the due-window size. Both d_1 and d_2 are decision variables in this study. A job completed within the due-window is regarded as on time and is not penalized. If a job is completed on or before the due-window starting time d_1, an earliness penalty is incurred. On the other hand, if a job is completed after the due-window finishing time d_2, a tardiness penalty is incurred. For a given schedule, we denote the completion time, earliness, and tardiness of job $[j]$ as $C_{[j]}$, $E_{[j]}$, and

$T_{[j]}$, respectively, Then, we have $E_{[j]} = \max\{0, d_1 - C_{[j]}\}$ and $T_{[j]} = \max\{0, C_{[j]} - d_2\}$.

Moreover, the machine may need maintenance to improve its production efficiency. Assume that at most one maintenance activity is allowed throughout the planning horizon. Maintenance can be performed immediately after the processing of any job is completed. However, the position and starting time of the maintenance activity in the production schedule are unknown in advance. To realistically model the problem, we further assume that the maintenance duration is a linear function of its starting time and is given by $f(t) = \mu + \sigma t$, where $\mu > 0$ is the basic maintenance time, $\sigma \geq 0$ is a maintenance factor, and t is the starting time of the maintenance activity. We aim to find jointly the optimal due-window location, the optimal location of the maintenance activity, and the optimal job sequence π to minimize the following cost function:

$$f(d_1, D, \pi) = Z = \sum_{j=1}^{n}\left(\alpha E_j + \beta T_j + \gamma d_1 + \delta D\right) \tag{2}$$

where $\alpha > 0$, $\beta > 0$, $\gamma > 0$, and $\delta > 0$ are the earliness, tardiness, due-window starting time, and due-window size costs per unit time, respectively. Using the three-field notation of Graham et al.(1979), we denote the problem under study as $1\left|ma, s_{psd}\right|\sum_{j=1}^{n}\left(\alpha E_j + \beta T_j + \gamma d_1 + \delta D\right)$, where ma in the second field denotes the maintenance activity.

3 Preliminary analysis

It is clear that an optimal schedule exists that starts at time zero and contains no idle time between consecutive jobs.

Lemma 1. For any given sequence π, there exists an optimal common due-window such that the starting time d_1 equals to $C_{[k]}$,where $k = \lceil (n(\delta - \gamma))/\alpha \rceil$, and the finishing time d_2 equals to $C_{[k+m]}$, where $(k+m) = \lceil (n(\beta - \delta))/\beta \rceil$.

Proof. The proof is similar to that of Lemma 3 in Mosheiov and Sarig [8].

The following lemmas are useful for solving the problem.

Lemma 2. Let there be two sequences of non-negative numbers x_i and y_i. The sum of the products of the corresponding elements $\sum_{j=1}^{n} x_i y_i$ is the least if the sequences are monotonic in the opposite sense.

Proof. See Hardy etal. [5].

4 An optimal solution for $1\left|am, s_{psd}\right|\sum_{j=1}^{n}\left(\alpha E_j+\beta T_j+\gamma d_1+\delta D\right)$

By Lemma 1,we can determine the optimal locations of the common due- window starting time d_1 and finishing time d_2. Let k and m be defined as in Lemma.

If the maintenance activity is performed after the ith job which is processed before the due-window ($i.e., i<k$), then the total cost is given by

$$f\left(d,D,\pi\right)=Z=\sum_{j=1}^{n}\left(\alpha E_j+\beta T_j+\gamma d_1+\delta D\right)$$

$$=\alpha\sum_{j=1}^{n}E_j+\beta\sum_{j=1}^{n}T_j+\gamma n d_1+\delta n D$$

$$=\alpha\left[\sum_{j=1}^{i}\left(d_1-C_{[j]}\right)+\sum_{j=i+1}^{k}\left(d_1-C_{[j]}\right)\right]+\beta\sum_{j=k+m+1}^{n}\left(C_{[j]}-d_2\right)+\gamma n\left[C_{[k]}+\left(u+\sigma C_{[i]}\right)\right]$$

$$+\delta n\left(C_{[k+m]}-C_{[k]}\right)$$

$$=\alpha\left[\sum_{j=1}^{k}\left(d_1-C_{[j]}\right)+\mathrm{i}\left(u+\sigma C_{[i]}\right)\right]+\beta\sum_{j=k+m+1}^{n}\left(C_{[j]}-d_2\right)+\gamma n\left[C_{[k]}+\left(u+\sigma C_{[i]}\right)\right]$$

$$+\delta n\left(C_{[k+m]}-C_{[k]}\right)$$

$$=\alpha\sum_{j=1}^{k}\left(d_1-C_{[j]}\right)+\beta\sum_{j=k+m+1}^{n}\left(C_{[j]}-d_2\right)+\gamma n C_{[k]}+\delta n\left(C_{[k+m]}-C_{[k]}\right)$$

$$+\left(\mathrm{i}\alpha+\gamma n\right)\left(u+\sigma C_{[i]}\right)$$

$$=\Big[\left(bk-bj\right)\left(\alpha k+\gamma n-\beta n+\beta k+\beta m\right)+bm\left(\delta n-\beta n+\beta k+\beta m\right)$$

$$+\sigma\left(i\alpha+\gamma n\right)\left[b\left(i-j\right)+1\right]-\alpha\frac{\left(k-j+1\right)\left[b\left(k-j\right)+2\right]}{2}$$

$$+\beta\frac{\left(n-k-m\right)\left[b\left(k+m+n+1-2j\right)+2\right]}{2}\Bigg]\sum_{j=1}^{i}p_{[j]}$$

$$\Big[\left(bk-bj\right)\left(\alpha k+\gamma n-\beta n+\beta k+\beta m\right)+bm\left(\delta n-\beta n+\beta k+\beta m\right)$$

$$-\alpha\frac{\left(k-j+1\right)\left[b\left(k-j\right)+2\right]}{2}+\beta\frac{\left(n-k-m\right)\left[b\left(k+m+n+1-2j\right)+2\right]}{2}\Bigg]\sum_{j=i+1}^{k}p_{[j]}$$

$$+\left[\left(\delta n-\beta n+\beta k+\beta m\right)\left(bk+bm-bj+1\right)+\beta\left(n-k-m\right)\frac{2+b\left(k+m+n+1-2j\right)}{2}\right]\sum_{j=k+1}^{k+m}p_{[j]}$$

$$+\left[\beta\left(n-j+1\right)\frac{2+b\left(n-j\right)}{2}\right]\sum_{j=k+m+1}^{n}p_{[j]}+u\left(i\alpha+\gamma n\right) \qquad (3)$$

If the maintenance activity is performed after the ith job which is processed within the due-window (i.e., $k\le i<\left(k+m\right)$), then the total cost is given by

$$f\left(d,D,\pi\right)=Z=\sum_{j=1}^{n}\left(\alpha E_j+\beta T_j+\gamma d_1+\delta D\right)$$

$$=\alpha\sum_{j=1}^{k}\left(d_1-C_{[j]}\right)+\beta\sum_{j=k+m+1}^{n}\left(C_{[j]}-d_2\right)+\gamma nC_{[k]}+\delta n\left(C_{[k+m]}-C_{[k]}+\left(u+\sigma C_{[i]}\right)\right)$$

$$=\left[\left(bk-bj\right)\left(\alpha k+\gamma n-\beta n+\beta k+\beta m\right)+bm\left(\delta n-\beta n+\beta k+\beta m\right)+n\delta\left[b\left(i-j\right)+1\right]\right.$$

$$\left.-\alpha\frac{\left(k-j+1\right)\left[b\left(k-j\right)+2\right]}{2}+\beta\frac{\left(n-k-m\right)\left[b\left(k+m+n+1-2j\right)+2\right]}{2}\right]\sum_{j=1}^{k}p_{[j]}$$

$$+\left[\left(\delta n-\beta n+\beta k+\beta m\right)\left(bk+bm-bj+1\right)+n\delta\left[b\left(i-j\right)+1\right]\right.$$

$$\left.+\beta\left(n-k-m\right)\frac{2+b\left(k+m+n+1-2j\right)}{2}\right]\sum_{j=k+1}^{i}p_{[j]}$$

$$+\left[\left(\delta n-\beta n+\beta k+\beta m\right)\left(bk+bm-bj+1\right)\right.$$

$$\left.+\beta\left(n-k-m\right)\frac{2+b\left(k+m+n+1-2j\right)}{2}\right]\sum_{j=i+1}^{k+m}p_{[j]} \qquad (4)$$

If the maintenance activity is performed after the ith job which is processed after the due-window (i.e., $i\ge\left(k+m\right)$), then the total cost is given by

$$f\left(d,D,\pi\right)=Z=\sum_{j=1}^{n}\left(\alpha E_j+\beta T_j+\gamma d_1+\delta D\right)$$

$$=\alpha\sum_{j=1}^{k}\left(d_1-C_{[j]}\right)+\beta\left[\sum_{j=k+m+1}^{i}\left(C_{[j]}-d_2\right)+\sum_{j=i+1}^{n}\left(C'_{[j]}-d_2\right)\right]+\gamma nC_{[k]}+\delta n\left(C_{[k+m]}-C_{[k]}\right)$$

$$=\alpha\sum_{j=1}^{k}\left(d_1-C_{[j]}\right)+\beta\left[\sum_{j=k+m+1}^{i}\left(C_{[j]}-d_2\right)+\sum_{j=i+1}^{n}\left(C_{[j]}-d_2\right)+\left(n-i\right)\left(u+\sigma C_{[i]}\right)\right]+\gamma nC_{[k]}$$

$$+\delta n\left(C_{[k+m]}-C_{[k]}\right)$$

$$=\alpha\sum_{j=1}^{k}\left(d_1-C_{[j]}\right)+\beta\left[\sum_{j=k+m+1}^{i}\left(C_{[j]}-d_2\right)+\sum_{j=i+1}^{n}\left(C_{[j]}-d_2\right)\right]+\gamma nC_{[k]}++\delta n\left(C_{[k+m]}-C_{[k]}\right)$$

$$+\beta(n-i)\left(u+\sigma C_{[i]}\right)$$

$$=\left[(bk-bj)(\alpha k+\gamma n-\beta n+\beta k+\beta m)+bm(\delta n-\beta n+\beta k+\beta m)+\beta\sigma\left[b(i-j)+1\right]\right.$$

$$\left.-\alpha\frac{(k-j+1)\left[b(k-j)+2\right]}{2}+\beta\frac{(n-k-m)\left[b(k+m+n+1-2j)+2\right]}{2}\right]\sum_{j=1}^{k}p_{[j]}$$

$$+\left[(\delta n-\beta n+\beta k+\beta m)(bk+bm-bj+1)+\beta\sigma\left[b(i-j)+1\right]\right.$$

$$\left.+\beta(n-k-m)\frac{2+b(k+m+n+1-2j)}{2}\right]\sum_{j=k+1}^{k+m}p_{[j]}$$

$$+\left[\beta(n-j+1)\frac{2+b(n-j)}{2}+\beta\sigma\left[b(i-j)+1\right]\right]\sum_{j=k+m+1}^{i}p_{[j]}$$

$$+\left[\beta(n-j+1)\frac{2+b(n-j)}{2}\right]\sum_{j=i+1}^{n}p_{[j]}+\beta u(n-i) \tag{5}$$

Note that if $i=n$, then it means that no maintenance activity is necessary in the schedule. In addition, we let

$$\varpi=\begin{cases}(n\gamma+i\alpha)\mu & i<k\\ n\delta\mu & k\le i<(k+m)\\ (n-i)\beta\mu & i\ge(k+m)\end{cases}$$

Then, the total cost can be expressed as

$$f(d,D,\pi)=Z=\sum_{j=1}^{n}\left(\alpha E_j+\beta T_j+\gamma d_1+\delta D\right)=\sum_{j=1}^{n}\omega_j p_{[j]}+\varpi$$

For the three cases scheduled maintenance activities, ω_j can be determined by Eqs. (3)-(5) , respectively.

Based on the above analysis and Lemma 2, we propose an $O\left(n^2\log n\right)$ time algorithm to solve the problem.

Algorithm 1

Step 1. By Lemma 1, calculate the optimal positions of the due-window starting time $k=\lceil (n(\delta-\gamma))/\alpha\rceil$ and finishing time $(k+m)=\lceil (n(\beta-\delta))/\beta\rceil$.

Step 2. Set $i=1$.

Step 3. For $j=1,2,\cdots,n.$, calculate the positional weights ω_j according to Eqs.

(3)-(5).

Step 4. Renumber the jobs in non-decreasing order of their normal processing times p_j. By Lemma 2, match the smallest w_j value to the job with the largest p_j value, the second smallest w_j value to the job with the second largest p_j value, and so on..Then, obtain a local optimal schedule and the corresponding total cost..

Step 5. $i = i + 1$.If $i \le n$, then go to Step 3. Otherwise, go to Step6

Step 6. The optimal schedule is the one with the minimum total cost.

The time complexity of Steps 1 and 3 is $O(1)$ and the time complexity of Step 4 is $O(n \log n)$. Since the maintenance activity can be scheduled immediately after any one of the jobs, n different positions of the maintenance activity must be evaluated to obtain the global optimal solution in Step 5. Hence, the time complexity for solving $1\left|am, s_{psd}\right|\sum_{j=1}^{n}\left(\alpha E_j + \beta T_j + \gamma d_1 + \delta D\right)$ the problem is $O\left(n^2 \log n\right)$.

5 Conclusions

In this paper, we have investigated the single-machine common due-window assignment and scheduling problem with simultaneous considerations of past-sequence-dependent (p-s-d) setup times and a deteriorating maintenance activity. The maintenance duration is assumed to be a linear function of its starting time. Our goal is to find jointly the optimal location of the maintenance activity, the optimal location and size of the due- window, as well as the optimal job sequence to minimize the total earliness, tardiness, and due-window starting time and size costs.We propose a polynomial time algorithm for the problem. Further research may consider the problem with other models of maintenance time, in multi-machine settings, and optimizing other performance measures.

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