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Group scheduling and due date assignment on a single machine with convex resource-dependent processing times

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Abstract

This paper addresses a single-machine scheduling problem with simultaneous consideration of due date assignment and convex resource-dependent processing times under a group technology environment. The jobs of customers are classified into groups according to their production similarities in advance. The goal is to find the job schedule and the due date for each group that minimizes a cost function that includes the earliness, tardiness, due date assignment and resource alloction. The structual properties of the problem is studied and an important sepecial case is addressed.

Mathematics Subject Classification: 90Bxx

Keywords: Scheduling; due date assignment; resource allocation; group technology environment.

1 Introduction

Meeting due dates is an important objective of scheduling. Delivery date quotation by a supplier corresponds to due-date assignment and orders are referred to as jobs in scheduling. We mainly use the scheduling terminology following past researchers who have studied related problems. While traditional scheduling models consider due dates as externally given, in the modern flexible and integrated production system due dates are internally determined and take into consideration the system's ability to meet the quoted delivery dates. This is why increasing numbers of studies have viewed due-date assignment as part of the scheduling process, highlighting that the ability to control due dates is a major factor in improving system performance. The due-date assignment methods often used in manufacturing include the common due-date assignment (referred to as CON), slack due-date assignment (referred to as SLK), unrestricted due-date assignment (referred to as DIF), and so on. For research results on scheduling models considering due-date assignment and their practical applications, the reader may refer to Cheng and Gupta [3], Gordon et al. [5,6], and Lauff and Werner [9].

Recently, Li et al. [10] consider a single-machine scheduling problem involving both the due date assignment and job scheduling under a group technology environment. The jobs of customers are classified into groups according to their production similarities in advance. To achieve production efficiency and save time/money resource, all jobs of the same group are required to be processed contiguously on the machine. A sequence-independent setup time precedes the processing of each group. The due dates are assignable according to one of the following three due date assignment methods: FML-CON, FML-SLK and DIF, where FML-CON means that all jobs within the same group are assigned a common due date, FML-SLK means that all jobs within the same group are assigned an equal flow allowance, and DIF means that each job can be assigned a different due date with no restrictions. The objcetive is to determine an optimal combination of the due date assignment strategy and job schedule so as to minimize an objective function that includes earliness, tardiness, due date assignment and flow time costs. An $O(n\log n)$ time unified optimization algorithm is provided for all of the above three due date assignment methods. Shabtay et al. [17] consider a single-machine scheduling problem involving both the FML-CON due date assignment method and resource dependent processing times under a group technology environment. By resource dependent processing times, they mean that which job processing times are controllable by the allocation of a continuous and nonrenewable resource such as fuel, gas, catalyzer or manpower to compress the job operation times. The resource allocation function is either linear or convex. The objective is to find the job schedule, the due date for each group and the resource alloction that minimizes an objective function which includes earliness, tardiness,due date assignment and resource allocation. We also extend the analysis to address the case in which the job processing times are resource dependent. For this case we include the total weighted resource consumption and the makespan penalties to the objective function. For other reslults on the scheduling model on due date assignment and resouce dependent processing times, the reader is referred to [1-2,4,7-8,11-16,18-21].

In this paper, we consider the single-machine scheduling problem involving both the FML-SLK due date assignment method and resource dependent processing times under a group technology environment. The rest of this paper is organized as follows. In the following section we we formulate the problem. In Section 3 we develop some structual properties of the problem and present a polynomial-time algorithm for a special case where the number of jobs in each group is identical. We conclude the paper and suggest topics for future research in the last section.

1.2 Problem formulation

The problem can be stated as follows: There are n jobs to be processed without interruption on a single machine that can deal with only one job at a time. All the jobs are available for processing at time zero. The jobs are divided into m job families $G_1, G_2, \ldots, G_m$. Each group G_i, for $1 \le i \le m$, consists of a set of $\{J_{i1}, J_{i2}, \ldots, J_{in_i}\}$ of n_i jobs with $n = \sum_{i=1}^{m} n_i$ .The jobs within each group are consecutively sequenced, i.e., job families are not allowed to interweave in order to take advantage of their similarities in the production process. A sequence-independent machine setup time s_i proceeds the processing of the first job of group $G_i, i = 1, 2, \ldots, m$. Each job J_{ij}, has a processing time p_{ij} and a due date d_{ij}, in which p_{ij} can be compressed according to the following convex resource consumption function

$$p_{ij}(u_{ij}) = \left(\frac{w_{ij}}{u_{ij}}\right)^k \qquad (1)$$

where u_{ij} is the amount of resource allocated to job J_{ij}, w_{ij} is a positive parameter which represents the workload of job J_{ij} and k is a positive constant, and d_{ij} is assignable according to the FML-SLK due date assignment method in which all jobs of group G_i are assigned an equal flow allowance that reflects equal waiting time (equal slacks), i.e., $d_{ij} = p_{ij}(u_{ij}) + slk_i$, where $slk_i \ge 0$. The goal is to find an optimal schedule π^*, the optimal slack vector $slk^* = (slk_1^*, slk_2^*, \ldots, slk_m^*)$, and the optimal resource allocation matrix $u = (u_{ij})$ for $i = 1, \ldots, m$ and $j = 1, \ldots, n_i$ which all together minimizes the following objective function:

$$Z(\pi, slk, u) = \sum_{i=1}^{m} \sum_{j=1}^{n_i} \left(\alpha d_{ij} + \beta E_{ij} + \gamma T_{ij} + v_{ij} u_{ij}\right) + \delta C_{\max} \qquad (2)$$

where C_{ij} is the completion time of J_{ij}; $E_{ij} = \{d_{ij} - C_{ij}, 0\}$ is the earliness and $T_{ij} = \{C_{ij} - d_{ij}, 0\}$ is the tardiness of J_{ij} for $i = 1, \ldots, m$ and $j = 1, \ldots, n_i$; α, β, γ and δ are nonnegative constant which denote the cost of one unit of due date, earliness, tardiness and operation time, respectively; and v_i is the cost of one unit of resource allocated to job J_{ij}.

3 Preliminary analysis

Let us define $[i]$ as the index of the group in the ith position in the group sequence, and $i[j]$ and $[i,j]$ as the index of the job in the i th position in group G_i and the job which is in the j th position in i th group, respectivley.

Lemma 3.1. There exists an optimal slack allowance $slk_{[i]}^*$ of group $G_{[i]}, (i=1,\ldots,m)$ for the problem with constant processing times such that:

$$slk_{[i]}^* = \begin{cases} C_{[i,l_i^*-1]}, & l_i^* = \left\lceil \frac{n_i(\gamma-\alpha)}{\gamma+\beta} \right\rceil, \alpha<\gamma \\ 0 & otherwise \end{cases} \qquad (3)$$

and $C_{[i,l^*-1]} = \sum_{k=1}^{i-1} p_{[k]} + \sum_{k=1}^{i} s_{[i]} + \sum_{l=1}^{l_i^*-1} p_{[i,l]}(u_{[i,l]})$, where $p_{[k]}$ denotes the total processing time of jobs in group $G_{[k]}$.

Proof. It is similar to that of Lemma 3.5 in Li et al. [10].

The following result is obvious.

Lemma 3.2. An optimal schedule does not include idle times.

First,we analyze the case where $\alpha<\gamma$. According to lemma 3.1, an optimal slack allowance $slk_{[i]}^*$ can be determined according to Eq. (4). For this case, we have

$$E_{[i,j]} = \begin{cases} \sum_{l=j}^{l_i^*-1} p_{[i,l]}(u_{[i,l]}) & 1 \le j \le l_i^*-1 \\ 0 & l_i^* \le j \le n_i \end{cases} \qquad (4)$$

$$T_{[i,j]} = \begin{cases} \sum_{l=l_i^*}^{j-1} p_{[i,l]}(u_{[i,l]}) & l_i^*+1 \le j \le n_i \\ 0 & 1 \le j \le l_i^* \end{cases} \qquad (5)$$

$$d_{[i,j]}^* = p_{[i,j]}(u_{[i,l]}) + slk_{[i]}^* = p_{[i,j]}(u_{[i,l]}) + \sum_{k=1}^{i-1} p_{[k]} + \sum_{k=1}^{i} s_{[k]} + \sum_{l=1}^{l^*-1} p_{[i,l]}(u_{[i,l]}) \qquad (6)$$

By substituting Eqs.（4）-（6）into Eq. (2),we obtain a new expression for the objective function value under an optimal due date assignment strategy:

$$\begin{aligned} Z(\pi,slk,u) \quad = \quad & \alpha \sum_{i=1}^{m} n_{[i]} \times \left[\sum_{k=1}^{i-1} p_{[k]} + \sum_{k=1}^{i} s_{[k]} \right] \\ + \quad & \sum_{i=1}^{m} \sum_{j=1}^{l_i^*-1} \left(\delta + (n_i+1)\alpha + j\beta \right) p_{i[j]}(u_{i[j]}) \\ + \quad & \sum_{i=1}^{m} \sum_{j=l_i^*}^{n_i} \left(\delta + \alpha + (n_i-j)\gamma \right) p_{i[j]}(u_{i[j]}) + \sum_{i=1}^{m} \sum_{j=1}^{n_i} v_{i[j]} u_{i[j]} \end{aligned} \qquad (7)$$

Next,we analyze the opposite case where $\alpha \ge \gamma$, for this case, the following holds for $i = 1, \ldots, m$:

$$E_{[i,j]} = 0, \ for 1 \le j \le n_i \qquad (8)$$

$$T_{[i,j]} = \sum_{k}^{i-1} p_{[k]} + \sum_{k}^{i} s_{[k]} + \sum_{l=1}^{j-1} p_{[i,l]}(u_{[i,l]}), \ \ for 1 \le j \le n \qquad (9)$$

$$d^*_{[i,j]} = p_{[i,j]}(u_{[i,j]}) \qquad (10)$$

By substituting Eqs.（8）-（10）into Eq. (2), we obtain a new expression for the objective function value under an optimal due date assignment strategy:

$$Z(\pi, slk, u) = \gamma \sum_{i=1}^{m} n_{[i]} \times \left[\sum_{k=1}^{i-1} p_{[k]} + \sum_{k=1}^{i} s_{[i]} \right] + \sum_{i=1}^{m} \sum_{j=1}^{n_i} \left(\delta + \alpha + \left(n_i - j \right) \gamma \right) p_{i[j]}(u_{i[j]}) + \sum_{i=1}^{m} \sum_{j=1}^{n_i} v_{i[j]} u_{i[j]} \qquad (11)$$

Let

$$\zeta = \min(\alpha, \gamma), \qquad (12)$$

and for $i = 1, \ldots, m$ let

$$\varpi_{ij} = \begin{cases} \delta + (n_i + 1)\alpha + j\beta & 1 \le j \le l_i^* - 1 \\ \delta + \alpha + (n_i - j)\gamma & l_i^* \le j \le n_i \end{cases} \qquad (13)$$

Then the objective function for both cases (the case where $\alpha \ge \gamma$ and the case where $\alpha < \gamma$) can be represented in a unified form as follows:

$$Z(\pi, slk, u) = \zeta \sum_{i=1}^{m} \left(n_{[i]} \times \left(\sum_{k=1}^{i-1} p_{[k]} + \sum_{k=1}^{i} s_{[k]} \right) \right) + \sum_{i=1}^{m} \sum_{j=1}^{n_i} \varpi_{ij} p_{i[j]}(u_{i[j]}) + \sum_{i=1}^{m} \sum_{j=1}^{n_i} v_{i[j]} u_{i[j]}$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{n_{[i]}} \left(\zeta \times \sum_{r=i+1}^{m} n_{[i]} + \varpi_{[i]j} + \delta \right) p_{[i,j]}(u_{[i,j]}) + \zeta \sum_{i=1}^{m} \left(n_{[i]} \times \sum_{r=1}^{i} s_{[i]} \right) + \sum_{i=1}^{m} \sum_{j=1}^{n_{[i]}} v_{[i,j]} u_{[i,j]} \qquad (14)$$

By substituting Eq. (1) into the objective function in Eq. (14), we obtain the following expression

$$Z(\pi, slk, u) = \sum_{i=1}^{m} \sum_{j=1}^{n_{[i]}} \left(\zeta \times \sum_{r=i+1}^{m} n_{[i]} + \varpi_{[i]j} + \delta \right) \left(\frac{w_{[i,j]}}{u_{[i,j]}} \right)^k + \zeta \sum_{i=1}^{m} \left(n_{[i]} \times \sum_{r=1}^{i} s_{[i]} \right) + \sum_{i=1}^{m} \sum_{j=1}^{n_{[i]}} v_{[i,j]} u_{[i,j]} \qquad (15)$$

For given job sequence, the optimal resource allocation can be determined by the following result.

Lemma 3.3. The optimal resource allocation as a function of job sequence π is

$$u_{ij}^{*}(\pi)=\left(\frac{k\times\left(\zeta\times\sum_{r=i+1}^{m}n_{[r]}+\varpi_{[i]j}+\delta\right)}{v_{[i,j]}}\right)^{1/(k+1)}\times w_{[i,j]}^{k/(k+1)} \quad (16)$$

Proof. By taking the derivative of the objective function given by Eq. (15) with respect to $u_{[i,j]}$, for $i=1,\ldots,m$, $j=1,\ldots,n_i$, equating it to zero and solving it for $u_{[i,j]}^{*}$, we obtain Eq. (16). Since the objective is a convex function, Eq. (16) provides necessary and sufficient conditions for optimality.

By substituting Eq. (16) into Eq. (15), we obtain the following new expression for the cost function under an optimal resource allocation and due date assignment

$$\begin{aligned}Z(\pi, slk,\mu^{*})&=\left(k^{-k/(k+1)}+k^{1/(k+1)}\right)\\&\times\sum_{i=1}^{m}\sum_{j=1}^{n_i}\theta_{[i,j]}\times\left(\zeta\times\sum_{r=i+1}^{m}n_{[r]}+\varpi_{[i]j}+\delta\right)+\zeta\times\sum_{i=1}^{m}\left(n_{[i]}\times\sum_{r=1}^{i}s_{[r]}\right)\end{aligned} \quad (17)$$

where $\theta_{ij}=\left(w_{ij}\times v_{ij}\right)^{k/(k+1)}, i=1,\ldots,m; j=1,\ldots,n_i$ (18)

The optimal job sequence within group G_i denoted by π_i^{*} can be obtained by applying lemma 3.4.

Lemma 3.4. The optimal job sequence within group G_i, π_i^{*} is obtained by matching the elements of ϖ_{ij} with θ_{ij} in opposite orders.

Proof. The proof is similar to that of Lemma 8 in Shabtay et al. [16].

In view of Lemma 3.4, it remains to finding the optimal sequence of job families that minimizes Eq.(18).The complexity of this problem remains an open question. In what follows, we concentrates on a special case where the number of jobs in each group is identical, i.e., $n_1=n_2=\cdots=nm=n/m=\tilde{n}$. In this case, the l_i^{*} values as given in Eq. (3) become identical for all families,i.e., $l_i^{*}=l^{*}=\left\lceil\frac{\tilde{n}(\gamma-\alpha)}{\gamma+\beta}\right\rceil$ for $i=1,\ldots,m$. Hence, the ϖ_{ij} values given in Eq.(13) are independent of the ith index and can be rewritten as

$$\varpi_j=\begin{cases}\delta+(\tilde{n}+1)\alpha+j\beta & 1\le j\le l^{*}-1\\ \delta+\alpha+(\tilde{n}-j)\gamma & l^{*}\le j\le\tilde{n}\end{cases} \quad (19)$$

Therefore, the objective function in (17) becomes

$$\begin{aligned}Z(\pi, slk,u^{*})&=\left(k^{-k/(k+1)}+k^{1/(k+1)}\right)\\&\times\sum_{i=1}^{m}\sum_{j=1}^{\tilde{n}}\theta_{[i,j]}\times\left(\zeta\tilde{n}(m-i)+\varpi_j+\delta\right)+\zeta\times\sum_{i=1}^{m}\left(\tilde{n}\times\sum_{r=1}^{i}s_{[r]}\right)\end{aligned} \quad (20)$$

From Eq.(20) it is clear that the penalty of job J_{ij} depends solely on the position of group G_i in the group sequence and the position of it in group G_i, and is independent of the families preceding or succeeding group G_i in the group sequence. In addition, it follows from Lemma 3.4 that the optimal job sequence in each group can be predetermined. Let c_{il} be the minimal penalty incurred by assigning group G_i to position l in the group sequence. Then, by Eq.(20), we have

$$c_{il} = \left(k^{-k/(k+1)} + k^{1/(k+1)}\right) \times \sum_{j=1}^{\tilde{n}} \theta_{i[j]} \times \left(\zeta \tilde{n}\left(m-l\right) + \varpi_j + \delta\right)^{1/(k+1)} + \zeta \tilde{n}\left(m-l+1\right) \times s_i \quad (21)$$

Let us now define $x_{il} = 1$,if group G_i is assigned to position l in the gorup sequence and $x_{il} = 0$ otherwise.The sequencing problem of determining the optimal group sequence can be formulated as the following linear assignment problem:

(P1) $\min \sum_{i=1}^{m} \sum_{l=1}^{m} c_{il} \times x_{il}$

s.t.

$\sum_{i=1}^{m} x_{il} = 1$, for $l = 1, \ldots, m$,

$\sum_{l=1}^{m} x_{il} = 1$, for $i = 1, \ldots, m$,

$x_{il} = 0$ or 1 for $l = 1, \ldots, m$.

The first set of constraints in the formulation ensures that each group will be assigned only to one position,the second set ensures that each position will be assigned only once,and the penalty for each assignment under an optimal resource allocation appears in the objective function.

Summing up the above analysis, our algorithm for the problem with the case where the number of jobs in each group is identical can be formally described as follows.

Algorithm 1.

Step 1. Calculate $l^* = \max\left\{\left[\frac{\tilde{n} \times (\gamma - \alpha)}{\gamma + \beta}\right], 0\right\}$, and θ_{ij} and $\varpi_{ij} = \varpi_j$ according to Eqs. (18) and (19), respectively, for $i = 1, \ldots, m,\ j = 1, \ldots, n_i$.

Step 2. Determine the optimal job sequence for each group accodring to Lemma 3.3.

Step 3. Calculate all c_{il} values according to Eq.(21) for i, $l = 1, \ldots, m$.

Step 4. Determine the optimal group sequence by solving the linear assignment problem P1.

Step 5. Determine the optimal resource allocation matrix $\mu^* = (u_{ij})$ accoding to Eq. (16).

Step 6. If $\alpha < \gamma$ then assign the due dates according to Eq.(6) and job processing times $p^* = (p(u_{ij}^*))$;otherwise, assign the due dates according to Eq.(10).

Theorem 2. Algorithm 1 solves the problem with the case where the number of jobs in each group is identical in $o(\max((n\log n), m \times \max(n, m^2)))$ time.

Proof. The correctness of the algorithm follows directly from lemmas 3.1-3.4.Step 1 is performed in constant time. Step 2 requires $o\left(\sum_{i=1}^{m} n_i \log n_i\right) = o(n\log n)$ time and Step 3 takes $o(nm)$ time.Solving a linear assignment problem in Step 4 requires $o(m^3)$ time,and determining the optimal resource allocation matrix in Step 5 requires $o(n)$ time. In Step 6 we have to determining the optimal processing times,which requires $o(n)$ time, similar to the time needed to calculate the due date values.Thus,the overall complexity of algorithm 2 is . Since $m = o(n)$ the complexity is bounded by $o(n^3)$.

4 Summary and future research

In this paper, we have studied the problem of scheduling groups of jobs on a single machine with FML-SLK due date assignment method and convex resource dependent processing time under a group technology restriction, where the objective is to find the the job schedule and the due date for each group that minimizes an cost function that based on the earliness, tardiness, due date assignment and resource alloction. The structual properties of the problem is studeide and a sepecial case where the number of jobs in each group is identical is shown to solvble in $o(\max((n\log n), m \times \max(n, m^2)))$ time.

For future research, it would be interesting to extend our problems to the cases involving multiple agents and in other machine settings.

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