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A Note on Fuzzy Metric Spaces

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Abstract

In this paper, we obtain sufficiently many of fuzzy metric spaces from a metric space. Also, we give some results on FH-bounded (FR-bounded and FM-bounded) fuzzy metric spaces. Further, we obtain sufficiently many of FH-bounded (FR-bounded and FM-bounded) fuzzy metric spaces from a metric spaces. Finally, we present similar results for product fuzzy metric spaces.

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1. Introduction

In 1965, the concept of fuzzy Sets was introduced by zadeh [1]. Since then many authors have expansively developed the theory of fuzzy Sets and applications–Especially, Deng [2], Erceg [3], Kavela and Seokkala [4].kramosil and Michalek[5] have introduced the concept of fuzzy metric spaces in different ways. Recently, many authors have also studied the fixed point theory in these spaces. For more details see [6]. In this work, we obtain sufficiently many of fuzzy metric spaces. Author in [7], has introduced the concept of FM –roundness, FH- roundness by classic selection properties. We show also that one can obtain sufficiently many of FH -bounded (FR-bounded and FM-bounded) fuzzy metric spaces from a metric spaces (X,d).

First, we begin with the following definitions.

Definition1.1. Let X be a nonempty set. A function of X into [0,1] is called a fuzzy set in X.

Definition1.2.A binary operation*:[0,1]× [0,1]→ [0,1] is a continuous t-norm if it satisfies in the following conditions:

(i)* is associative and commutative;

(ii)* is continuous;

(iii)a*1= afor each $a \in [\circ,1]$;

(iv) if $a \le c$ and $b \le d$, then $a * b \le *d$ for each $a,b,c,d \in [\circ,1]$

Example1.3.Consider $* : [1,\circ] \times [\circ,1] \to [1,\circ]$ defined by $a * b = ab$ or $a * b = \min\{a,b\}$.Then * is a continuous t-norm.

Definition1.4. Let X be a nonempty set. * a continuous t-norm and M a fuzzy set on $X^2 \times (0,\alpha)$ which satisfies in the following conditions: $(x,y,z \in X, t,s > \circ)$.

(i) $M(x,y,t) > \circ$;

(ii) $M(x,y,t) = \circ \Leftrightarrow x = y$;

(iii) $M(x,y,t) = M(y,x,t)$;

(iv) $M(x,y,t) * M(y,z,s) \le M(x,z,t+s)$;

(v) $M(x,y,\circ) : (\circ,\infty) \to [\circ,1]$ is continuous.

Then the triple (X, M,*)is called a fuzzy metric spaces.

Let $(X,M,*)$be a fuzzy metric spaces for given $t > \circ, r > \circ$ *and* $x \in X$. An open ball $B(x,r,t)$ is defined as follow:

$$B(x,r,t) = \{y \in X : M(x,y,t) > 1 - r\}$$

Where r is radius and x is the centre of $\beta(x,r,t)$.

Let τ be the set of all $A \subset X$ with $x \in A$ if and only if there exist $t > \circ$ *and* $\circ < r < 1$ such that $B(x,r,t) \subset A$.then τ is a topology on X (induced by fuzzy metric M). This topology is Hausdorff .The collection

$$\{B(x,r,t) : x \in X, r \in (\circ,1), t > \circ\}$$

Is a base of the topology on X. Notice that the collection

$$\left\{B(x,\frac{1}{n},\frac{1}{n}) : x \in X, n \in N\right\}$$

Is also a base of the topology.

Recall that a topology space has the Manger (Rothberger) [Hurewicz] covering property if for each sequence (X_n) of open cover of X there is a sequence $(v_n)[(W_n)]and\ X = \bigcup_{n\in N}\bigcup v_n (X = \bigcup_{n\in N} v_n)$ [each $x \in X$ belongs to $\bigcup_{w_n}$ for all but finitely many n].

Definiton1.5. A fuzzy metric metric spaces $(X, M, *)$ is said to be:

FM: F-Manger- bounded (or FM-bounded);

FR: F-Rothberger- bounded (or FR- bounded);

FH: FM-Hurewicz-bounded (or FH-bounded);

If for each sequence (r_n) of elements of $(\circ,1)$ and each $t > \circ$ there is a sequence

FM: (A_n) of finite subsets of X such that $X = \bigcup_{n\in N}\bigcup_{a\in A_n} N(a, r_n, t)$;

FR: (x_n) of elements of such that $X = \bigcup_{n\in N} B(x_n, r_n, t)$;

FH: (A_n) of finite subsets of X such that for each $x \in X$ there is $n_\circ \in N$ such that such that $X = \bigcup_{\alpha\in A_n} B(a, r_n, t)$ for all $n \geq n_\circ$.

Evidently,

FH-bounded⇨FM-bounded

and

FR-bounded⇨FM-bounded

Definiton1.6. Let $(X, M_X, *) and (Y, M_Y, *)$ be fuzzy metric spaces and let $Z = X \times Y$. For $z_1 = (x_1, y_1), z_2 = (x_2, y_2) \in Z\ and\ t > \circ$ define

$$M_Z(Z_1, Z_1, t) = M_X(x_1, y_1, t) * M_Y(x_2, y_2, t)$$

The $(M_Z, *)$ is a fuzzy metric on Z and the triple $(Z, M_Z, *)$ is called

The product metric spaces of X and Y.

2. Main Results

We begin this section with the following lemma.

Lemma2.1. Let (x, d) be a metric spaces and $r \in IR^+$ .then the function $d : X \times X \to IR$ defined by

$$\tilde{d}_r(x,y) = \frac{d(x,y)}{1+rd(x,y)}$$

Is also a metric on X.

Proof. We only verify the triangle inequality. Define $f : IR^+ \to IR^+$ by

$$f(t) = \frac{t}{1+rt}(t \in IR^+)$$

Obviously F is continuous on IR^+.we have

$$f'(t) = \frac{t}{(1+rt)^2}(t \in IR^+)$$

So, f is strictly increasing on IR^+. Also, we have

$$d(x,y) \leq d((x,z)+d(z,y))$$

This means

$$\begin{aligned}\tilde{d}_r(x,y) &= \frac{d(x,y)}{1+rd(x,y)} \leq \frac{d(x,z)+d(z,y)}{1+r(d(x,z)+d(z,y))} \\ &\leq \frac{d(x,z)}{1+rd(x,z)} + \frac{d(z,y)}{1+rd(z,y)} \\ &= \tilde{d}_r(x,z) + \tilde{d}_r(z,y)\end{aligned}$$

As required.□

The preceding lemma shows that we can obtain infinite many of metric spaces from a given metric space(X,d).

Now we immediately obtain the following result.

Theorem2.2. One can obtain infinite many of fuzzy metric spaces from a given metric spaces(X,d).

Proof. It is well known that d induces a fuzzy metric. By Lemma 2.1, the proof is completed.□

Lemma2.3. Let (X,d) be a metric spaces with the Rothberger property. Consider the standard fuzzy metric M_d on X induced by d defined by

$$M_d(x,y,t) = \frac{t}{t+d(x,y)}(x,y \in X, t > \circ)$$

and denote

$$a * b = ab(a,b \in [\circ,1])$$

Then the fuzzy metric sequence $(X, M_d, *)$ is FR-bounded.

Proof. Let (r_n) be a sequence in $(\circ,1)$ and let $t > \circ$. As (X,d) has the Rothberger covering property, her is a sequence (x_n) in X such

$$X = \bigcup k(x_n, r_n)$$

Where $k(x,r) = \{y \in X : d(x,y) < \varepsilon\}$. Suppose $a \in X$. There exist $n \in \mathrm{N}$ and $x_n \in X$ such that $d(a, x_n) < r_n$

$$M_d(a, x_n, t) = \frac{t}{t + d(a, x_n)} > \frac{t}{t + r_n} = 1 - \frac{r_n}{t + r_n} > 1 - r_n$$

Therefore $a \in B(x_n, r_n, t)$.□

In [8], the previous lemma has been established for the case which (X, d) has the menger property. In a similar manner, one can prove lemma2.2 for which (X, d) has the Hurewicz property.

Theorem2.4.For a given metric space (X, d), we can obtain infinite many of FR- bounded (FM-bounded, FH-bounded) fuzzy metric spaces.

Proof. The result follows from theorem2.2 and lemma2.3.□

Theorem2.5. Let (X, d) and (Y, P) be metric spaces with Hurewicz property then we can obtain infinite many of FH-bounded product fuzzy metric space

Proof. We know from [8, theorem4.4] that the product of two FH-bounded space is also FH-bounded. Now, the result follows from therem2.4.□

Theorem 2.6.let (x, d) and (Y, P) be metric space with menger and Hurewicz property ties, respectively. Then we can obtain in finite many of FM-bounded property fuzzy metric space.

Proof. From [8, theorem4.5],the product of a FM-bounded fuzzy and a FH-bounded fuzzy metric space is FM-bounded. Now, the proof is completed by theorem2.4□.

Indeed, the product $(z, M_z, *)$ of two FR-bounded spaces $(X, M_x, *)$ and $(Y, M_Y, *)$ is also FR-bounded. The next theorem repeats this fact.

Theorem2.7. The product $(z, M_z, *)$ of two FR-bounded space $(X, M_x, *)$ and (Y, M_Y) is also FR-bounded.

Proof. Let $(r_n) \subset (\circ,1)$ $and\, t > \circ$ .for given $n \in N$, pick $\delta_n \in (\circ,1)$ such that $(1-\delta_n) * (1-\delta_n) > 1-r_n$. By assumptions on X and Y, we conclude there exist two sequence (x_n) and (y_n) in X and Y, respectively, such that $X = \bigcup_{n \in N} B(x_n, \delta_n, t/2)$ and $Y = (y_n, \delta_n, t/2)$ .Set $Z_n = (x_n, y_n) \in X_x Y = Z$.

Suppose $z = (x, y) \in Z$. Then there exists $n_\circ \in N$ so that $x \in B(x_{n_\circ}, \delta_{n_\circ}, t/2)$ and $y \in B(y_{n_\circ}, \delta_{n_\circ}, t/2)$. Now, we have

$$M_z(z, z_{,n}, t) = M_x x, x_{n_\circ}, t) * M_Y(y, y_{n_\circ}, t) > (1-\delta_{n_\circ}) * (1-\delta_{n_\circ}) > 1-r_{n_\circ}$$

Which implies $z \in B(z_{n_\circ}, r_{n_\circ}, t)$.□

Now, by theorem2.7 and theorem2.4 we obtain the following result.

Corollary2.8. let (x, d) and (Y, P) be property. Then we can obtain infinite many of FR-bounded product fuzzy metric space.

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