

Bipolar fuzzy n-fold KU-ideal of KU-algebras

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Abstract

In this paper, we apply the notion of a bipolar fuzzy n-fold KU-ideal of KU-algebras. We introduce the concept of a bipolar fuzzy n-fold KU-ideal and investigate several properties. Also, we give relations between a bipolar fuzzy n-fold KU-ideal and n-fold KU-ideal. The image and the pre-image of bipolar fuzzy n-fold KU-ideals in KU-algebras are defined and how the image and the pre-image of bipolar fuzzy n-fold KU-ideals in KU-algebras become bipolar fuzzy n-fold KU-ideals are studied. Moreover, the product of bipolar fuzzy n-fold KU-ideals in Cartesian product KU-algebras is given.

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1 Introduction

In 1956, Zadeh [14] introduced the notion of fuzzy sets. At present this concept has been applied to many mathematical branches. There are several kinds of fuzzy sets extensions in the fuzzy set theory, for example, intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets etc. Lee [5] introduced an extension of fuzzy

sets named bipolar-valued fuzzy sets. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval $[0, 1]$ to $[-1, 1]$. Bipolar-valued fuzzy sets have membership degrees that represent the degree of satisfaction to the property and its counter property. In bipolar-valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree $[-1, 0)$ indicates that elements somewhat satisfy the implicit counter property. In [5, 6, 9 and 12], the authors introduced bipolar-valued fuzzy set on different structures. In [10] and [11] constructed a new algebraic structure which is called KU-algebras. And introduced the concept of homomorphisms for such algebras and investigated some related properties. Mostafa et al [7] introduced the notion of fuzzy KU-ideals of KU-algebras and then they investigated several basic properties which are related to fuzzy KU-ideals. Akram et al and Yaqoob et al [1, 13] introduced the notion of cubic sub-algebras and ideals in KU-algebras. They discussed relationship between a cubic subalgebra and a cubic KU-ideal. Muhiuddin [9] applied the bipolar-valued fuzzy set theory to KU-algebras, and introduced the notions of bipolar fuzzy KU-subalgebras and bipolar fuzzy KU-ideals in KU-algebras. He considered the specifications of a bipolar fuzzy KU-subalgebra, a bipolar fuzzy KU-ideal in KU-algebras and discussed the relations between a bipolar fuzzy KU-subalgebra, a bipolar fuzzy KU-ideal and provided conditions for a bipolar fuzzy KU-subalgebra to be a bipolar fuzzy KU-ideal. Gulistan et al [4] studied (α, β) -fuzzy KU-ideals in KU-algebras and discussed some special properties. Mostafa and Kareem [8] introduced n-fold KU-ideals and obtained some related results. Akram et al [1] introduced the notion of interval-valued $(\tilde{\theta}, \tilde{\delta})$ -fuzzy KU-ideals of KU-algebras and obtained some related properties. In this paper, we will introduce a generalization of bipolar fuzzy KU-ideal of KU-algebras [9]. Therefore, a few properties similar to the properties of bipolar fuzzy KU-ideal in KU-algebras can be obtained. Also, few results of bipolar fuzzy n-fold KU-ideals of KU-algebra under homomorphism have been discussed. Moreover, some algorithms for folding theory have been constructed.

2 Preliminaries

Now we review some concepts related to KU-algebra and bipolar fuzzy logic.

Definition 2.1. [10,11] Let X be a nonempty set with a binary operation $*$ and a constant $\mathbf{0}$, then $(X, *, \mathbf{0})$ is called a KU -algebra, if for all $x, y, z \in X$ the following axioms are holds:

$$(ku_1) \quad (x * y) * [(y * z) * (x * z)] = 0,$$

$$(ku_2) \quad x * \mathbf{0} = 0,$$

$$(ku_3) \quad \mathbf{0} * x = x,$$

$$(ku_4) \quad x * y = \mathbf{0} \text{ and } y * x = 0 \text{ implies } x = y,$$

$$(ku_5) x * x = \mathbf{0},$$

On a KU-algebra $(X, *, \mathbf{0})$ we can define a binary relation \leq on X by putting:
 $x \leq y \Leftrightarrow y * x = \mathbf{0}$. Then (X, \leq) is a partially ordered set and $\mathbf{0}$ is its smallest element. Thus $(X, *, \mathbf{0})$ satisfies the following conditions: for all $x, y, z \in X$

$$(ku_1): (y * z) * (x * z) \leq (x * y)$$

$$(ku_2): 0 \leq x$$

$$(ku_3): x \leq y, y \leq x \text{ implies } x = y,$$

$$(ku_4): y * x \leq x.$$

Theorem 2.2. [7]: In a KU-algebra $(X, *, 0)$, the following axioms are satisfied:
 For all $x, y, z \in X$,

$$(1): x \leq y \text{ imply } y * z \leq x * z,$$

$$(2): x * (y * z) = y * (x * z), \text{ for all } x, y, z \in X,$$

$$(3): ((y * x) * x) \leq y.$$

Definition 2.3[10]. A non-empty subset S of a KU-algebra $(X, *, 0)$ is called a KU-sub algebra of X if $x * y \in S$ whenever $x, y \in S$.

Definition 2.4[10,11]. A non-empty subset I of a KU-algebra $(X, *, 0)$ is called an ideal of X if for any $x, y \in X$,

$$(i) \mathbf{0} \in I,$$

$$(ii) x * y, x \in I \text{ imply } y \in I.$$

Definition 2.5. [7] A subset I of a KU-algebra X is said to be an KU-ideal of X , if

$$(I_1) 0 \in I$$

$$(I_2) \forall x, y, z \in X, \text{ if } x * (y * z) \in I \text{ and } y \in I, \text{ imply } x * z \in I.$$

For any elements x and y of a KU-algebra X , $x^n * y$ denotes $x * (x * \dots * (x * y))$,

where n is positive integer and x occurs n times.

Definition 2.6[8]. A nonempty subset I of a KU-algebra X is called an n -fold KU-ideal of X if

$$(1) 0 \in I$$

(2) $\forall x, y, z \in X$ there exists a natural number n such that $x^n * z \in I$ whenever $x^n * (y * z) \in I$ and $y \in I$.

Obviously, $\{0\}$ and X itself are n -fold KU-ideal of X for every positive integer n .

We will refer to X is a KU-algebra unless otherwise indicated.

Definition 2.7. [5] A bipolar valued fuzzy subset B in a nonempty set X is an object having the form $B = \{(x, \mu^-(x), \mu^+(x)) \mid x \in X\}$ where $\mu^- : X \rightarrow [-1, 0]$ and $\mu^+ : X \rightarrow [0, 1]$ are mappings. The positive membership degree $\mu^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar-valued fuzzy set $B = \{(x, \mu^-(x), \mu^+(x)) \mid x \in X\}$, and the negative membership degree $\mu^-(x)$ denotes the satisfaction degree of x to some implicit counter-property of a bipolar-valued fuzzy set $B = \{(x, \mu^-(x), \mu^+(x)) \mid x \in X\}$. For simplicity, we shall use the symbol $B = (x, \mu^-, \mu^+)$ for bipolar fuzzy set $B = \{(x, \mu^-(x), \mu^+(x)) \mid x \in X\}$, and use the notion of bipolar fuzzy sets instead of the notion of bipolar-valued fuzzy sets.

Definition 2.8[5]. Let $B = (x, \mu^-, \mu^+)$ be a bipolar fuzzy set and $(s, t) \in [-1, 0] \times [0, 1]$.

The set $B_s^- = \{x \in X : \mu^-(x) \leq s\}$ and $B_t^+ = \{x \in X : \mu^+(x) \geq t\}$ which are called the negative s -cut and the positive t -cut of $B = (x, \mu^-, \mu^+)$, respectively.

Definition 2.9[9]. A bipolar fuzzy set $B = (x, \mu^-, \mu^+)$ in a KU-algebra $(X, *, 0)$ is called a bipolar fuzzy KU-subalgebra of X if it satisfies the following condition: For all $x, y \in X$,

$$\mu^-(x * y) \leq \max\{\mu^-(x), \mu^-(y)\}, \mu^+(x * y) \geq \min\{\mu^+(x), \mu^+(y)\}.$$

Lemma 2.10. If $B = (x, \mu^-, \mu^+)$ is a bipolar fuzzy KU-subalgebra of X , then $\mu^-(0) \leq \mu^-(x)$ and $\mu^+(0) \geq \mu^+(x)$.

Proof: by using (ku_5) and Definition 2.9. the proof is complete.

Example 2.11. Let $X = \{0, 1, 2, 3\}$ be a set with the following table:

*	0	1	2	3
0	0	1	2	3
1	0	0	0	2
2	0	2	0	1
3	0	0	0	0

Using the algorithms in Appendix A, we can prove that $(X, *, 0)$ is a KU- algebra.

Define a bipolar fuzzy set $B = (x, \mu^-, \mu^+)$ by

$B = \{(0, -0.7, 0.8), (1, -0.6, 0.6), (2, -0.4, 0.5), (3, -0.3, 0.2)\}$. It is easy to verify that

$B = (x, \mu^-, \mu^+)$ is a bipolar fuzzy KU-subalgebra of X .

Defintion2.12. A bipolar fuzzy set $B = (x, \mu^-, \mu^+)$ in X is called a bipolar fuzzy ideal of X if it satisfies the following condition: for all $x, y \in X$

(b₁) $\mu^-(0) \leq \mu^-(x)$ and $\mu^+(0) \geq \mu^+(x)$,

(b₂) $\mu^-(y) \leq \max\{\mu^-(x * y), \mu^-(x)\}$, $\mu^+(y) \geq \min\{\mu^+(x * y), \mu^+(x)\}$.

Defintion2.13. [9] A bipolar fuzzy set $B = (x, \mu^-, \mu^+)$ in X is called a bipolar fuzzy KU-ideal of X if it satisfies the following condition: for all $x, y, z \in X$

(B₁) $\mu^-(0) \leq \mu^-(x)$ and $\mu^+(0) \geq \mu^+(x)$,

(B₂)

$\mu^-(x * z) \leq \max\{\mu^-(x * (y * z)), \mu^-(y)\}$, $\mu^+(x * z) \geq \min\{\mu^+(x * (y * z)), \mu^+(y)\}$.

Proposition 2.14. Every a bipolar fuzzy KU-ideal of X is a bipolar fuzzy ideal of X .

Proof: clear.

Proposition 2.15. If $B = (x, \mu^-, \mu^+)$ is a bipolar fuzzy KU-ideal of X and $x \leq y$, then $\mu^-(x) \leq \mu^-(y)$ and $\mu^+(x) \geq \mu^+(y)$.

Proof:

If $x \leq y$, then $y * x = 0$ and by (ku_3) $0 * x = x$, since $B = (x, \mu^-, \mu^+)$ is a bipolar fuzzy KU-ideal of X , we get

$$\begin{aligned} \mu^-(0 * x) &= \mu^-(x) \leq \max\{\mu^-(0 * (y * x)), \mu^-(y)\} = \max\{\mu^-(0 * 0), \mu^-(y)\} \\ &= \max\{\mu^-(0), \mu^-(y)\} = \mu^-(y). \end{aligned}$$

And

$$\begin{aligned} \mu^+(0 * x) &= \mu^+(x) \geq \min\{\mu^+(0 * (y * x)), \mu^+(y)\} = \min\{\mu^+(0 * 0), \mu^+(y)\} \\ &= \min\{\mu^+(0), \mu^+(y)\} = \mu^+(y). \end{aligned}$$

Theorem 2.16.[6] Every a bipolar fuzzy KU-ideal of X is a bipolar fuzzy KU-subalgebra of X .

3- Bipolar fuzzy n-fold KU-ideals

Now, we give the definition and properties of a bipolar fuzzy n-fold KU-ideal of X , where n is positive integer.

Definition 3.1. A bipolar fuzzy set $B = (x, \mu^-, \mu^+)$ in X is called a bipolar fuzzy n-fold KU-ideal of X if it satisfies the following condition:

$$\begin{aligned} \text{(Bf}_1\text{)} \quad & \mu^-(0) \leq \mu^-(x) \text{ and } \mu^+(0) \geq \mu^+(x), \\ \text{(Bf}_2\text{)} \quad & \mu^-(x^n * z) \leq \max\{\mu^-(x^n * (y * z)), \mu^-(y)\}, \\ & \mu^+(x^n * z) \geq \min\{\mu^+(x^n * (y * z)), \mu^+(y)\}. \end{aligned}$$

For all $x, y, z \in X$.

Proposition 3.2. Every a bipolar fuzzy n-fold KU-ideal of X is a bipolar fuzzy ideal of X .

Proof : Let $B = (x, \mu^-, \mu^+)$ be a bipolar fuzzy n-fold KU-ideal of X . If we put $x = 0$ in (Bf₂), then

$$\mu^-(z) \leq \max\{\mu^-(y * z), \mu^-(y)\} \text{ and } \mu^+(z) \geq \min\{\mu^+(y * z), \mu^+(y)\}.$$

Hence $B = (x, \mu^-, \mu^+)$ is a bipolar fuzzy ideal of X .

Proposition 3.3. Every a bipolar fuzzy n-fold KU-ideal of X is satisfies the following condition. For all $x, y \in X$

$$\mu^-(x^n * y) \leq \mu^-(y) \text{ and } \mu^+(x^n * y) \geq \mu^+(y).$$

Proof: Let $B = (x, \mu^-, \mu^+)$ be a bipolar fuzzy n-fold KU-ideal of X . If we put $y = z$ in (Bf₂), and by using ku_2 and ku_5 . We get:

$$\begin{aligned} \mu^-(x^n * y) & \leq \max\{\mu^-(x^n * (y * y)), \mu^-(y)\} = \max\{\mu^-(x^n * 0), \mu^-(y)\} \\ & = \max\{\mu^-(x * (x * \dots * (x * 0))), \mu^-(y)\} = \max\{\mu^-(0), \mu^-(y)\} = \mu^-(y). \end{aligned}$$

And

$$\begin{aligned} \mu^+(x^n * y) & \geq \min\{\mu^+(x^n * (y * y)), \mu^+(y)\} = \min\{\mu^+(x^n * 0), \mu^+(y)\} \\ & = \min\{\mu^+(x * (x * \dots * (x * 0))), \mu^+(y)\} = \min\{\mu^+(0), \mu^+(y)\} = \mu^+(y). \end{aligned}$$

For all $x, y \in X$.

Proposition 3.4. Let $B = (x, \mu^-, \mu^+)$ be a bipolar fuzzy n-fold KU-ideal of X . If the inequality $x^n * y \leq z$ holds in X , then

$$\mu^-(y) \leq \max\{\mu^-(x^n), \mu^-(z)\} \text{ and } \mu^+(y) \geq \min\{\mu^+(x^n), \mu^+(z)\} \text{ for all } x, y, z \in X.$$

Proof: Assume that the inequality $x^n * y \leq z$ holds in X , then $z * (x^n * y) = 0$ and by (Bf₂)

$$\begin{aligned} \mu^-(x^n * y) &\leq \max\{\mu^-(x^n * (z * y)), \mu^-(z)\} = \max\{\mu^-(z * (x^n * y)), \mu^-(z)\} \\ &= \max\{\mu^-(0), \mu^-(z)\} = \mu^-(z) \dots \dots \dots (1) \end{aligned}$$

Now $\mu^-(0 * y) = \mu^-(y) \leq \max\{\mu^-(0 * (x^n * y)), \mu^-(x^n)\}$
 $= \max\{\mu^-(x^n * y), \mu^-(x^n)\} \leq \max\{\mu^-(z), \mu^-(x^n)\}$ (by using (1))

i.e $\mu^-(y) \leq \max\{\mu^-(x^n), \mu^-(z)\}$.

Similarly

$$\begin{aligned} \mu^+(x^n * y) &\geq \min\{\mu^+(x^n * (z * y)), \mu^+(z)\} = \min\{\mu^+(z * (x^n * y)), \mu^+(z)\} \\ &= \min\{\mu^+(0), \mu^+(z)\} = \mu^+(z) \dots \dots \dots (2) \end{aligned}$$

Now, $\mu^+(0 * y) = \mu^+(y) \geq \min\{\mu^+(0 * (x^n * y)), \mu^+(x^n)\} = \min\{\mu^+(x^n * y), \mu^+(x^n)\}$
 $\geq \min\{\mu^+(z), \mu^+(x^n)\}$ (by using (2))

i.e $\mu^+(y) \geq \min\{\mu^+(x^n), \mu^+(z)\}$.

Theorem 3.5. If a bipolar fuzzy KU-subalgebra $B = (x, \mu^-, \mu^+)$ of X satisfies the condition in Proposition 3.4, then $B = (x, \mu^-, \mu^+)$ is a bipolar fuzzy n-fold KU-ideal of X .

Proof: Let $B = (x, \mu^-, \mu^+)$ be a bipolar fuzzy KU-subalgebra that satisfies the condition in Proposition 3.4, by Lemma 2.10. We have $\mu^-(0) \leq \mu^-(x)$ and $\mu^+(0) \geq \mu^+(x)$ for all $x \in X$. By Th.2.2 (3) we have, $(x^n * (y * z)) * (x^n * z) \leq y$ for all $x, y, z \in X$, it follows (from Proposition 3.4), that $\mu^-(x^n * z) \leq \max\{\mu^-(x^n * (y * z)), \mu^-(y)\}$ and $\mu^+(x^n * z) \geq \min\{\mu^+(x^n * (y * z)), \mu^+(y)\}$ for all $x, y, z \in X$. Therefore $B = (x, \mu^-, \mu^+)$ is a bipolar fuzzy n-fold KU-ideal of X .

Proposition 3.6. If $B = (x, \mu^-, \mu^+)$ is a bipolar fuzzy n-fold KU-ideal of X , then $\mu^-(x^n * (y * z)) \leq \mu^-(x^n * z)$ and $\mu^+(x^n * (y * z)) \geq \mu^+(x^n * z)$.

Proof: By ku_3, ku_5 and Th. (2.2), we get

$$\begin{aligned} (x^n * z) * (x^n * (y * z)) &= x^n * ((x^n * z) * (y * z)) = x^n * (y * (x^n * z) * z) = y * (x^n * ((x^n * z) * z)) \\ &= y * ((x^n * z) * (x^n * z)) = y * 0 = 0, \text{ and by using the property of the binary relation } \leq, \text{ we have} \\ (x^n * (y * z)) &\leq (x^n * z), \text{ and hence by Proposition 2.15, we get } \mu^-(x^n * (y * z)) \leq \mu^-(x^n * z) \text{ and} \\ \mu^+(x^n * (y * z)) &\geq \mu^+(x^n * z). \end{aligned}$$

Proposition 3.7. If $B = (x, \mu^-, \mu^+)$ is a bipolar fuzzy n-fold KU-ideal of X , then the sets $J = \{x \in X : \mu^+(x) = \mu^+(0)\}$ and $K = \{x \in X : \mu^-(x) = \mu^-(0)\}$ are n-fold KU-ideal of X .

Proof: since $0 \in X$, $\mu^+(0) = \mu^+(0)$ and $\mu^-(0) = \mu^-(0)$ implies $0 \in J$ and $0 \in K$, so $J \neq \phi, K \neq \phi$. Let $(x^n * (y * z)) \in J$ and $y \in J$ implies

$$\mu^+(x^n * (y * z)) = \mu^+(0) \text{ and}$$

$$\mu^+(y) = \mu^+(0). \text{ Since}$$

$$\mu^+(x^n * z) \geq \min\{\mu^+(x^n * (y * z)), \mu^+(y)\} = \mu^+(0) \Rightarrow \mu^+(x^n * z) \geq \mu^+(0) \text{ but}$$

$\mu^+(0) = \mu^+(x^n * z)$, it follows that $(x^n * z) \in J$, for all $x, y, z \in X$. Hence J is n-fold KU-ideal of X . Similarly we can prove K is n-fold KU-ideal of X .

Theorem 3.8. For a bipolar fuzzy set $B = (x, \mu^-, \mu^+)$ in X , the following are equivalent:

(1) $B = (x, \mu^-, \mu^+)$ is a bipolar fuzzy n-fold KU-ideal of X .

(2) $B = (x, \mu^-, \mu^+)$ is satisfies the following:

(i) $(\forall s \in [-1,0])(B_s^- \neq \phi \Rightarrow B_s^- \text{ is an n-fold KU-ideal of } X)$.

(ii) $(\forall t \in [0,1])(B_t^+ \neq \phi \Rightarrow B_t^+ \text{ is an n-fold KU-ideal of } X)$.

Proof: (1) \Rightarrow (2). (i) Let $s \in [-1,0]$ be such that $B_s^- \neq \phi$. Then there exists $y \in B_s^-$, and so $\mu^-(y) \leq s$. It follows from (Bf₁) that $\mu^-(0) \leq \mu^-(y) \leq s$ so that $0 \in B_s^-$.

Let $x, y, z \in X$ such that $(x^n * (y * z)) \in B_s^-$ and $y \in B_s^-$. Then $\mu^-(x^n * (y * z)) \leq s$ and $\mu^-(y) \leq s$. Using (Bf₂), we have

$$\mu^-(x^n * z) \leq \max\{\mu^-(x^n * (y * z)), \mu^-(y)\} \leq s \text{ which implies that } (x^n * z) \in B_s^-.$$

Therefore B_s^- is an n-fold KU-ideal of X .

(ii) Assume that $B_t^+ \neq \phi$ for $t \in [0,1]$, and let $a \in B_t^+$. Then $\mu^+(a) \geq t$, and so

$$\mu^+(0) \geq \mu^+(a) \geq t \text{ by (Bf}_1\text{)}, \text{ thus } 0 \in B_t^+.$$

Let $x, y, z \in X$ be such that $(x^n * (y * z)) \in B_t^+$ and $y \in B_t^+$. Then $\mu^+(x^n * (y * z)) \geq t$ and $\mu^+(y) \geq t$. It follows from (Bf₂) that $\mu^+(x^n * z) \geq \min\{\mu^+(x^n * (y * z)), \mu^+(y)\} \geq t$ so that $(x^n * z) \in B_t^+$.

Hence B_t^+ is an n-fold KU-ideal of X .

(2) \Rightarrow (1) Assume that there exists $a \in X$ such that $\mu^-(0) \geq \mu^-(a)$. Taking

$$s_0 = \frac{1}{2}(\mu^-(0) + \mu^-(a)), \text{ for some } s_0 \in [-1,0] \text{ implies } \mu^-(a) < s_0 < \mu^-(0). \text{ This is}$$

a contradiction, and thus $\mu^-(0) \leq \mu^-(y)$ for all $y \in X$. Suppose that

$$\mu^-(x^n * z) \leq \max\{\mu^-(x^n * (y * z)), \mu^-(y)\} \text{ for some } x, y, z \in X \text{ and let}$$

$s_1 = \frac{1}{2}(\mu^-(x^n * z) + \max\{\mu^-(x^n * (y * z)), \mu^-(y)\})$. Then $\max\{\mu^-(x^n * (y * z)), \mu^-(y)\} < s_1 < \mu^-(x^n * z)$, which is a contradiction. Therefore $\mu^-(x^n * z) \leq \max\{\mu^-(x^n * (y * z)), \mu^-(y)\}$ for all $x, y, z \in X$. Now, if $\mu^+(0) < \mu^+(y)$ for some $y \in X$, then $\mu^+(0) < t_0 < \mu^+(y)$ for some $t_0 \in (0,1]$. This is a contradiction. Thus $\mu^+(0) \geq \mu^+(y)$ for all $y \in X$. If $\mu^+(x^n * z) \geq \min\{\mu^+(x^n * (y * z)), \mu^+(y)\}$ for some $x, y, z \in X$, then there exists $t_1 \in (0,1]$ such that $\mu^+(x^n * z) < t_1 \leq \min\{\mu^+(x^n * (y * z)), \mu^+(y)\}$. It follows that $x^n * (y * z) \in B_{t_1}^+$ and $y \in B_{t_1}^+$ but $(x^n * z) \notin B_{t_1}^+$, a contradiction. Consequently, $\mu^+(x^n * z) \geq \min\{\mu^+(x^n * (y * z)), \mu^+(y)\}$ for all $x, y, z \in X$. Therefore $B = (x, \mu^-, \mu^+)$ is a bipolar fuzzy n-fold KU-ideal of X .

4- Bipolar fuzzy n-fold KU-ideal under homomorphism

Definition 4.1[11]. Let $(X, *, 0)$ and $(X', *', 0')$ be KU-algebras, a homomorphism is a map $f : X \rightarrow X'$ satisfying $f(x * y) = f(x) *' f(y)$ for all $x, y \in X$. Note that if $f : X \rightarrow X'$ is a KU-homomorphism, then $f(0) = 0'$ and for any $x \in X$ we have $f(x^n) = (f(x))^n$.

Definition 4.2. Let $f : X \rightarrow X'$ be a homomorphism of KU-algebras. For any bipolar fuzzy set $B = (x, \mu^-, \mu^+)$, we define a new bipolar fuzzy set $B_f = (x, \mu_f^-, \mu_f^+)$ in X by $\mu_f^-(x) = \mu^-(f(x))$ and $\mu_f^+(x) = \mu^+(f(x))$, for any $x \in X$.

Theorem 4.3. Let $(X, *, 0)$ and $(X', *', 0')$ be KU-algebras and f a homomorphism from X onto X' . Then $B = (x', \mu^-, \mu^+)$ is a bipolar fuzzy n-fold KU-ideal of X' if and only if $B_f = (x, \mu_f^-, \mu_f^+)$ is a bipolar fuzzy n-fold KU-ideal of X .

Proof:(\Rightarrow) For $x' \in X'$ there exists $x \in X$ such that $f(x) = x'$, we have

$$\mu_f^-(0) = \mu^-(f(0)) = \mu^-(0') \leq \mu^-(x') = \mu^-(f(x)) = \mu_f^-(x)$$

$$\text{and } \mu_f^+(0) = \mu^+(f(0)) = \mu^+(0') \geq \mu^+(x') = \mu^+(f(x)) = \mu_f^+(x).$$

Let $x, z \in X$, $y' \in X'$ then there exists $y \in X$ such that $f(y) = y'$. We have

$$\mu_f^-(x^n * z) = \mu^-(f(x^n * z)) = \mu^-(f(x^n) * f(z)) \leq \max\{\mu^-(f(x^n) * (y' * f(z))), \mu^-(y')\} = \max\{\mu^-(f(x^n) * (f(y) * f(z))), \mu^-(f(y))\} = \max\{\mu_f^-(x^n * (y * z)), \mu_f^-(y)\}.$$

And

$$\begin{aligned}\mu_f^+(x^n * z) &= \mu^+(f(x^n * z)) = \mu^+(f(x^n) * f(z)) \geq \min\{\mu^+(f(x^n) * (y' * f(z))), \mu^+(y')\} = \\ &= \min\{\mu^+(f(x^n) * (f(y) * f(z))), \mu^+(f(y))\} = \min\{\mu_f^+(x^n * (y * z)), \mu_f^+(y)\}.\end{aligned}$$

Hence $B_f = (x, \mu_f^-, \mu_f^+)$ is a bipolar fuzzy n-fold KU-ideal of X .

(\Leftarrow) Since $f : X \rightarrow X'$ onto, for $x, y, z \in X'$ there exists $a, b, c \in X$ such that $f(a) = x, f(b) = y$ and $f(c) = z$.

Now,

$$\begin{aligned}\mu^-(x^n * z) &= \mu^-((f(a))^n * f(c)) = \mu^-(f(a^n * c)) = \mu_f^-(a^n * c) \leq \max\{\mu_f^-(a^n * (b * c)), \mu_f^-(b)\} \\ &= \max\{\mu^-(f(a^n) * (f(b) * f(c))), \mu^-(f(b))\} = \max\{\mu^-(x^n * (y * z)), \mu^-(y)\}\end{aligned}$$

And

$$\begin{aligned}\mu^+(x^n * z) &= \mu^+((f(a))^n * f(c)) = \mu^+(f(a^n * c)) = \mu_f^+(a^n * c) \geq \min\{\mu_f^+(a^n * (b * c)), \mu_f^+(b)\} \\ &= \min\{\mu^+(f(a^n) * (f(b) * f(c))), \mu^+(f(b))\} = \min\{\mu^+(x^n * (y * z)), \mu^+(y)\}\end{aligned}$$

Hence $B = (x, \mu^-, \mu^+)$ is a bipolar fuzzy n-fold KU-ideal of X' .

Definition 4.4. Let f be a mapping from the set X to a set Y . If $B = (x, \mu^-, \mu^+)$ is a bipolar fuzzy set of KU-algebra X then the bipolar fuzzy set of Y defined by

$$f(\mu^+)(y) = B^+(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu^+(x), & \text{if } f^{-1}(y) = \{x \in X : f(x) = y\} \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

Is said to be the image of μ^+ under f .

$$\text{And } f(\mu^-)(y) = B^-(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \mu^-(x), & \text{if } f^{-1}(y) = \{x \in X : f(x) = y\} \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

Is said to be the image of μ^- under f .

Definition 4.5. Let $B = (x, \mu^-, \mu^+)$ be a bipolar fuzzy set of KU-algebra X then we say that μ^+ has ‘Sup’ property, if for any subset $T \subseteq X$ there exists $x_0 \in T$ such that $\mu^+(x_0) = \sup_{t \in T} \mu^+(t)$ and we say that μ^- has ‘inf’ property, if for any subset $S \subseteq T$ there exists $y_0 \in T$ such that $\mu^-(y_0) = \inf_{s \in S} \mu^-(s)$.

Theorem 5.6. Let $f : X \rightarrow X'$ be an onto homomorphism of KU-algebras. If $B = (x, \mu^-, \mu^+)$ is a bipolar fuzzy n-fold KU-ideal of X with μ^+ has ‘Sup’ property and μ^- has ‘inf’ property then the image of B under f is also a bipolar fuzzy n-fold KU-ideal of X' .

Proof : By definition $B^+(x') = f(\mu^+)(x') := \sup_{x \in f^{-1}(x')} \mu^+(x)$, and

$B^-(x') = f(\mu^-)(x') := \inf_{x \in f^{-1}(x')} \mu^-(x)$ for all $x' \in X'$ and $\sup \phi := 0$. We have to

prove that

$$B^+((x')^n * z') \geq \min\{B^+((x')^n * (y' * z')), B^+(y')\} \text{ and}$$

$$B^-((x')^n * z') \leq \max\{B^-((x')^n * (y' * z')), B^-(y')\} \quad \forall x', y', z' \in X' .$$

Let $f : X \rightarrow X'$ be an onto a homomorphism of KU-algebras, $B = (x, \mu^-, \mu^+)$ be a fuzzy bipolar n -fold KU-ideal of X with “sup” property and ‘inf ‘property of X . Since $B = (x, \mu^-, \mu^+)$ is a fuzzy bipolar n -fold KU-ideal of X , Then we have $\mu^+(0) \geq \mu^+(x)$ and $\mu^-(0) \leq \mu^-(x)$ for all $x \in X$. Note that $0 \in f^{-1}(0')$, where $0, 0'$ are the zero of X and X' respectively, Thus

$$B^+(0') = \sup_{t \in f^{-1}(0')} \mu^+(t) = \mu^+(0) \geq \mu^+(x), \text{ and}$$

$$B^-(0') = \inf_{t \in f^{-1}(0')} \mu^-(t) = \mu^-(0) \leq \mu^-(x), \text{ for all } x \in X, \text{ since } B = (x, \mu^-, \mu^+) \text{ have}$$

“sup” and ‘inf ‘properties of X . Which implies that

$$B^+(0') \geq \sup_{t \in f^{-1}(x')} \mu^+(t) = B^+(x'), \text{ and } B^-(0') \leq \inf_{t \in f^{-1}(x')} \mu^-(t) = B^-(x'), \text{ for any}$$

$x' \in X'$.

For any $x', y', z' \in X'$, let $x_0 \in f^{-1}(x')$, $y_0 \in f^{-1}(y')$, $z_0 \in f^{-1}(z')$ be such that

$$\mu^+((x_0)^n * z_0) = \sup_{t \in f^{-1}((x')^n * z')} \mu^+(t) , \mu^+(y_0) = \sup_{t \in f^{-1}(y')} \mu^+(t),$$

$$\mu^-((x_0)^n * z_0) = \inf_{t \in f^{-1}((x')^n * z')} \mu^-(t) , \mu^-(y_0) = \inf_{t \in f^{-1}(y')} \mu^-(t)$$

And

$$\begin{aligned} \mu^+((x_0)^n * (y_0 * z_0)) &= B^+\{f((x_0)^n * (y_0 * z_0))\} = B^+((x')^n * (y' * z')) \\ &= \sup_{((x_0)^n * (y_0 * z_0)) \in f^{-1}((x')^n * (y' * z'))} \mu^+((x_0)^n * (y_0 * z_0)) = \sup_{t \in f^{-1}((x')^n * (y' * z'))} \mu^+(t), \end{aligned}$$

$$\begin{aligned} \mu^-((x_0)^n * (y_0 * z_0)) &= B^-\{f((x_0)^n * (y_0 * z_0))\} = B^-((x')^n * (y' * z')) \\ &= \inf_{((x_0)^n * (y_0 * z_0)) \in f^{-1}((x')^n * (y' * z'))} \mu^-((x_0)^n * (y_0 * z_0)) = \inf_{t \in f^{-1}((x')^n * (y' * z'))} \mu^-(t). \end{aligned}$$

Then

$$\begin{aligned}
 B^+((x')^n * z') &= \sup_{t \in f^{-1}((x')^n * z')} \mu^+(t) = \mu^+((x_0)^n * z_0) \\
 &\geq \min\{\mu^+((x_0)^n * (y_0 * z_0)), \mu^+(y_0)\} = \\
 &\min\left\{ \sup_{t \in f^{-1}((x')^n * (y' * z'))} \mu^+(t), \sup_{t \in f^{-1}(y')} \mu^+(t) \right\} = \\
 &\min\{B^+((x')^n * (y' * z')), B^+(y')\}, \text{ and } . \\
 B^-((x')^n * z') &= \inf_{t \in f^{-1}((x')^n * z')} \mu^-(t) = \mu^-((x_0)^n * z_0) \\
 &\leq \max\{\mu^-((x_0)^n * (y_0 * z_0)), \mu^-(y_0)\} = \\
 &\max\left\{ \inf_{t \in f^{-1}((x')^n * (y' * z'))} \mu^-(t), \inf_{t \in f^{-1}(y')} \mu^-(t) \right\} = \max\{B^-((x')^n * (y' * z')), B^-(y')\}.
 \end{aligned}$$

Hence B under f is a bipolar fuzzy n-fold KU-ideal of X' .

5- Product of bipolar fuzzy n-fold KU-ideal of KU-algebra

In this section, product of bipolar fuzzy n-fold KU-ideals in KU-algebra is defined and some results are studied.

Definition 5.1. Let $B_1 = (x, \mu_1^-, \mu_1^+)$ and $B_2 = (y, \mu_2^-, \mu_2^+)$ be two bipolar fuzzy sets of X . The Cartesian product $B_1 \times B_2 = ((x, y), \mu_1^- \times \mu_2^-, \mu_1^+ \times \mu_2^+)$ is defined by $(\mu_1^- \times \mu_2^-)(x, y) = \max\{\mu_1^-(x), \mu_2^-(y)\}$ and $(\mu_1^+ \times \mu_2^+)(x, y) = \min\{\mu_1^+(x), \mu_2^+(y)\}$, where $\mu_1^- \times \mu_2^- : X \times X \rightarrow [-1, 0]$ and $\mu_1^+ \times \mu_2^+ : X \times X \rightarrow [0, 1]$ for all $x, y \in X$.

Proposition 5.2. Let $B_1 = (x, \mu_1^-, \mu_1^+)$ and $B_2 = (y, \mu_2^-, \mu_2^+)$ be two bipolar fuzzy n-fold KU-ideals of KU-algebra X , then $B_1 \times B_2$ is bipolar fuzzy n-fold KU-ideal of $X \times X$.

Proof. For any $(x, y) \in X \times X$, we have

$$\begin{aligned}
 (\mu_1^- \times \mu_2^-)(0, 0) &= \max\{\mu_1^-(0), \mu_2^-(0)\} \leq \max\{\mu_1^-(x), \mu_2^-(y)\} = (\mu_1^- \times \mu_2^-)(x, y) \text{ and} \\
 (\mu_1^+ \times \mu_2^+)(0, 0) &= \min\{\mu_1^+(0), \mu_2^+(0)\} \geq \min\{\mu_1^+(x), \mu_2^+(y)\} = (\mu_1^+ \times \mu_2^+)(x, y).
 \end{aligned}$$

Let $(x_1, x_2), (y_1, y_2)$ and $(z_1, z_2) \in X \times X$, then

$$\begin{aligned} (\mu_1^- \times \mu_2^-)(x_1^n * z_1, x_2^n * z_2) &= \max\{\mu_1^-(x_1^n * z_1), \mu_2^-(x_2^n * z_2)\} \leq \\ &\max\{\max\{\mu_1^-(x_1^n * (y_1 * z_1)), \mu_1^-(y_1)\}, \max\{\mu_2^-(x_2^n * (y_2 * z_2)), \mu_2^-(y_2)\}\} \\ &= \max\{\max\{\mu_1^-(x_1^n * (y_1 * z_1)), \mu_2^-(x_2^n * (y_2 * z_2))\}, \max\{\mu_1^-(y_1), \mu_2^-(y_2)\}\} \\ &= \max\{(\mu_1^- \times \mu_2^-)\{(x_1^n * (y_1 * z_1)), (x_2^n * (y_2 * z_2))\}, (\mu_1^- \times \mu_2^-)(y_1, y_2)\} \end{aligned}$$

And

$$\begin{aligned} (\mu_1^+ \times \mu_2^+)(x_1^n * z_1, x_2^n * z_2) &= \min\{\mu_1^+(x_1^n * z_1), \mu_2^+(x_2^n * z_2)\} \geq \\ &\min\{\min\{\mu_1^+(x_1^n * (y_1 * z_1)), \mu_1^+(y_1)\}, \min\{\mu_2^+(x_2^n * (y_2 * z_2)), \mu_2^+(y_2)\}\} \\ &= \min\{\min\{\mu_1^+(x_1^n * (y_1 * z_1)), \mu_2^+(x_2^n * (y_2 * z_2))\}, \min\{\mu_1^+(y_1), \mu_2^+(y_2)\}\} \\ &= \min\{(\mu_1^+ \times \mu_2^+)\{(x_1^n * (y_1 * z_1)), (x_2^n * (y_2 * z_2))\}, (\mu_1^+ \times \mu_2^+)(y_1, y_2)\}. \end{aligned}$$

Hence $B_1 \times B_2$ is bipolar fuzzy n-fold KU-ideal of $X \times X$.

Definition 5.3. Let $B = (x, \mu^-, \mu^+)$ be a bipolar fuzzy set of KU-algebra X . The operator $\oplus B$ and $\otimes B$ are defined as $\oplus B = (\mu^+, (\mu^+)^c)$ and $\otimes B = ((\mu^-)^c, \mu^-)$ in X , where $(\mu^+)^c = 1 - \mu^+$ and $(\mu^-)^c = 1 - \mu^-$.

Lemma 5.4. If $B_1 = (x, \mu_1^-, \mu_1^+)$ and $B_2 = (y, \mu_2^-, \mu_2^+)$ are bipolar fuzzy n-fold KU-ideals of X , then $\oplus(B_1 \times B_2) = (\mu_1^+ \times \mu_2^+, (\mu_1^+)^c \times (\mu_2^+)^c)$ is a bipolar fuzzy n-fold KU-ideal of $X \times X$.

Proof: now,

$$\begin{aligned} (\mu_1^+ \times \mu_2^+)(x, y) &= \min\{\mu_1^+(x), \mu_2^+(y)\}. \text{ That implies,} \\ 1 - ((\mu_1^+)^c \times (\mu_2^+)^c)(x, y) &= \min\{1 - (\mu_1^+)^c(x), 1 - (\mu_2^+)^c(y)\} \Rightarrow \\ 1 - \min\{1 - (\mu_1^+)^c(x), 1 - (\mu_2^+)^c(y)\} &= ((\mu_1^+)^c \times (\mu_2^+)^c)(x, y) \Rightarrow \\ ((\mu_1^+)^c \times (\mu_2^+)^c)(x, y) &= \max\{(\mu_1^+)^c(x), (\mu_2^+)^c(y)\}. \end{aligned}$$

Hence (by Proposition 5.2) $\oplus(B_1 \times B_2)$ is a bipolar fuzzy n-fold KU-ideal of $X \times X$.

Lemma 5.5. If $B_1 = (x, \mu_1^-, \mu_1^+)$ and $B_2 = (y, \mu_2^-, \mu_2^+)$ are bipolar fuzzy n-fold KU-ideal of X , then $\otimes(B_1 \times B_2) = ((\mu_1^-)^c \times (\mu_2^-)^c, \mu_1^- \times \mu_2^-)$ is a bipolar fuzzy n-fold KU-ideal of $X \times X$.

Proof: Since $(\mu_1^- \times \mu_2^-)(x, y) = \max\{\mu_1^-(x), \mu_2^-(y)\}$. That implies,

$$\begin{aligned} 1 - ((\mu_1^-)^c \times (\mu_2^-)^c)(x, y) &= \max\{1 - (\mu_1^-)^c(x), 1 - (\mu_2^-)^c(y)\} \Rightarrow \\ 1 - \max\{1 - (\mu_1^-)^c(x), 1 - (\mu_2^-)^c(y)\} &= ((\mu_1^-)^c \times (\mu_2^-)^c)(x, y) \Rightarrow \\ ((\mu_1^-)^c \times (\mu_2^-)^c)(x, y) &= \min\{(\mu_1^-)^c(x), (\mu_2^-)^c(y)\}. \end{aligned}$$

Hence $\otimes (B_1 \times B_2) = ((\mu_1^-)^c \times (\mu_2^-)^c, \mu_1^- \times \mu_2^-)$ is a bipolar fuzzy n-fold KU-ideal of $X \times X$.

Conclusion: we have studied the bipolar fuzzy foldedness of KU-ideal in KU-algebras. Also we discussed a few results of bipolar fuzzy n-fold KU-ideal of KU-algebras under homomorphism, the image and the pre-image of bipolar fuzzy n-fold KU-ideals in KU - algebras are defined. How the image and the pre-image of bipolar fuzzy n-fold KU-ideals in KU-algebras become bipolar fuzzy n-fold KU-ideals are studied. Moreover, the product of bipolar fuzzy n-fold KU-ideals to product bipolar fuzzy n-fold KU-algebras is established. Furthermore, we construct some algorithms for folding theory applied to KU-ideals in KU-algebras. The main purpose of our future work is to investigate the foldedness of other types of fuzzy ideals with special properties such as an intuitionistic (interval value) fuzzy n-fold KU-ideal of KU-algebras.

Appendix A

Algorithm for KU-algebras

Input (X : set, $*$: binary operation)

Output (“ X is a KU-algebra or not”)

Begin

If $X = \emptyset$ then go to (1.);

EndIf

If $0 \notin X$ then go to (1.);

EndIf

Stop: =false;

$i := 1$;

While $i \leq |X|$ and not (Stop) do

If $x_i * x_i \neq 0$ then

Stop: = true;

EndIf

$j := 1$

While $j \leq |X|$ and not (Stop) do

If $((y_j * x_i) * x_i) \neq 0$ then

Stop: = true;

EndIf

EndIf

$k := 1$

```

While  $k \leq |X|$  and not (Stop) do
  If  $(x_i * y_j) * ((y_j * z_k) * (x_i * z_k)) \neq 0$  then
    Stop: = true;
  EndIf
EndIf While
EndIf While
EndIf While
If Stop then
  (1.) Output ("  $X$  is not a KU-algebra")
Else
  Output ("  $X$  is a KU-algebra")
EndIf
End

```

Algorithm for fuzzy subsets

```

Input (  $X$  : KU-algebra,  $A : X \rightarrow [0,1]$  );
Output ("  $A$  is a fuzzy subset of  $X$  or not")
Begin
  Stop: =false;
   $i := 1$ ;
  While  $i \leq |X|$  and not (Stop) do
    If  $(A(x_i) < 0)$  or  $(A(x_i) > 1)$  then
      Stop: = true;
    EndIf
  EndIf While
  If Stop then
    Output ("  $A$  is a fuzzy subset of  $X$  ")
  Else
    Output ("  $A$  is not a fuzzy subset of  $X$  ")
  EndIf
End

```

Algorithm for n -fold KU-ideals

```

Input (  $X$  : KU-algebra,  $I$  : subset of  $X, n \in N$  );
Output ("  $I$  is an  $n$ -fold KU-ideal of  $X$  or not");

```

```

Begin
If  $I = \phi$  then go to (1.);
EndIf
If  $0 \notin I$  then go to (1.);
EndIf
Stop: =false;
 $i := 1$ ;
While  $i \leq |X|$  and not (Stop) do
 $j := 1$ 
While  $j \leq |X|$  and not (Stop) do
 $k := 1$ 
While  $k \leq |X|$  and not (Stop) do
If  $(x_i^n * (y_j * z_k)) \in I$  and  $y_j \in I$  then
If  $(x_i^n * z_k) \notin I$  then
Stop: = true;
EndIf
EndIf
EndIf While
EndIf While
EndIf While
If Stop then
Output (“ $I$  is an n-fold KU-ideal of  $X$ ”)
Else
(1.) Output (“ $I$  is not an n-fold KU-ideal of  $X$ ”)
EndIf
End

```

Algorithm for fuzzy n-fold KU-ideals

```

Input ( $X$  : KU-algebra,  $*$  : binary operation,  $A$  : fuzzy subset of  $X$ );
Output (“ $A$  is a fuzzy n-fold KU-ideal of  $X$  or not”)
Begin
Stop: =false;
 $i := 1$ ;

```



```

While  $i \leq |X|$  and not (Stop) do
  If  $A(0) < A(x_i)$  then
    Stop: = true;
  EndIf
   $j := 1$ 
  While  $j \leq |X|$  and not (Stop) do
     $k := 1$ 
    While  $k \leq |X|$  and not (Stop) do
      If  $A(x_i^n * z_k) < \min(A(x_i^n * (y_j * z_k)), A(y_j))$  then
        Stop: = true;
      EndIf
    EndIf While
  EndIf While
EndIf While
If Stop then
  Output (“  $A$  is not a fuzzy  $n$ -fold KU-ideal of  $X$  ”)
  Else
    Output (“  $A$  is a fuzzy  $n$ -fold KU-ideal of  $X$  ”)
  EndIf
End

```

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