Mathematica Aeterna, Vol. 4, 2014, no. 7, 795 - 797

A Brief Introduction to Hyper Orlicz Functions

E.Safaee

Email-Address:elham_safaee73@yahoo.com

Department of Mathematics,Mobarakeh Branch,Islamic Azad University,Mobarakeh,Iran

Abstract. In the short work, we introduce the concept of hyperOrliczfunction.Then we prove some results on spaces which there is a HyperOrlicz function on them. Finally, we obtain all results in[1] when $(X,\sum,\mu)$ is a measure space, where $X$is a locally compact normed space.

2010MSC: 46B10,46B25,46B40.

Keywords: HyperOrlicz function, HyperOrlicz space, Locally compact space.

1.Introduction

Recently, many mathematician have worked on Orlicz spaces. These spaces have a few applications in various branches of mathematics . For example, in [1] an application of Orlicz spaces in partial differential equations has been presented. For a detailed account on Orlicz spaces the reader should consult [1] or [2].

There are another version of Orlicz spaces; For instance, authors in [3] studied some basic properties of weak Orlicz spaces and their applications to harmonic analysis. They discussed the absolute continuity of the quasi-norm and its normality, then prove the boundedness of several maximal operators. They also established a kind of Marcinkiewicz-type interpolation theorem between weak Orlicz spaces.

In this short work, we introduce the notion of hyperOrlicz functions and hyperOrlicz spaces and get some results on them.

2.Main Results

First of all, we introduce the notion of a hyperOrlicz function.

Definition2.1. Let X be a normed space. A function $\phi: X \to [0,\infty]$ is said to be a hyper Orliczfunction(for summary h-Orlicz function) if ϕ is even, convex, continuous on X, ϕ vanish only at 0 and ϕ is not identically equal to zero.

By H-Orl(X), we denote the set of all h-Orlicz functions on X.

We wish to search spaces which there is a h-Orlicz function on them.

Proposition2.2. Let Xbe a n-dimensional space. Then H-Orl(X)$\neq \emptyset$.

Proof. First note that $X \cong \mathbb{R}^k$ for some $k \in \mathbb{N}$. It is easy to show that the function $\phi: \mathbb{R}^k \to [0, \infty]$ defined by

$$\phi(x_1, \dots, x_k) = (x_1^2 + \cdots + x_k^2)^{\frac{1}{2}}$$

Is a h-Orlicz function. There is a linear bijection $f: X \to \mathbb{R}^k$. Define the function $\phi o f: X \to [0, \infty]$. It can be seen easily that $\phi o f$ is a h-Orlicz function. Therefore $\mathrm{H} - \mathrm{Orl(X)} \neq \emptyset$.■

It follows from the fact any norm is a h-Orlicz function that:

Theorem2.3. Let X be a normable topological space.Then$\mathrm{H} - \mathrm{Orl(X)} \neq \emptyset$.

Corollary2.4. Let X be a normed space. Then $\mathrm{H} - \mathrm{Orl(X)} \neq \emptyset$.

The following theorem gives an important information on cardinal of the set of h-orlicz functions on a normed space.

Theorem2.5. Let X be a normed space. Then

$$\mathrm{Card}\big(\mathrm{H} - \mathrm{Orl(X)}\big) = c$$

Where c is the cardinal of continuum.

Proof. Let $\|.\|$ be the original norm on X.Define

$$\mathcal{N} = \{P: X \to \mathbb{R} | P(x) = k\|x\| \text{where k is a positive real number}\}$$

It is clear that $\mathcal{N}$ is a family of norms on X.Now by Corollary2.5, $\mathcal{N}$ is a family of h-orlicz functions also. Finally

$$\mathrm{Card}\big(\mathrm{H} - \mathrm{Orl(X)}\big) = c$$

As desired.■

Theorem2.6. If X is reflexive space with $\mathrm{H} - \mathrm{Orl(X)} \neq \emptyset$, then

$$H - Orl(X^{(2n)}) \neq \emptyset$$

For any $n \in \mathbb{N}$; Where $X^{(2n)}$ denotes the $(2\mathrm{n}) - \mathrm{th}$ dual of X.

Proof. First note that $X^{(n)}$ is reflexive for any $n \in \mathbb{N}$. Since X is reflexive, so the canonical embedding $\tau: X \to X^{**}$ is surjective and thus is an isometrically isomorphism. Therefore by the proof proposition2.3,

$$H - Orl(X^{**}) \neq \emptyset$$

In exactly the same way, the canonical embedding $\tau: X^{(2n-2)} \to X^{(2n)}$ is surjective, for any $n \geq 2$, and thus is an isometrically isomorphism and so

$$H - Orl(X^{(2n)}) \neq \emptyset$$

As claimed.■

Corollary2.7. Let X be a n-dimensional space. Then $H - Orl(X^{(n)}) \neq \emptyset$ for any $n \in \mathbb{N}$; Where $X^{(n)}$ denotes the nth dual of X.

Proof. It follows from this fact that $X^{(n)}$ is reflexive[5], for every $n = 0,1,2, \ldots$. Note that $X^0 = X$.■

Problem2.8. Find a formula for complementary function ψ of a h-Orliczfunction ϕ.

Remark2.9. Notice that if one can solve the above problem, then:

Let X be a locally compact normed space. Suppose further, ϕ is a h-Orlicz function on X. We define a convex function $I_\phi: L^0(\mu) \to [0, \infty]$ by

$$I_\phi(f) = \int \phi(f(x)) d\,\mu(x)$$

Where $(X, \sum, \mu)$ is a measure space, μ is the haar measure on X and

$$L^0(\mu) = \{f: X \to X | f \text{ is measurable}\}$$

By using of the above definitions and notations one can prove all results in [1], for h-Orlicz functions.

Remark2.10. It is well-known and easy to show that a normed space X is locally compact if and only if it has finite dimension(for example, see[4]); Therefore, in Remark2.9, X must be a finite-dimensional space.

References

[1]A.Wróblewska-Kamińska, An application of Orlicz spaces in partial differential equations, University of Warsaw,Faculty of Mathematics, Informatics and Mechanics, PhD dissertation, 2012.

[2]Y. Cui, H.Hudzik and J. Li, Some fundamental properties for duals of Orlicz spaces, Nonlinear Analysis 73(2010), 2353-2360.

[3] L. PeiDe& Wang MaoFa, Weak Orlicz spaces: Some basic properties and their applications to harmonic analysis, Science China Mathematics, 56(4) (2013),789–802.

[4]W.Rudin,Functionalanalysis,McGraw-Hill, New York,1973.

Received: August, 2014