Mathematica Aeterna, Vol. 4, 2014, no. 8, 967 - 977

On trans hyperbolic Sasakian manifolds

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Abstract

After finding some basic results on trans hyperbolic Sasakian manifold, we find explicit formulae for Ricci operator, Ricci tensor and curvature tensor in a three-dimensional trans hyperbolic Sasakian manifold. It is also proved that, in a three-dimensional trans hyperbolic Sasakian manifold $Q\varphi = \varphi Q$ if $grad\beta = -\varphi(grad\alpha)$. Finally we find expression for Ricci tensor in three-dimensional trans hyperbolic Sasakian manifold in case of the manifold being η-Einstein or Ricci semi symmetric.

Mathematics Subject Classification: 53C20, 53C25
Keywords: trans hyperbolic Sasakian manifold, Ricci tensor, Ricci operator, curvature tensor, η-Einstein, Ricci semi symmetric

1 Introduction

In a Gray-Hervella classification of almost Hermitian manifolds [7], there appears a class W_4 of Hermitian manifolds which are closely related to locally conformal Kähler manifolds. An almost contact metric structure on a manifold M is called a trans-Sasakian structure [13] if the product manifold $M \times R$ belongs to the class W_4. The class $C_6 \oplus C_5$ [11] coincides with the class of trans-Sasakian structures of type (α, β). It is known that a trans-Sasakian structure of type $(0, 0)$, $(\alpha, 0)$ and $(0, \beta)$ are cosympletic [1], α-Sasakian [2, 16] and β-Kenmotsu [2, 10] respectively. Thus trans-Sasakian manifold of type (α, β) is a generalization of Sasakian, α -Sasakian, Kenmotsu and β-Kenmotsu manifolds. An almost contact metric structure (ϕ, ξ, η, g) on M is called a trans-Sasakian structure [13] if $(M \times R, J, G)$ belongs to the class W_4, where J is the almost complex structure on $M \times R$ defined by

$$J\left(X, f\frac{d}{dt}\right) = \left(\phi X - f\xi, \eta\left(X\right) f\frac{d}{dt}\right), \quad (1)$$

for all vector fields on M and smooth functions f on $M \times R$,and G is the product metric on $M \times R$. This may be expressed by the condition

$$(\nabla_X \phi)Y = \alpha\left\{g\left(X, Y\right)\xi - \eta\left(Y\right)X\right\} + \beta\left\{g\left(\varphi X, Y\right)\xi - \eta\left(Y\right)\varphi X\right\}, \quad (2)$$

where ∇ is Levi-Civita connection of Riemannian metric g and α and β are smooth functions on M. Recently it was studied by several geometers. Jeong-Sik Kim et al [8] has studied generalized Ricci-recurrent trans-Sasakian manifold. While in [4], Ricci-operator, Ricci-tensor and curvature tensor are found for three-dimensional trans-Sasakian manifold. Prasad et al [15] studied some curvature tensors on trans-Sasakian manifold.

On the other hand almost contact hyperbolic (f, g, η, ξ)-structure was introduced by Upadhyay and dube[18]. Further it was studied by number of authors [17, 14, 9, 3] etc. The purpose of the present paper is to study Ricci tensor in a trans-hyperbolic Sasakian manifold. Section-2 is devoted to some necessary preliminaries. In section-3 some basic results are given. The Ricci operator, the Ricci-tensor and the curvature tensor in a three-dimensional trans hyperbolic Sasakian manifold are found in section-4. In the section, it is also proved that If $grad\beta = -\varphi(grad\alpha)$ in a three-dimensional trans hyperbolic Sasakian manifold, then the Ricci operator and the structure tensor commute i.e., $Q\varphi = \varphi Q$. Finally in section-5 and 6, we find expression for Ricci tensor in three-dimensional trans hyperbolic Sasakian manifold in case of the manifold being η-Einstein or Ricci semi symmetric.

2 Preliminary

Let us consider a $(2n + 1)$-dimensional complete real differentiable manifold M with fundamental tensor field φ of type $(1, 1)$, fundamental time like vector field ξ, a 1-form η, such that for every vector field X, we have

$$\varphi^2 = I + \eta \circ \xi, \quad (3)$$

$$\eta(\xi) = -1(\xi \texttt{ is time like vector field}), \quad (4)$$

$$\varphi(\xi) = 0, \quad (5)$$

$$\eta \circ \varphi = 0 \quad (6)$$

$$rank(\varphi) = 2n, \quad (7)$$

where I is the identity endomorphism of the tangent bundle of M. Then M is called almost hyperbolic contact manifold [18]. An almost hyperbolic contact manifold M is said to be an almost hyperbolic contact metric manifold if a semi-Riemannian metric g satisfies

$$\begin{aligned} g(\phi X, \phi Y) &= -g(X,Y) - \eta(X).\eta(Y), &(8)\\ g(\phi X, Y) &= -g(X, \phi Y), &(9)\\ g(X,\xi) &= \eta(X). &(10)\end{aligned}$$

The structure (φ, ξ, η, g) on M is called almost hyperbolic contact metric structure. An almost hyperbolic contact metric manifold M is called a trans hyperbolic Sasakian manifold [3] if equation (2) is satisfied in it. From equation (2), it follows that

$$\begin{aligned} \nabla_X \xi &= -\alpha\varphi X - \beta[X + \eta(X)\xi], &(11)\\ (\nabla_X \eta)Y &= -\alpha g(\varphi X, Y) + \beta g(\varphi X, \varphi Y). &(12)\end{aligned}$$

Let $\{e_1, e_2, ..., e_{2n}, e_{(2n+1)} = \xi\}$ is a local orthonormal basis of vector fields in a $(2n+1)$-dimensional semi-Riemannian manifold. In a semi-Riemannian manifold, the semi-Riemannian metric g satisfies

$$g(e_i, e_j) = \varepsilon_i \delta_{ij} (\texttt{summation with respect to} i), \tag{13}$$

here ε_i is signature of the basis. For spacelike, null and timelike vector fields, the signature $\varepsilon_i = g(e_i, e_i)$ is defined as follows:

1. e_i is spacelike then $\varepsilon_i = g(e_i, e_i) > 0$,

2. e_i is null then $\varepsilon_i = g(e_i, e_i) = 0$,

3. e_i is timelike then $\varepsilon_i = g(e_i, e_i) < o$.

Here note that e_i is non-zero vector field and a zero vector field is always spacelike.

In an orthonormal basis of an almost hyperbolic contact metric manifold only ξ is timelike and remaining all are spacelike. Thus Ricci tensor and scaler curvature of an almost hyperbolic contact metric manifold are defined as follows.

$$\begin{aligned} S(X,Y) &= \sum_{i=1}^{2n+1} \varepsilon_i g(R(e_i, X)Y, e_i)\\ &= \sum_{i=1}^{2n} g(R(e_i, X)Y, e_i) - g(R(\xi, X)Y, \xi), &(14)\end{aligned}$$

$$\tau = \sum_{i=1}^{2n+1} \varepsilon_i S(e_i, e_i) = \sum_{i=1}^{2n} S(e_i, e_i) - S(\xi, \xi). \tag{15}$$

3 Some basic results on trans hyperbolic Sasakian manifold

We begin with the following lemma.

Lemma 3.1 *In a trans hyperbolic Sasakian manifold, we have*

$$\begin{aligned} R(X,Y)\xi &= (\alpha^2+\beta^2)(\eta(Y)X-\eta(X)Y) \\ &\quad 2\alpha\beta(\eta(Y)\varphi X-\eta(X)\varphi Y)+(Y\alpha)\varphi X \\ &\quad -(X\alpha)\varphi Y+(Y\beta)\varphi^2 X-(X\beta)\varphi^2 Y, \end{aligned} \tag{16}$$

where R is the curvature tensor.

Proof: In a Riemannian manifold, it is known that

$$R(X,Y)\xi = \nabla_X\nabla_Y\xi - \nabla_Y\nabla_X\xi - \nabla_{[X,Y]}\xi. \tag{17}$$

Taking account of equations (2), (11) and (17), we get the result. •

Now in view of $g(R(X,Y)\xi, Z) = g(R(\xi,Z)X, Y)$, equation (16) implies

$$\begin{aligned} R(\xi,X)Y &= (\alpha^2+\beta^2)(g(X,Y)\xi-\eta(Y)X) \\ &\quad 2\alpha\beta\,(\eta(Y)\varphi X-g(\varphi X,Y)\xi)+(Y\alpha)\varphi X \\ &\quad -(grad\alpha)g(\varphi X,X)-(Y\beta)\varphi^2 X \\ &\quad +(grad\beta)(g(X,Y)+\eta(X)\eta(Y)). \end{aligned} \tag{18}$$

On taking $Y=\xi$ in the above equation, we have

$$\begin{aligned} R(\xi,X)\xi &= (\alpha^2+\beta^2-\xi\beta)(X+\eta(X)\xi) \\ &\quad -(2\alpha\beta-\xi\alpha)\varphi X. \end{aligned} \tag{19}$$

Again from equation (16), we have

$$\begin{aligned} R(\xi,X)\xi &= (\alpha^2+\beta^2-\xi\beta)(X+\eta(X)\xi) \\ &\quad +(2\alpha\beta-\xi\alpha)\varphi X. \end{aligned} \tag{20}$$

In view of equations (19) and (20), we have following theorem:

Theorem 3.2 *In a trans hyperbolic Sasakian manifold, we have*

$$R(\xi,X)\xi = (\alpha^2+\beta^2-\xi\beta)(X+\eta(X)\xi), \tag{21}$$

$$2\alpha\beta-\xi\alpha = 0. \tag{22}$$

Now in view of equation (22), we have following corollary:

Corollary 3.3 *A trans hyperbolic Sasakian manifold of type (α, β) with α a non-zero constant is always hyperbolic α-Sasakian.*

Using equations (14), (16) and (21), we can state following proposition.

Proposition 3.4 *In a trans hyperbolic Sasakian manifold, we have*

$$\begin{aligned} S(X,\xi) &= \Big(2n(\alpha^2+\beta^2)-\xi\beta\Big)\,\eta(X) \\ &\quad +(2n-1)(X\beta)-(\varphi X)\alpha, \end{aligned} \tag{23}$$

$$\begin{aligned} Q\xi &= \Big(2n(\alpha^2+\beta^2)-\xi\beta\Big)\,\xi \\ &\quad +(2n-1)(grad\beta)+\varphi(grad\alpha), \end{aligned} \tag{24}$$

where S is Ricci tensor and Q is Ricci operator.

Remark 3.5 *If in a trans hyperbolic Sasakian manifold of kind (α, β) $\varphi(grad\alpha)+(2n-1)grad\beta = 0$, then*
$\xi\beta = g(\xi, grad\beta) = -\frac{1}{(2n-1)}g(\xi, \varphi(grad\alpha)) = 0.$
Hence

$$S(X,\xi) = 2n(\alpha^2+\beta^2)\eta(X), \tag{25}$$

$$Q\xi = 2n(\alpha^2+\beta^2)\xi. \tag{26}$$

Remark 3.6 *If in a trans hyperbolic Sasakian manifold of kind (α, β) $\varphi(grad\alpha)+(2n-1)grad\beta = (2n-1)(\xi\beta)$, then*

$$S(X,\xi) = 2n(\alpha^2+\beta^2+\xi\beta)\eta(X), \tag{27}$$

$$Q\xi = 2n(\alpha^2+\beta^2+\xi\beta)\xi. \tag{28}$$

4 Three dimensional trans hyperbolic Sasakian manifold

We begin with the definition:

Definition 4.1 *The Weyl conformal curvature tensor C of type $(1,3)$ of an $(2n+1)$-dimensional manifold M is defined by*

$$\begin{aligned} C(X,Y)Z &= R(X,Y)Z - \frac{1}{2n-1}[S(Y,Z)X - S(X,Z)Y \\ &\quad + g(Y,Z)QX - g(X,Z)QY] \\ &\quad + \frac{\tau}{(2n)(2n-1)}[g(Y,Z)X - g(X,Z)Y], \end{aligned} \tag{29}$$

where R, S, Q, τ denotes respectively the Riemannian curvature tensor, Ricci-tensor of type $(0,2)$, the Ricci-operator and the scalar curvature of the manifold.

In a three-dimensional trans hyperbolic Sasakian manifold, we have from Proposition **3.4**

$$S(X,\xi) = (2(\alpha^2+\beta^2) - \xi\beta)\eta(X) + X\beta - (\phi X)\alpha, \tag{30}$$

$$S(\xi,\xi) = -2(\alpha^2+\beta^2-\xi\beta), \tag{31}$$

$$Q\xi = (2(\alpha^2+\beta^2) - \xi\beta)\xi + grad\beta + \phi(grad\alpha). \tag{32}$$

Lemma 4.2 *In a three-dimensional trans hyperbolic Sasakian manifold, the Ricci-operator is given by*

$$\begin{aligned} QX &= \left(\frac{\tau}{2} + \xi\beta - (\alpha^2+\beta^2)\right) X \\ &\quad + \left(\frac{\tau}{2} + \xi\beta - 3(\alpha^2+\beta^2)\right)\eta(X)\xi \\ &\quad -(\varphi(grad\alpha) + grad\beta)\eta(X) - (X\beta - (\varphi X)\alpha)\xi. \end{aligned} \tag{33}$$

Proof: We know that the Weyl conformal curvature tensor vanishes in three-dimensional Riemannian manifold, therefore from equation (29)

$$\begin{aligned} R(X,Y)Z &= g(Y,Z)QX - g(X,Z)QY + S(Y,Z)X \\ &\quad -S(X,Z)Y - \frac{\tau}{2}(g(Y,Z)X - g(X,Z)Y). \end{aligned} \tag{34}$$

On taking $X = Z = \xi$ and using equations (4) and (10), we have

$$\begin{aligned} R(\xi,Y)\xi &= \eta(Y)Q\xi + QY + S(Y,\xi)\xi - S(\xi,\xi)Y \\ &\quad -\frac{\tau}{2}(Y + \eta(Y)\xi). \end{aligned} \tag{35}$$

Using equations (21), (30), (31) and (32) in the above equation, we get the equation (33). •

Theorem 4.3 *In a three-dimensional trans hyperbolic Sasakian manifold, the Ricci-tensor and curvature tensor are given respectively as*

$$\begin{aligned} S(X,Y) &= \left(\frac{\tau}{2} + \xi\beta - (\alpha^2+\beta^2)\right) g(X,Y) \\ &\quad + \left(\frac{\tau}{2} + \xi\beta - 3(\alpha^2+\beta^2)\right)\eta(X)\eta(Y) \\ &\quad -((Y\beta) - (\varphi Y)\alpha)\eta(X) \\ &\quad -((X\beta) - (\varphi X)\alpha)\eta(Y), \end{aligned} \tag{36}$$

and

$$\begin{aligned} R(X,Y)Z &= \left(\frac{\tau}{2}+2\xi\beta-2(\alpha^2+\beta^2)\right)(g(Y,Z)X-g(X,Z)Y \\ &+g(Y,Z)\{\left(\frac{\tau}{2}+\xi\beta-3(\alpha^2+\beta^2)\right)\eta(X)\xi \\ &-((X\beta)-(\varphi X)\alpha)\xi-(\varphi(grad\alpha)+grad\beta)\eta(X)\} \\ &-g(X,Z)\{\left(\frac{\tau}{2}+\xi\beta-3(\alpha^2+\beta^2)\right)\eta(Y)\xi \\ &-((Y\beta)-(\varphi Y)\alpha)\xi-(\varphi(grad\alpha)+grad\beta)\eta(Y)\} \\ &+\{\left(\frac{\tau}{2}+\xi\beta-3(\alpha^2+\beta^2)\right)\eta(Y)\eta(Z) \\ &-((Y\beta)-(\varphi Y)\alpha)\eta(Z)-((Z\beta)-(\varphi Z)\alpha)\eta(Y)\}X \\ &+\{\left(\frac{\tau}{2}+\xi\beta-3(\alpha^2+\beta^2)\right)\eta(X)\eta(Z) \\ &-((X\beta)-(\varphi X)\alpha)\eta(Z) \\ &-((Z\beta)-(\varphi Z)\alpha)\eta(X)\}Y. \end{aligned} \tag{37}$$

Proof: Using equation (33) and taking account of $S(X,Y)=g(QX,Y)$, we get the equation (36) and using equations (33),(36) in the equation (34), we have the equation (37). •

Proposition 4.4 *Let M be a three-dimensional trans hyperbolic Sasakian manifold of type (α,β). If $grad\beta=-\varphi(grad\alpha)$, then the Ricci operator and the structure tensor commute i.e., $Q\varphi=\varphi Q$.*

Proof: If $grad\beta=-\varphi(grad\alpha)$, then

$$\begin{aligned} X\beta &= g(X,grad\beta) \\ &= g(X,-\varphi(grad\beta)) \\ &= g(\varphi X,(grad\beta)) \\ &= (\varphi X)\beta \ \texttt{i.e.,} \\ X\beta &= (\varphi X)\beta. \end{aligned} \tag{38}$$

From equation (38), we can get

$$\xi\beta=0 \tag{39}$$

In virtue of equations (38) and (39), equation (33) reduces to

$$\begin{aligned} QX &= \left(\frac{\tau}{2}-(\alpha^2+\beta^2)\right)X \\ &+\left(\frac{\tau}{2}-3(\alpha^2+\beta^2)\right)\eta(X)\xi. \end{aligned} \tag{40}$$

Replace X by φX in equation (40) and using (6), we get

$$(Q\varphi)X = \left(\frac{\tau}{2} - (\alpha^2 + \beta^2)\right)\varphi X. \tag{41}$$

Now operating φ in the equation (40) and using equation (5), we get

$$(\varphi Q)X = \left(\frac{\tau}{2} - (\alpha^2 + \beta^2)\right)\varphi X. \tag{42}$$

In view of equations (41) and (42), we have the result. •

5 Three dimensional η-Einstein trans hyperbolic Sasakian manifold

In this section we prove following theorem.

Theorem 5.1 *In a three-dimensional η-Einstein trans hyperbolic Sasakian manifold, the Ricci-tensor S is*

$$\begin{aligned} S(X,Y) &= \left(\frac{\tau}{2} - \xi\beta + (\alpha^2 + \beta^2)\right) g(X,Y) \\ &\quad + \left(\frac{\tau}{2} + \xi\beta - (\alpha^2 + \beta^2)\right)\eta(X)\eta(Y). \end{aligned} \tag{43}$$

Proof: For an η-Einstein manifold, the Ricci tensor S is of the form

$$S(X.Y) = ag(X.Y) + b\eta(X)\eta(Y), \tag{44}$$

where a and b are the smooth functions. Contracting equation (44), we get

$$\tau = a + b. \tag{45}$$

Now taking $X = Y = \xi$ in the equation (36), we get

$$b - a = 2(\xi\beta - (\alpha^2 + \beta^2)). \tag{46}$$

First we find the values of a and b from (46) and (45) and put in equation (44), we get the result. •

6 Three dimensional Ricci-semi-symmetric trans hyperbolic Sasakian manifold

A Riemannian manifold M is said to be Ricci-semi-symmetric if

$$R(X,Y).S = 0. \tag{47}$$

The above condition is equivalent to

$$S(R(X,Y)U,V) + S(U,R(X,Y)V) = 0. \tag{48}$$

In particular we have

$$S(R(\xi,X)\xi,Y) + S(\xi,R(\xi,X)Y) = 0. \tag{49}$$

Taking account of equations (18) and (21) in the above equation, we have

$$\begin{aligned}(\alpha^2+\beta^2-\xi\beta)S(X,Y) &= -(\alpha^2+\beta^2-\xi\beta)\eta(X)S(\xi,Y)\\ &\quad -(\alpha^2+\beta^2)g(X,Y)S(\xi,\xi)\\ &\quad +(\alpha^2+\beta^2)\eta(Y)S(\xi,X)\\ &\quad +2\alpha\beta\eta(Y)S(\xi,\varphi X)\\ &\quad +2\alpha\beta g(\varphi X,Y)S(\xi,\xi)\\ &\quad +g(\varphi X,Y)S(\xi,grad\alpha)\\ &\quad -(Y\alpha)S(\xi,\varphi X)+(Y\beta)S(\xi,\varphi^2X)\\ &\quad +g(\varphi X,\varphi Y)S(\xi,grad\beta).\end{aligned} \tag{50}$$

Since S is symmetric, from the above equation we also have

$$\begin{aligned}(\alpha^2+\beta^2-\xi\beta)S(X,Y) &= -(\alpha^2+\beta^2-\xi\beta)\eta(Y)S(\xi,X)\\ &\quad -(\alpha^2+\beta^2)g(X,Y)S(\xi,\xi)\\ &\quad +(\alpha^2+\beta^2)\eta(X)S(\xi,Y)\\ &\quad +2\alpha\beta\eta(X)S(\xi,\varphi Y)\\ &\quad -2\alpha\beta g(\varphi X,Y)S(\xi,\xi)\\ &\quad -g(\varphi X,Y)S(\xi,grad\alpha)\\ &\quad -(X\alpha)S(\xi,\varphi Y)+(X\beta)S(\xi,\varphi^2Y)\\ &\quad +g(\varphi X,\varphi Y)S(\xi,grad\beta).\end{aligned} \tag{51}$$

Adding (50) and (51), we get

$$\begin{aligned}2(\alpha^2+\beta^2-\xi\beta)S(X,Y) &= \xi\beta(\eta(Y)S(\xi,X)+\eta(Y)S(\xi,X))\\ &\quad -2(\alpha^2+\beta^2)g(X,Y)S(\xi,\xi)\\ &\quad +2\alpha\beta(\eta(Y)S(\xi,\varphi X)+\eta(X)S(\xi,\varphi Y))\\ &\quad -(X\alpha)S(\xi,\varphi Y)+(X\beta)S(\xi,\varphi^2Y)\\ &\quad -(Y\alpha)S(\xi,\varphi X)+(Y\beta)S(\xi,\varphi^2X)\\ &\quad +2g(\varphi X,\varphi Y)S(\xi,grad\beta).\end{aligned} \tag{52}$$

Using equations (3), (30), (31 and (33), we have

$$(\alpha^2+\beta^2-\xi\beta)S(X,Y) \;=\; \{(2(\alpha^2+\beta^2)-\xi\beta)^2-2(\alpha^2+\beta^2)^2$$

$$
\begin{aligned}
&-\|grad\beta\|^2-\varphi(grad\alpha)\beta\}g(X,Y)\\
&+\{4\alpha^2\beta^2-\|grad\beta\|^2\\
&-\varphi(grad\alpha)\beta\}\eta(X)\eta(Y)\\
&+\{(Y\beta-\frac{1}{2}(\varphi Y)\alpha)\xi\beta+\alpha\beta(\varphi Y)\beta\}\eta(X)\\
&+\{(X\beta-\frac{1}{2}(\varphi X)\alpha)\xi\beta+\alpha\beta(\varphi X)\beta\}\eta(Y)\\
&+(X\alpha)(Y\alpha)+(X\beta)(Y\beta)\\
&-\frac{1}{2}\{(X\alpha)(\varphi Y)\beta+(X\beta)(\varphi Y)\alpha\\
&+(Y\alpha)(\varphi X)\beta+(Y\beta)(\varphi X)\alpha\}. \qquad (53)
\end{aligned}
$$

Hence we have following theorem:

Theorem 6.1 *In a three-dimensional Ricci-semi-symmetric trans hyperbolic Sasakian manifold, the Ricci-tensor S satisfies the equation (53).*

ACKNOWLEDGEMENTS. The authors wish to express his sincere thanks and gratitude to the referee for accepting the paper.

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Received: October, 2014