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The Explicit Solution to the Countable Systems of Linear Ordinary Differential Equations with Constant Coefficients

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Abstract

In this paper, we have derived an explicit solution to an infinite countable system of linear ordinary differential equations with constant coefficients. We applied some mathematical techniques and Mathematical Induction method and have found the solution to the system under consideration when the coefficients matrix is bidiagonal.

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1 Introduction

Infinite linear system of ordinary differential equations (ODEs) can model a wide range of problems in science and engineering. Unlike finite linear systems of ODEs, which are widely studied by numerous analytical and numerical approaches [1, 2, 3, 4, 5, 6, 7, 8], infinite linear systems which are considered to be a very important special case of ODEs in banach spaces are still suffering from the lack of wide and satisfactorlily developments up to now. Although, very important researchers for studying those systems have appeared. The stability and existence of the solution of infinite systems have been studied and applied in mechanics [9, 10, 11, 12]. Theories of branching proceses, neural nets, and dissociation of polymers are also studied by considering infinte systems of ODEs [13, 14, 15, 16]. Properties of infinite systems of the first order with auxiliary boundary conditions are studied in [17]. Those systems can also play a very important rule in solving some problems involving parabolic partial differtianal equations [18, 19]. Many applications of the system under consideration are found in [12, 20, 21].

In this paper, we will find an explict solution of the following system of ODEs:

$$\dot{n}(t) = An(t). \tag{1}$$

where A is an infinite lower bi-diagonal matrix given by:

$$A = \begin{pmatrix} -\sigma_1 & 0 & 0 & \cdots & \cdots & 0 & \cdots \\ a_1 & -\sigma_2 & 0 & \cdots & \cdots & 0 & \cdots \\ 0 & a_2 & -\sigma_3 & \ddots & \cdots & 0 & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \cdots \\ 0 & \cdots & 0 & a_k & -\sigma_k & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \end{pmatrix},$$

and $n(t)$ is an infinite column vector in the form:

$$n(t) = \begin{pmatrix} n_1(t) \\ n_2(t) \\ \vdots \\ n_k(t) \\ \vdots \end{pmatrix}.$$

with $n_k(0)$ is given as an initial condition, the coefficients a_i, σ_j are constants, and $\sigma_i \neq \sigma_j$ if $i \neq j$.
In the next section, an important lemma is proved and used to find the solution of system (1).

2 Analysis and Main Results

In the following, we present and prove a lemma that is used to derive the solution of system (1).

Lemma 2.1 Let $A = \{\lambda_i \in R : i = 1, 2, ...\}$ be an infinite sequence, such that for $i \neq j$ and $i, j \in N$, $\lambda_i \neq \lambda_j$. Let $\{\mu_1, \mu_2, ..., \mu_n\}$ be any finite subset of size $n \geq 2$ of A. Then, for all $n \geq 2$,

$$\begin{aligned} & \frac{1}{(\mu_2-\mu_1)(\mu_3-\mu_1)\ldots(\mu_n-\mu_1)} \\ & +\frac{1}{(\mu_1-\mu_2)(\mu_3-\mu_2)\ldots(\mu_n-\mu_2)} \\ & +\ldots\ldots \\ & +\frac{1}{(\mu_1-\mu_{(n-1)})(\mu_2-\mu_{(n-1)})\ldots(\mu_n-\mu_{(n-1)})} \\ = & \frac{-1}{(\mu_1-\mu_n)(\mu_2-\mu_n)\ldots(\mu_{(n-1)}-\mu_n)}. \end{aligned} \tag{2}$$

Proof :
We will prove this lemma by Mathematical Induction. Let S be the set of all $n \in N$ for which the formula is true.
First we prove (2) for $n = 2$. For any $i \neq j$,

$$\frac{1}{(\mu_i-\mu_j)} = \frac{-1}{(\mu_j-\mu_i)}.$$

Therefore $2 \in S$.
We assume that it is true for $n = m$, that is, let $\{\mu_1, \mu_2, ..., \mu_m\}$ be any subset of size m of S. We have

$$\begin{aligned} & \frac{1}{(\mu_2-\mu_1)(\mu_3-\mu_1)\ldots(\mu_m-\mu_1)} \\ & +\frac{1}{(\mu_1-\mu_2)(\mu_3-\mu_2)\ldots(\mu_m-\mu_2)} \\ & +\ldots\ldots \\ & +\frac{1}{(\mu_1-\mu_{(m-1)})(\mu_2-\mu_{(m-1)})\ldots(\mu_m-\mu_{(m-1)})} \\ = & \frac{-1}{(\mu_1-\mu_m)(\mu_2-\mu_m)\ldots(\mu_{(m-1)}-\mu_m)}. \end{aligned} \tag{3}$$

Multiplying both sides of (3) by

$$(\mu_1 - \mu_m)(\mu_2 - \mu_m)\ldots(\mu_{(m-1)} - \mu_m),$$

we get

$$\begin{aligned}
&\frac{(\mu_2 - \mu_m)(\mu_3 - \mu_m)\ldots(\mu_{(m-1)} - \mu_m)}{(\mu_2 - \mu_1)(\mu_3 - \mu_1)\ldots(\mu_{(m-1)} - \mu_1)}\\
&+\frac{(\mu_1 - \mu_m)(\mu_3 - \mu_m)\ldots(\mu_{(m-1)} - \mu_m)}{(\mu_1 - \mu_2)(\mu_3 - \mu_2)\ldots(\mu_{(m-1)} - \mu_2)}\\
&+\ldots\ldots\\
&+\frac{(\mu_1 - \mu_m)(\mu_2 - \mu_m)\ldots(\mu_{(m-2)} - \mu_m)}{(\mu_1 - \mu_{(m-1)})(\mu_2 - \mu_{(m-1)})\ldots(\mu_{(m-2)} - \mu_{(m-1)})} = 1. \qquad (4)
\end{aligned}$$

Now consider $n = m + 1$. In other words, we have to prove the identity

$$\begin{aligned}
&\frac{1}{(\mu_2 - \mu_1)(\mu_3 - \mu_1)\ldots(\mu_{(m+1)} - \mu_1)}\\
&+\frac{1}{(\mu_1 - \mu_2)(\mu_3 - \mu_2)\ldots(\mu_{(m+1)} - \mu_2)}\\
&+\ldots\ldots\\
&+\frac{1}{(\mu_1 - \mu_m)(\mu_2 - \mu_m)\ldots(\mu_{(m+1)} - \mu_m)}\\
= \quad &\frac{-1}{(\mu_1 - \mu_{(m+1)})(\mu_2 - \mu_{(m+1)})\ldots(\mu_m - \mu_{(m+1)})}. \qquad (5)
\end{aligned}$$

Multiply both sides of (5) by

$$(\mu_1 - \mu_{(m+1)})(\mu_2 - \mu_{(m+1)})\ldots(\mu_m - \mu_{(m+1)}),$$

we get,

$$\begin{aligned}
&\frac{(\mu_2 - \mu_{(m+1)})(\mu_3 - \mu_{(m+1)})\ldots(\mu_m - \mu_{(m+1)})}{(\mu_2 - \mu_1)(\mu_3 - \mu_1)\ldots(\mu_m - \mu_1)}\\
&+\frac{(\mu_1 - \mu_{(m+1)})(\mu_3 - \mu_{(m+1)})\ldots(\mu_m - \mu_{(m+1)})}{(\mu_1 - \mu_2)(\mu_3 - \mu_2)\ldots(\mu_m - \mu_2)}\\
&+\ldots\ldots\\
&+\frac{(\mu_1 - \mu_{(m+1)})(\mu_2 - \mu_{(m+1)})\ldots(\mu_{(m-1)} - \mu_{(m+1)})}{(\mu_1 - \mu_m)(\mu_2 - \mu_m)\ldots(\mu_{(m-1)} - \mu_m)} = 1. \qquad (6)
\end{aligned}$$

We move the right hand side of (3) to the left hand side to get

$$\begin{aligned}&\frac{1}{(\mu_2-\mu_1)(\mu_3-\mu_1)\ldots(\mu_m-\mu_1)}\\&+\frac{1}{(\mu_1-\mu_2)(\mu_3-\mu_2)\ldots(\mu_m-\mu_2)}\\&+\ldots\ldots\\&+\frac{1}{(\mu_1-\mu_{(m-1)})(\mu_2-\mu_{(m-1)})\ldots(\mu_m-\mu_{(m-1)})}\\&+\frac{1}{(\mu_1-\mu_m)(\mu_2-\mu_m)\ldots(\mu_{(m-1)}-\mu_m)}=0.\end{aligned}\tag{7}$$

Then we multiply (7) by the factor

$$(\mu_2-\mu_{(m+1)})(\mu_3-\mu_{(m+1)})\ldots(\mu_m-\mu_{(m+1)})$$

After the multiplication we keep the first term in the left hand side and move the rest of the terms to the right hand side; we get,

$$\begin{aligned}&\frac{(\mu_2-\mu_{(m+1)})(\mu_3-\mu_{(m+1)})\ldots(\mu_m-\mu_{(m+1)})}{(\mu_2-\mu_1)(\mu_3-\mu_1)\ldots(\mu_m-\mu_1)}=\\&-(\mu_2-\mu_{(m+1)})\frac{(\mu_3-\mu_{(m+1)})(\mu_4-\mu_{(m+1)})\ldots(\mu_m-\mu_{(m+1)})}{(\mu_1-\mu_2)(\mu_3-\mu_2)\ldots(\mu_m-\mu_2)}\\&-(\mu_3-\mu_{(m+1)})\frac{(\mu_2-\mu_{(m+1)})(\mu_4-\mu_{(m+1)})\ldots(\mu_m-\mu_{(m+1)})}{(\mu_1-\mu_3)(\mu_2-\mu_3)\ldots(\mu_m-\mu_3)}\\&-\ldots\ldots\\&-(\mu_{(m-1)}-\mu_{(m+1)})\frac{(\mu_2-\mu_{(m+1)})(\mu_3-\mu_{(m+1)})\ldots(\mu_m-\mu_{(m+1)})}{(\mu_1-\mu_{(m-1)})(\mu_2-\mu_{(m-1)})\ldots(\mu_m-\mu_{(m-1)})}\\&-(\mu_m-\mu_{(m+1)})\frac{(\mu_2-\mu_{(m+1)})(\mu_3-\mu_{(m+1)})\ldots(\mu_{(m-1)}-\mu_{(m+1)})}{(\mu_1-\mu_m)(\mu_2-\mu_m)\ldots(\mu_{(m-1)}-\mu_m)}.\end{aligned}\tag{8}$$

The left hand side of (8) is the first term of left hand side of (6). We replace the first term of left hand side of (6) by the right hand side of (8). Then, the left hand side of (6) becomes:

$$\begin{aligned}&[-(\mu_2-\mu_{(m+1)})+(\mu_1-\mu_{(m+1)})]\frac{(\mu_3-\mu_{(m+1)})\ldots(\mu_m-\mu_{(m+1)})}{(\mu_1-\mu_2)(\mu_3-\mu_2)\ldots(\mu_m-\mu_2)}\\&[-(\mu_3-\mu_{(m+1)})+(\mu_1-\mu_{(m+1)})]\frac{(\mu_2-\mu_{(m+1)})\ldots(\mu_m-\mu_{(m+1)})}{(\mu_1-\mu_3)(\mu_2-\mu_3)\ldots(\mu_m-\mu_3)}\\&+\ldots\ldots\end{aligned}$$

$$+[-(\mu_{(m-1)}-\mu_{(m+1)})+(\mu_1-\mu_{(m+1)})]\frac{(\mu_2-\mu_{(m+1)})\ldots(\mu_m-\mu_{(m+1)})}{(\mu_1-\mu_{(m-1)})\ldots(\mu_m-\mu_{(m-1)})}$$

$$+[-(\mu_m-\mu_{(m+1)})+(\mu_1-\mu_{(m+1)})]\frac{(\mu_2-\mu_{(m+1)})\ldots(\mu_{(m-1)}-\mu_{(m+1)})}{(\mu_1-\mu_m)(\mu_2-\mu_m)\ldots(\mu_{(m-1)}-\mu_m)}$$

$$\begin{aligned}
= &\frac{(\mu_3-\mu_{(m+1)})(\mu_4-\mu_{(m+1)})\ldots(\mu_m-\mu_{(m+1)})}{(\mu_3-\mu_2)(\mu_4-\mu_2)\ldots(\mu_m-\mu_2)}\\
&+\frac{(\mu_2-\mu_{(m+1)})(\mu_4-\mu_{(m+1)})\ldots(\mu_m-\mu_{(m+1)})}{(\mu_2-\mu_3)(\mu_4-\mu_3)\ldots(\mu_m-\mu_3)}\\
&+\ldots\ldots\\
&+\frac{(\mu_2-\mu_{(m+1)})(\mu_3-\mu_{(m+1)})\ldots(\mu_m-\mu_{(m+1)})}{(\mu_2-\mu_{(m-1)})(\mu_3-\mu_{(m-1)})\ldots(\mu_m-\mu_{(m-1)})}\\
&+\frac{(\mu_2-\mu_{(m+1)})(\mu_3-\mu_{(m+1)})\ldots(\mu_{(m-1)}-\mu_{(m+1)})}{(\mu_2-\mu_m)(\mu_3-\mu_m)\ldots(\mu_{(m-1)}-\mu_m)}. \qquad (9)
\end{aligned}$$

Since the induction hypothesis is assumed true for any subset of A of size m, and expression (9) deals with $\{\mu_2, \mu_3, ..., \mu_{(m+1)}\}$ which has size m, therefore by equation (4), expression (9) equals 1. This proves that (6) holds. Therefore $m+1 \in S$. This is the end of the proof.

Theorem 2.2 The solution of system (1) is given by

$$\begin{aligned}
n_k(t) \;=\; & n_k(0)e^{-\sigma_k t} + a_{k-1}n_{k-1}(0)\Big[\frac{e^{-\sigma_k t}}{\sigma_{k-1}-\sigma_k} + \frac{e^{-\sigma_{k-1} t}}{\sigma_k-\sigma_{k-1}}\Big]\\
&+a_{k-1}a_{k-2}n_{k-2}(0)\Big[\frac{e^{-\sigma_{k-2} t}}{(\sigma_{k-1}-\sigma_{k-2})(\sigma_k-\sigma_{k-2})}\\
&+\frac{e^{-\sigma_{k-1} t}}{(\sigma_{k-2}-\sigma_{k-1})(\sigma_k-\sigma_{k-1})} + \frac{e^{-\sigma_k t}}{(\sigma_{k-2}-\sigma_k)(\sigma_{k-1}-\sigma_k)}\Big]\\
&+\ldots\ldots\\
&+a_{k-1}a_{k-2}\ldots a_1 n_1(0)\Big[\frac{e^{-\sigma_1 t}}{(\sigma_2-\sigma_1)(\sigma_3-\sigma_1)\ldots(\sigma_k-\sigma_1)}\\
&+\frac{e^{-\sigma_2 t}}{(\sigma_1-\sigma_2)(\sigma_3-\sigma_2)\ldots(\sigma_k-\sigma_2)}\\
&+\ldots\ldots\\
&+\frac{e^{-\sigma_k t}}{(\sigma_1-\sigma_k)(\sigma_2-\sigma_k)\ldots(\sigma_{k-1}-\sigma_k)}\Big]. \qquad (10)
\end{aligned}$$

Proof :

We will prove this also by Mathematical Induction. To find $n_1(t)$, from the first equation of the system we have

$$\dot{n_1}(t) \;=\; -\sigma_1 n_1(t).$$

Solve for $n_1(t)$, we have

$$n_1(t) \quad = \quad n_1(0)e^{-\sigma_1 t}.$$

To solve for $n_2(t)$, from the second equation of the system, we have

$$\dot{n_2}(t) \quad = \quad a_1 n_1(t) - \sigma_2 n_2(t).$$

Since

$$\frac{d}{dt}(n_2(t)e^{\sigma_2 t}) \quad = \quad a_1 n_1(t)e^{\sigma_2 t}$$

we have

$$\begin{aligned} n_2(t) &= n_2(0)e^{-\sigma_2 t} + a_1 e^{-\sigma_2 t}\int_0^t n_1(0)e^{-\sigma_1 \tau}e^{\sigma_2 \tau}d\tau \\ &= n_2(0)e^{-\sigma_2 t} + a_1 n_1(0)e^{-\sigma_2 t}\Big[\frac{1-e^{-(\sigma_1-\sigma_2)t}}{(\sigma_1-\sigma_2)}\Big] \\ &= n_2(0)e^{-\sigma_2 t} + a_1 n_1(0)\Big[\frac{e^{-\sigma_2 t}}{(\sigma_1-\sigma_2)} + \frac{e^{-\sigma_1 t}}{(\sigma_2-\sigma_1)}\Big]. \end{aligned}$$

Now, we assume that the formula (10) is true for $n_k(t)$, so we have

$$\begin{aligned} n_k(t) \quad = \quad & n_k(0)e^{-\sigma_k t} + a_{k-1}n_{k-1}(0)\Big[\frac{e^{-\sigma_k t}}{(\sigma_{k-1}-\sigma_k)} + \frac{e^{-\sigma_{k-1} t}}{(\sigma_k-\sigma_{k-1})}\Big] \\ & + a_{k-1}a_{k-2}n_{k-2}(0)\Big[\frac{e^{-\sigma_{k-2} t}}{(\sigma_{k-1}-\sigma_{k-2})(\sigma_k-\sigma_{k-2})} \\ & + \frac{e^{-\sigma_{k-1} t}}{(\sigma_{k-2}-\sigma_{k-1})(\sigma_k-\sigma_{k-1})} + \frac{e^{-\sigma_k t}}{(\sigma_{k-2}-\sigma_k)(\sigma_{k-1}-\sigma_k)}\Big] \\ & + \ldots\ldots \\ & + a_{k-1}a_{k-2}\ldots a_{k-i}n_{k-i}(0)\Big[\frac{e^{-\sigma_{k-i} t}}{(\sigma_{k-i+1}-\sigma_{k-i})(\sigma_{k-i+2}-\sigma_{k-i})\ldots(\sigma_k-\sigma_{k-i})} \\ & + \frac{e^{-\sigma_{k-i+1} t}}{(\sigma_{k-i}-\sigma_{k-i+1})(\sigma_{k-i+2}-\sigma_{k-i+1})\ldots(\sigma_k-\sigma_{k-i+1})} \\ & + \ldots\ldots \\ & + \frac{e^{-\sigma_k t}}{(\sigma_{k-i}-\sigma_k)(\sigma_{k-i+1}-\sigma_k)\ldots(\sigma_{k-1}-\sigma_k)}\Big] \\ & + \ldots\ldots \\ & + a_{k-1}a_{k-2}\ldots a_1 n_1(0)\Big[\frac{e^{-\sigma_1 t}}{(\sigma_2-\sigma_1)(\sigma_3-\sigma_1)\ldots(\sigma_k-\sigma_1)} \\ & + \frac{e^{-\sigma_2 t}}{(\sigma_1-\sigma_2)(\sigma_3-\sigma_2)\ldots(\sigma_k-\sigma_2)} \end{aligned}$$

$$+\ldots\ldots$$
$$+\frac{e^{-\sigma_k t}}{(\sigma_1-\sigma_k)(\sigma_2-\sigma_k)\ldots(\sigma_{k-1}-\sigma_k)}\Big]. \tag{11}$$

We are going to prove it is true for $n_{k+1}(t)$. From the $(k+1)^{th}$ equation of the system we have

$$\begin{aligned}\dot{n}_{k+1}(t) &= a_k n_k(t) - \sigma_{k+1} n_{k+1}(t),\\ n_{k+1}(t) &= n_{k+1}(0)e^{-\sigma_{k+1}t} + a_k e^{-\sigma_{k+1}t}\int_0^t n_k(\tau)e^{\sigma_{k+1}\tau}d\tau. \end{aligned} \tag{12}$$

To evaluate the integral of $n_k(\tau)$ in (12), we only need to evaluate the integral of the ith term of $n_k(t)$ in (11):

$$\begin{aligned}
& a_k e^{-\sigma_{k+1}t}\int_0^t\Big(a_{k-1}a_{k-2}\ldots a_{k-i}n_{k-i}(0)\Big[\\
& \frac{e^{(-\sigma_{k-i}+\sigma_{k+1})\tau}}{(\sigma_{k-i+1}-\sigma_{k-i})(\sigma_{k-i+2}-\sigma_{k-i})\ldots(\sigma_k-\sigma_{k-i})}\\
& +\frac{e^{(-\sigma_{k-i+1}+\sigma_{k+1})\tau}}{(\sigma_{k-i}-\sigma_{k-i+1})(\sigma_{k-i+2}-\sigma_{k-i+1})\ldots(\sigma_k-\sigma_{k-i+1})}\\
& +\ldots\ldots\\
& +\frac{e^{(-\sigma_k+\sigma_{k+1})\tau}}{(\sigma_{k-i}-\sigma_k)(\sigma_{k-i+1}-\sigma_k)\ldots(\sigma_{k-1}-\sigma_k)}\Big]\Big)d\tau\\
= \;& a_k a_{k-1}a_{k-2}\ldots a_{k-i}n_{k-i}(0)\Big(\\
& \frac{e^{-\sigma_{k-i}t}}{(\sigma_{k-i+1}-\sigma_{k-i})(\sigma_{k-i+2}-\sigma_{k-i})\ldots(\sigma_k-\sigma_{k-i})(\sigma_{k+1}-\sigma_{k-i})}\\
& +\frac{e^{-\sigma_{k-i+1}t}}{(\sigma_{k-i}-\sigma_{k-i+1})(\sigma_{k-i+2}-\sigma_{k-i+1})\ldots(\sigma_k-\sigma_{k-i+1})(\sigma_{k+1}-\sigma_{k-i+1})}\\
& +\ldots\ldots\\
& +\frac{e^{-\sigma_k t}}{(\sigma_{k-i}-\sigma_k)(\sigma_{k-i+1}-\sigma_k)\ldots(\sigma_{k-1}-\sigma_k)(\sigma_{k+1}-\sigma_k)}\\
& -e^{-\sigma_{k+1}t}\Big[\frac{1}{(\sigma_{k-i+1}-\sigma_{k-i})(\sigma_{k-i+2}-\sigma_{k-i})\ldots(\sigma_k-\sigma_{k-i})(\sigma_{k+1}-\sigma_{k-i})}\\
& +\frac{1}{(\sigma_{k-i}-\sigma_{k-i+1})(\sigma_{k-i+2}-\sigma_{k-i+1})\ldots(\sigma_k-\sigma_{k-i+1})(\sigma_{k+1}-\sigma_{k-i+1})}\\
& +\ldots\ldots\\
& +\frac{1}{(\sigma_{k-i}-\sigma_k)(\sigma_{k-i+1}-\sigma_k)\ldots(\sigma_{k-1}-\sigma_k)(\sigma_{k+1}-\sigma_k)}\Big]\Big).
\end{aligned} \tag{13}$$

Applying *lemma* 1, the last term between two brackets in (13) equals

$$\frac{-1}{(\sigma_{k-i}-\sigma_{k+1})(\sigma_{k-i+1}-\sigma_{k+1})\dots(\sigma_k-\sigma_{k+1})}.$$

Therefore, the right hand side of equation (13) can be simplified as

$$\begin{aligned}&a_k a_{k-1} a_{k-2}\dots a_{k-i} n_{k-i}(0)\Big(\frac{e^{-\sigma_{k-i}t}}{(\sigma_{k-i+1}-\sigma_{k-i})(\sigma_{k-i+2}-\sigma_{k-i})\dots(\sigma_{k+1}-\sigma_{k-i})}\\&+\frac{e^{-\sigma_{k-i+1}t}}{(\sigma_{k-i}-\sigma_{k-i+1})(\sigma_{k-i+2}-\sigma_{k-i+1})\dots(\sigma_{k+1}-\sigma_{k-i+1})}\\&+\dots\dots\\&+\frac{e^{-\sigma_k t}}{(\sigma_{k-i}-\sigma_k)(\sigma_{k-i+1}-\sigma_k)\dots(\sigma_{k+1}-\sigma_k)}\\&+\frac{e^{-\sigma_{k+1}t}}{(\sigma_{k-i}-\sigma_{k+1})(\sigma_{k-i+1}-\sigma_{k+1})\dots(\sigma_k-\sigma_{k+1})}\Big).\end{aligned}$$

This completes the proof.

3 Conclusions

In this paper, an explicit solution to an infinite countable system of Ordinary Linear Differential Equations with constant coefficients is found. For this purpose, a theorem is proved. A very important lemma, which can be used in a very wide range of Mathematical fields, is used to prove this theorem. We also were able to prove the lemma.

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