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Properties of Fuzzy Ideals with Thresholds (α, β) of Semigroups

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Abstract

In this paper the notions of fuzzy ideals, fuzzy interior ideals, fuzzy prime ideals, fuzzy semiprime ideals and fuzzy ideal extensions with thresholds (α, β) of a semigroup have been introduced and some important characterizations have been obtained.

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1 Introduction

Uncertainty is an attribute of information and uncertain data are presented in various domains. The most appropriate theory for dealing with uncertainties was introduced by Zadeh[14] in 1965 by defining fuzzy set which has opened up keen insights and applications in vast range of scientific fields. Rosenfeld[8] pioneered the study of fuzzy algebraic structures by introducing the notions of fuzzy groups and showed that many results in groups can be extended in an elementary manner to develop algebraic concepts. After that Kuroki[4, 5, 6, 7] started the study of fuzzy ideal theory in semigroups. Recently, Yuan[13] introduced the concept of fuzzy subgroups with thresholds. A fuzzy subgroup with thresholds α and β is also called a (α, β)-fuzzy subgroup. Using that concept, Yao[10, 11, 12] continued to make researches on (α, β)-fuzzy normal subgroups, (α, β)-fuzzy quotient subgroups and (α, β)-fuzzy subrings. Here we want to introduce fuzzy ideal theory with thresholds of a semigroup.

2 Main Results

These are the main results of the paper.

Unless or otherwise mentioned throughout this paper S stands for semigroup. Throughout this paper we assume that $0 \leq \alpha < \beta \leq 1$. We recall the following which will be required in the sequel.

Definition 2.1. [14] A function μ from a non-empty set S to the unit interval $[0, 1]$ is called a fuzzy subset of S.

Definition 2.2. [4]Let μ be a fuzzy subset of S, then we denote $\mu_{\lambda} := \{x \in S : \mu(x) \geq \lambda\}$ for all $\lambda \in [0, 1]$.

Definition 2.3. [1] Let S be a semigroup. A left(right) ideal of S is a nonempty subset A of S such that $SA \subseteq A(AS \subseteq A)$. A two-sided ideal(or simply an ideal) of S is a subset of S which is both a left and right ideal of S.

Definition 2.4. Let S be a semigroup. A non-empty fuzzy subset μ of S is called a fuzzy left ideal(fuzzy right ideal) with thresholds (α, β) of S if $\mu(xy) \vee \alpha \geq \mu(y) \wedge \beta$(resp. $\mu(xy) \vee \alpha \geq \mu(x) \wedge \beta$) for all $x, y \in S$.

Definition 2.5. Let S be a semigroup. A non-empty fuzzy subset μ of S is called a fuzzy ideal with thresholds (α, β) of S if it is a fuzzy left ideal and fuzzy right ideal with thresholds (α, β) of S.

Remark 1. Any fuzzy ideal with thresholds $(0, 1)$ of S is a fuzzy ideal with thresholds (α, β) of S, where $\alpha, \beta \in [0, 1]$ and $\alpha \leq \beta$, but the converse does not hold in general as shown in the following example.

Example 1. Let $S = \{0, a, b, c\}$ with the following cayley table:

.	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	0	b	b
c	0	0	b	b

Then S is a semigroup. We define a fuzzy subset $\mu : S \to [0, 1]$ as $\mu(0) = \mu(a) = r, \mu(b) = \mu(c) = \alpha$, where $0 < \alpha \leq \beta < 1, r \in [0, \alpha)$. Clearly, μ is a fuzzy ideal with thresholds (α, β) of S, but is not a fuzzy ideal with thresholds $(0, 1)$ of S, since $r = \mu(ab) < \mu(a) \vee \mu(b) = \alpha$.

Theorem 2.6. *Let μ be a fuzzy subset of S. Then the following are equivalent: (1) μ is a fuzzy left ideal(fuzzy right ideal, fuzzy ideal) with thresholds (α, β) of S; (2) μ_λ is a left ideal(resp. right ideal, ideal) of S, for any $\lambda \in (\alpha, \beta]$, where $\mu_\lambda \neq \phi$.*

Proof. (1) $\Rightarrow$ (2) : Let μ be a fuzzy left ideal with thresholds (α, β) of S and $\lambda \in (\alpha, \beta]$ such that $\mu_\lambda \neq \phi$. Let $x \in S$ and $y \in \mu_\lambda$. Then $\mu(y) \geq \lambda$. By hypothesis $\mu(xy) \vee \alpha \geq \mu(y) \wedge \beta \geq \lambda \wedge \beta = \lambda$. Since $\alpha \leq \lambda$, $\mu(xy) \geq \lambda$ and consequently, $xy \in \mu_\lambda$. Hence μ_λ is a left ideal of S.

(2) $\Rightarrow$ (1) : Let μ_λ is a left ideal of S, for any $\lambda \in [\alpha, \beta)$ such that $\mu_\lambda \neq \phi$. Let there exist $x_0, y_0 \in S$ such that $\mu(x_0 y_0) \vee \alpha < \mu(y_0) \wedge \beta$. Now consider $\mu(y_0) \wedge \beta = \lambda$. Then $\lambda \in (\alpha, \beta]$ and $\mu(y_0) \geq \lambda$. So by hypothesis μ_λ is a left ideal of S. But $\mu(x_0 y_0) < \lambda$. Hence $x_0 y_0 \notin \mu_\lambda$ whereas $y_0 \in \mu_\lambda$. This is a contradiction. So $\mu(xy) \vee \alpha \geq \mu(y) \wedge \beta$ for all $x, y \in S$. Hence μ be a fuzzy left ideal with thresholds (α, β) of S. Similarly we can prove the other cases also. □

Proposition 2.7. *Let S be a semigroup and μ_1 and μ_2 be two fuzzy left ideals(fuzzy right ideals, fuzzy ideals) with thresholds (α, β) of S. Then $\mu_1 \cap \mu_2$, if it is non-empty, is a fuzzy left ideal(resp. fuzzy right ideal, fuzzy ideal) with thresholds (α, β) of S.*

Proof. Let μ_1 and μ_2 be two fuzzy left ideals with thresholds (α, β) of S such that $\mu_1 \cap \mu_2$ is non-empty. Let $x, y \in S$. Then

$$\begin{aligned}
(\mu_1 \cap \mu_2)(xy) \vee \alpha &= (\mu_1(xy) \wedge \mu_2(xy)) \vee \alpha \\
&= (\mu_1(xy) \vee \alpha) \wedge (\mu_2(xy) \vee \alpha) \\
&\geq (\mu_1(y) \wedge \beta) \wedge (\mu_2(y) \wedge \beta) \\
&= (\mu_1(y) \wedge \mu_2(y) \wedge \beta) \\
&= (\mu_1 \cap \mu_2)(y) \wedge \beta.
\end{aligned}$$

Hence $\mu_1 \cap \mu_2$ is a fuzzy left ideal with thresholds (α, β) of S. Similarly we can prove the other cases also. □

Definition 2.8. [1] Let S be a semigroup. A subsemigroup of S is a non-empty subset A of S such that $A^2 \subseteq A$.

Definition 2.9. [1] Let S be a semigroup. A subsemigroup A of S is called a bi-ideal of S if $ASA \subseteq A$.

Definition 2.10. Let S be a semigroup. A non-empty fuzzy subset μ of S is called a fuzzy subsemigroup with thresholds (α, β) of S if $\mu(xy) \vee \alpha \geq (\mu(x) \wedge \mu(y)) \wedge \beta$ for all $x, y \in S$.

Definition 2.11. Let S be a semigroup. A fuzzy subsemigroup μ with thresholds (α, β) of S is called a fuzzy bi-ideal with thresholds (α, β) of S if $\mu(xyz) \vee \alpha \geq (\mu(x) \wedge \mu(z)) \wedge \beta$ for all $x, y \in S$.

By routine verification we can prove the following.

Theorem 2.12. *Let S be a semigroup and μ be a fuzzy subset of S. Then the following are equivalent:* (1) *μ is a fuzzy subsemigroup*(*fuzzy bi-ideal*) *with thresholds* (α, β) *of* S; (2) μ_λ *is a subsemigroup*(*resp. bi-ideal*) *of* S, *for any* $\lambda \in (\alpha, \beta]$, *where* $\mu_\lambda \neq \phi$.

Definition 2.13. [1] Let S be a semigroup. A subsemigroup A of S is called an interior ideal of S if $SAS \subseteq A$.

Definition 2.14. Let S be a semigroup. A fuzzy subsemigroup μ with thresholds (α, β) of S is called a fuzzy interior ideal with thresholds (α, β) of S if $\mu(xay) \vee \alpha \geq \mu(a) \wedge \beta$ for all $x, a, y \in S$.

Theorem 2.15. *Let S be a semigroup and μ be a fuzzy subset of S. Then the following are equivalent:* (1) *μ is a fuzzy interior ideal with thresholds* (α, β) *of* S; (2) μ_λ *is an interior ideal of* S, *for any* $\lambda \in (\alpha, \beta]$, *where* $\mu_\lambda \neq \phi$.

Proof. (1) $\Rightarrow$ (2) : Let μ be a fuzzy interior ideal with thresholds (α, β) of S and $\lambda \in (\alpha, \beta]$ such that $\mu_\lambda \neq \phi$. Since μ is a fuzzy subsemigroup of S then by Theorem 2.12, μ_λ is a subsemigroup of S.

Now let $x, a, y \in S$ be such that $a \in \mu_\lambda$. Then $\mu(a) \geq \lambda$. By hypothesis $\mu(xay) \vee \alpha \geq \mu(a) \wedge \beta \geq \lambda \wedge \beta = \lambda$. Since $\alpha < \lambda$, so $\mu(xay) \geq \lambda$ and consequently, $xay \in \mu_\lambda$. Hence μ_λ is an interior ideal of S.

(2) $\Rightarrow$ (1) : Let μ_λ is a interior ideal of S, for any $\lambda \in (\alpha, \beta]$ such that $\mu_\lambda \neq \phi$. Then by Theorem 2.12 μ is a fuzzy subsemigroup of S. Let there exist $x_0, a, y_0 \in S$ such that $\mu(x_0ay_0) \vee \alpha < \mu(a) \wedge \beta$. Now consider $\mu(a) \wedge \beta = \lambda$. Then $\lambda \in (\alpha, \beta]$ and $\mu(a) \geq \lambda$. So by hypothesis μ_λ is an interior ideal of S. But $\mu(x_0ay_0) < \lambda$. Hence $x_0ay_0 \notin \mu_\lambda$ whereas $a \in \mu_\lambda$. This is a contradiction. So $\mu(xay) \vee \alpha \geq \mu(a) \wedge \beta$ for all $x, a, y \in S$. Hence μ is a fuzzy interior ideal with thresholds (α, β) of S. □

Definition 2.16. [1] A semigroup S is said to be regular if $x \in xSx$ for any $x \in S$.

Proposition 2.17. *Let S be a regular semigroup and μ be a fuzzy interior ideal with thresholds (α, β) of S. Then μ is a fuzzy ideal with thresholds (α, β) of S.*

Proof. Let $x, y \in S$. Then there exist $s \in S$ such that $x = xsx$. Then $\mu(xy) \vee \alpha = \mu((xsx)y) \vee \alpha$(since S is regular) $= \mu((xs)xy) \vee \alpha \geq \mu(x) \wedge \beta$. Hence μ is a fuzzy right ideal with thresholds (α, β) of S. Similarly we can show that μ is a fuzzy right ideal with thresholds (α, β) of S. Hence μ is a fuzzy ideal with thresholds (α, β) of S. $\square$

Theorem 2.18. *Let $f : S \to T$ be a homomorphism of semigroups and let μ be a fuzzy subsemigroup(fuzzy bi-ideal, fuzzy interior ideal, fuzzy ideal) with thresholds (α, β) of T. Then $f^{-1}(\mu)$ is a fuzzy subsemigroup(resp. fuzzy bi-ideal, fuzzy interior ideal, fuzzy ideal) with thresholds (α, β) of S, where $f^{-1}(\mu)(x) := \mu(f(x))$ for all $x \in S$.*

Proof. Let $x, y \in S$ and μ is a fuzzy subsemigroup with thresholds (α, β) of T. Then

$$\begin{aligned} f^{-1}(\mu)(xy) \vee \alpha &= \mu(f(xy)) \vee \alpha \\ &= \mu(f(x)f(y)) \vee \alpha (\text{since } f \text{ is a homomorphism}) \\ &\geq (\mu(f(x)) \wedge \mu(f(y))) \wedge \beta \\ &= (f^{-1}(\mu)(x) \wedge f^{-1}(\mu)(y)) \wedge \beta. \end{aligned}$$

Hence $f^{-1}(\mu)$ is a fuzzy subsemigroup with thresholds (α, β) of S. Similarly we can prove the other cases also. $\square$

Theorem 2.19. *Let $f : S \to T$ be a surjective homomorphism of semigroups and let μ be a fuzzy subsemigroup (fuzzy bi-ideal, fuzzy interior ideal, fuzzy ideal) with thresholds (α, β) of S. Then $f(\mu)$ is a fuzzy subsemigroup(fuzzy bi-ideal, fuzzy interior ideal, fuzzy ideal) with thresholds (α, β) of T, where $f(\mu)(y) := \sup_{x \in S} \{\mu(x) : f(x) = y\}$, for all $y \in T$.*

Proof. Let $x, y \in T$. Since f is a surjective homomorphism, $f^{-1}(x)$ and $f^{-1}(y)$ are non-empty. Then

$$\begin{aligned} f(\mu)(xy) \vee \alpha &= \sup_{t \in S}\{\mu(t) : f(t) = xy\} \vee \alpha \\ &= \sup_{t \in S}\{\mu(t) \vee \alpha : f(t) = xy\} \\ &\geq \sup_{m,n \in S}\{\mu(mn) \vee \alpha : f(m) = x, f(n) = y\} \\ &\geq \sup_{m,n \in S}\{(\mu(m) \wedge \mu(n)) \wedge \beta : f(m) = x, f(n) = y\} \end{aligned}$$

$$= ((\sup_{m\in S}\{\mu(m) : f(m) = x\}) \wedge (\sup_{n\in S}\{\mu(n) : f(n) = y\})) \wedge \beta$$
$$= (f(\mu)(x) \wedge f(\mu)(y)) \wedge \beta.$$

Hence $f(\mu)$ is a fuzzy subsemigroup with thresholds (α, β) of T. Similarly we can prove the other cases also. □

Definition 2.20. [1] Let S be a semigroup. An ideal P of S is said to be completely prime if for any two elements $a, b \in S$, $ab \in P$ implies that $a \in P$ or $b \in P$.

Definition 2.21. A fuzzy ideal μ with thresholds (α, β) of S is called a fuzzy prime ideal with thresholds (α, β) of S if $\mu(xy)\vee\alpha = (\mu(x)\vee\mu(y))\wedge\beta \; \forall x, y \in S$.

Theorem 2.22. *Let μ be a fuzzy subset of S. Then the following are equivalent: (1) μ is a fuzzy prime ideal with thresholds (α, β) of S; (2) μ_λ is a completely prime ideal of S, for any $\lambda \in [\alpha, \beta)$*

Proof. (1) $\Rightarrow$ (2) : Let μ be a fuzzy prime with thresholds (α, β) of S. Let $\lambda \in [\alpha, \beta)$ such that λ is non-empty. Let for $x, y \in S$, $xy \in \mu_\lambda$. Then $\mu(xy) \geq \lambda$. Since $\lambda \geq \alpha$, so $\mu(xy) \vee \alpha \geq \lambda$. Since μ is a fuzzy prime ideal with thresholds (α, β) of S, so $(\mu(x) \vee \mu(y)) \wedge \beta \geq \lambda = \lambda \wedge \beta$. Consequently, $(\mu(x) \vee \mu(y)) \geq \lambda$, which implies that $\mu(x) \geq \lambda$ or $\mu(y) \geq \lambda$. So $x \in \mu_\lambda$ or $x \in \mu_\lambda$. Hence μ_λ is a completely prime ideal of S.

(2) $\Rightarrow$ (1) : Let μ_λ is a completely prime ideal of S. Let $x, y \in S$ and $\mu(xy) \vee \alpha = \lambda$. Since $\lambda \in (\alpha, \beta]$, so $\mu(xy) \geq \lambda$. So $xy \in \mu_\lambda$. Since μ_λ is completely prime, so $x \in \mu_\lambda$ or $y \in \mu_\lambda$. This implies that $\mu(x) \geq \lambda$ or $\mu(y) \geq \lambda$. Then $\mu(x) \vee \mu(y) \geq \lambda$. Since $\lambda \in (\alpha, \beta]$, so $(\mu(x) \vee \mu(y)) \wedge \beta \geq \lambda = \mu(xy) \vee \alpha$......................(1). Since μ is a fuzzy ideal with thresholds (α, β) of S, so $\mu(xy) \vee \alpha \geq \mu(x) \wedge \beta$ and $\mu(xy) \vee \alpha \geq \mu(y) \wedge \beta$. Then $\mu(xy) \vee \alpha \geq (\mu(x) \wedge \beta) \vee (\mu(y) \wedge \beta) = (\mu(x) \vee \mu(y)) \wedge \beta$...................(2). Combining (1) and (2) we have $\mu(xy) \vee \alpha = (\mu(x) \vee \mu(y)) \wedge \beta$. Hence μ is a fuzzy prime ideal with thresholds (α, β) of S. □

Definition 2.23. [1] Let S be a semigroup. An ideal P of S is said to be semiprime if for an element $a \in S$, $a^2 \in P$ implies that $a \in P$.

Definition 2.24. Let S be a semigroup. A fuzzy ideal μ with thresholds (α, β) of S is called a fuzzy semiprime ideal with thresholds (α, β) of S if $\mu(x) \vee \alpha \geq \mu(x^2) \wedge \beta$ for all $x \in S$.

Theorem 2.25. *Let S be a semigroup and μ be a fuzzy subset of S. Then the following are equivalent: (1) μ is a fuzzy semiprime ideal with thresholds (α, β) of S; (2) μ_λ is a semiprime ideal of S, for any $\lambda \in (\alpha, \beta]$ where $\mu_\lambda \neq \phi$.*

Proof. (1) $\Rightarrow$ (2) : Let μ be a fuzzy semiprime ideal with thresholds (α, β) of S. Then by Definition 2.24 and Theorem 2.6, μ_λ is an ideal for any $\lambda \in (\alpha, \beta]$ where $\mu_\lambda \neq \phi$. Let $\lambda \in (\alpha, \beta]$ such that μ_λ is non-empty. Let $x \in S$ such that $x^2 \in \mu_\lambda$. Then $\mu(x^2) \geq \lambda$. Since $\beta \geq \lambda$, so $\mu(x^2) \wedge \beta \geq \lambda$. Since μ is a fuzzy semiprime ideal with thresholds (α, β) of S, so $\mu(x) \vee \alpha \geq \lambda = \lambda \vee \alpha$. Consequently, $\mu(x) \geq \lambda$. So $x \in \mu_\lambda$. Hence μ_λ is a completely semiprime ideal of S.

(2) $\Rightarrow$ (1) : Let μ_λ is a completely semiprime ideal of S, for any $\lambda \in (\alpha, \beta]$ where $\mu_\lambda \neq \phi$. Then by Definition 2.23 and Theorem 2.6, μ is a fuzzy ideal S. Let there exists $x \in S$ such that $\mu(x) \vee \alpha < \mu(x^2) \wedge \beta$. Now consider $\mu(x^2) \wedge \beta = \lambda$. Then $\lambda \in (\alpha, \beta]$ and $\mu(x^2) \geq \lambda$. So by hypothesis μ_λ is a semiprime ideal of S. But $\mu(x) < \lambda$. Hence $x \notin \mu_\lambda$ whereas $x^2 \in \mu_\lambda$. This is a contradiction. So $\mu(x) \vee \alpha \geq \lambda = \mu(x^2) \wedge \beta$ for all $x \in S$. Hence μ is a fuzzy semiprime ideal with thresholds (α, β) of S. □

Proposition 2.26. *Let S be a semigroup and μ_1 and μ_2 be two fuzzy semiprime ideals with thresholds (α, β) of S. Then $\mu_1 \cap \mu_2$, if it is non-empty, is a fuzzy semiprime ideal with thresholds (α, β) of S.*

Proof. Let μ_1 and μ_2 be two fuzzy semiprime ideals with thresholds (α, β) of S such that $\mu_1 \cap \mu_2$ is non-empty. Then by Definition 2.24 and Theorem 2.7, $\mu_1 \cap \mu_2$ is fuzzy ideal with thresholds (α, β) of S. Let $x \in S$. Then

$$\begin{aligned}
(\mu_1 \cap \mu_2)(x) \vee \alpha &= (\mu_1(x) \wedge \mu_2(x)) \vee \alpha \\
&= (\mu_1(x) \vee \alpha) \wedge (\mu_2(x) \vee \alpha) \\
&\geq (\mu_1(x^2) \wedge \beta) \wedge (\mu_2(x^2) \wedge \beta) \\
&= (\mu_1(x^2) \wedge \mu_2(x^2) \wedge \beta \\
&= (\mu_1 \cap \mu_2)(x^2) \wedge \beta.
\end{aligned}$$

Hence $\mu_1 \cap \mu_2$ is a fuzzy semiprime ideal with thresholds (α, β) of S. □

Definition 2.27. [9] Let S be a semigroup, μ be a fuzzy subset of S and $x \in S$, then the fuzzy subset $< x, \mu >: S \rightarrow [0, 1]$ defined by $< x, \mu > (y) = \mu(xy)$ for all $y \in S$ is called the extension of μ by x.

Proposition 2.28. *Let S be a commutative semigroup, μ be a fuzzy ideal with thresholds (α, β) of a S and $x \in S$. Then $< x, \mu >$ is a fuzzy ideal with thresholds (α, β) of S.*

Proof. Let μ be a fuzzy ideal with thresholds (α, β) of a commutative semigroup S and $p, q \in S$. Then $< x, \mu > (pq) \vee \alpha = \mu(xpq) \vee \alpha \geq \mu(xp) \wedge \beta =< x, \mu > (p) \wedge \beta$. So $< x, \mu >$ is a fuzzy ideal with thresholds (α, β) of S. Hence S being commutative $< x, \mu >$ is a fuzzy ideal with thresholds (α, β) of S. □

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