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Reflection of SV- Waves from the Free Surface of a Magneto-Thermoelastic Isotropic Elastic Half-Space under Initial Stress

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Abstract

The aim of this paper is to study the effect of temperature, magnetic field, relaxation time and initial stress on the reflection of plane SV-waves at the free surface of an isotropic elastic half-space under GL-theory. It is found that when SV-wave is incident on the free surface of the above medium, reflected SV-wave, reflected P-wave and a reflected thermal wave is obtained. We find that P-wave is affected due to the presence of thermal and magnetic field whereas SV-wave remains unaffected which is in accordance with the GL-theory since the temperature and magnetic field in an infinite space results only in irrotational changes. The effect of temperature, magnetic field, relaxation time and initial stresses on reflection coefficients of incident SV-wave are plotted under certain practical assumptions.

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Keywords: Reflection coefficients; Magnetic field; Relaxation time; GL- theory

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1 Introduction

The modelling of surface waves dispersion effects has become of growing interest to geotechnical engineers and geophysicists. The seismic waves usually studied in seismology and seismic surveying are those produced by earthquakes, explosions, or impacts. These waves are complex vibrations of limited duration having the nature of an impulse. The problem of propagation velocity of these complex vibrations requires additional research. When the wavelength of the harmonic component is significantly small compared with the heterogeneity, such as thickness of layers, the oscillations are propagated following the laws of geometrical optics. By knowing the reflection and refraction, the magneto-thermal elastic plane waves are the useful source for imagining the interior of the Earth. According to the conventional heat conduction theory, the thermal disturbances travel at infinite velocities. However, from the physical point of view, the above concept is unrealistic in the situation of very low temperature near absolute zero. The hyperbolic equations of motion are applicable in such cases and the elastic disturbances propagate with finite speeds. Thus, generalized thermoelasticity theories are proposed to examine modified thermoelastic models involving a hyperbolic type of heat equation.

Problem related to magneto-thermoelastic plane wave deals with the interactions among strain, temperature, and electromagnetic fields in transversely isotropic and anisotropic medium has many applications in geophysics, optics, electrical power engineering and seismology. Shekhar and Parvez [1] purposed plane waves propagating in transversely isotropic dissipative half space under the effect of rotation, magnetic field and stress. Othman and Song [2] discussed reflection of magneto-thermo-elastic wave by using generalized theory of elasticity. Mehditabar *et al.* [3] investigated magneto -thermo-elastic functionally graded conical shell. Othman [4] studied electro-magneto-thermoelastic thermal shock plane waves for a finite conducting half-space. Niraula and Noda [5] purposed non-linear electro-magneto-thermo-elasticity by deriving material constants. Niraula and Wang [6] studied the property of magneto-electro-elastic material with a penny-shaped crack subjected to temperature loading.

Kaur and Sharma [7] discussed reflection and transmission of thermoelastic plane waves at liquid-solid interface. Fractional order generalized electro-magneto-thermo-elasticity was given by Ya *et al.* [8]. Propagation of plane waves at the interface of an elastic solid half-space and a microstretch thermoelastic diffusion solid half-space was investigated by Kumar *et al.* [9]. Chakraborty [10] discussed reflection of plane elastic waves in half-space subjected to temperature and initial stress. Singh and Yadav [11] discussed the reflection of plane waves in a rotating transversely isotropic magneto-thermoelastic solid half-space. Singh and Bala [12] purposed the reflection of P and SV waves from the free surface of a two-temperature thermoelastic solid half-space.

This paper discusses the problem of reflection of magneto-thermoelastic

SV-waves under initial stress in transversely isotropic solid half space. Biot's equations are modified in terms of Green and Lindsay's theory of thermoelasticity. The governing equations are solved in light of modified heat equation to obtain reflection coefficients for P-wave, thermal wave and SV-wave. Results are plotted with MATLAB software to show the effect of temperature, magnetic field, relaxation time and initial stresses on the reflection of incident SV-wave.

2 Governing equations

The governing equations of linear, isotropic and homogenous magneto-thermoelastic solid with initial stress are

a. The stress-strain-temperature relation:

$$s_{ij} = -P(\delta_{ij} + \omega_{ij}) + \bar{\lambda}\, e_{PP}\, \delta_{ij} + 2\bar{\mu} e_{ij} - \frac{\alpha}{k_T}(\mathrm{T} + \alpha\dot{\mathrm{T}})\delta_{ij}, \tag{1}$$

where, s_{ij} are the components of stress tensor, P is initial pressure, δ_{ij} is the Kronecker delta, ω_{ij} are the components of small rotation tensor, $\bar{\lambda}, \bar{\mu}$ are the counterparts of Lame parameters, e_{ij} are the components of the strain tensor, α is the volume coefficient of thermal expansion, k_T is the isothermal compressibility, $\mathrm{T} = \Theta - \mathrm{T}_0$ is small temperature increment, Θ is the absolute temperature of the medium, T_0 is the reference uniform temperature of the body chosen such that $\left|\frac{\mathrm{T}}{\mathrm{T}_0}\right| \ll 1.$

The displacement-strain relation:

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{2}$$

where, u_i are the components of the displacement vector.

The small rotation-displacement relation:

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) \tag{3}$$

where, u_i are the components of the displacement vector.

b. The modified Fourier's law:

$$h_i + a^* \dot{h}_i = K \frac{\partial \mathrm{T}}{\partial x_i} \tag{4}$$

where, K is the thermal conductivity, $a, a^* \geq 0$ are the thermal relaxation times

c. The heat conduction equation :

$$K\left(\frac{\partial^2 \mathrm{T}}{\partial x^2} + \frac{\partial^2 \mathrm{T}}{\partial y^2}\right) = \rho c_p \left(\frac{\partial \mathrm{T}}{\partial t} + \tau \frac{\partial^2 \mathrm{T}}{\partial t^2}\right) + \gamma \mathrm{T}_0 \left(\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 v}{\partial y \partial t} + \tau_0 \delta_{ij}\left[\frac{\partial^3 u}{\partial x \partial t^2} + \frac{\partial^3 v}{\partial y \partial t^2}\right]\right) \tag{5}$$

where, K is the thermal conductivity, c_p is specific heat per unit mass at constant strain, τ_0 is the first relaxation time, τ is second relaxation time, δ_{ij} is the Kronecker delta, ρ is density and T is the incremental change of temperature from the initial state of the solid half space. Moreover the use of the relaxation times τ, τ_0 and a parameter δ_{ij} marks the aforementioned fundamental equations possible for the three different theories:

Classical Dynamical theory: $\tau = \tau_0 = 0, \delta_{ij} = 0$
Lord and Shulman's theory: $\tau = 0,\ \tau_0 > 0,\ \delta_{ij} = 1$
Green and Lindsay's theory: $\tau \geq \tau_0 > 0,\ \delta_{ij} = 0$

d. Maxwell's equations:

$$\vec{\nabla} \cdot \vec{\mathrm{E}} = 0, \vec{\nabla} \cdot \vec{\mathrm{B}} = 0, \vec{\nabla} \times \vec{\mathrm{E}} = -\frac{\partial \vec{\mathrm{B}}}{\partial t}, \vec{\nabla} \times \vec{\mathrm{B}} = \mu_e \varepsilon_e \frac{\partial \vec{\mathrm{E}}}{\partial t} \tag{6}$$

where, $\vec{\mathrm{E}}$, $\vec{B}$, μ_e and ε_e are electric field, magnetic field, permeability and permittivity of the medium.

e. The components of electric and magnetic field:

$$\vec{\mathrm{H}}(0,0,\mathrm{H}) = \vec{\mathrm{H}}_0 + \vec{h} \tag{7}$$

where, $\vec{h}$ is the perturbed magnetic field over $\vec{\mathrm{H}}_0$.

f. Maxwell stress components:

$$T_{ij} = \mu_e \left[H_i e_i + H_j e_j - \left(H_k e_k \right) \delta_{ij} \right] \text{ (where i, j, k =1, 2,3)} \tag{8}$$

where, H_i, H_j, H_k are the components of primary magnetic field, e_i, e_j, e_k are the stress components acting along x-axis, y-axis, z-axis respectively and δ_{ij} is the Kronecker delta.

Using Eq. (8), we get

$$T_{22} = \mu_e H_0^2 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \text{ and } T_{12} = 0 \tag{9}$$

The dynamical equations of motion for the propagation of wave have been derived by Biot [13] and in two dimensions these are given by

$$\frac{\partial s_{11}}{\partial x} + \frac{\partial s_{12}}{\partial y} - P \frac{\partial \omega}{\partial y} + B_x = \rho \frac{\partial^2 u}{\partial t^2} \tag{10}$$

$$\frac{\partial s_{12}}{\partial x} + \frac{\partial s_{22}}{\partial y} - P \frac{\partial \omega}{\partial x} + B_y = \rho \frac{\partial^2 v}{\partial t^2} \tag{11}$$

where, s_{11}, s_{22} and s_{12} are incremental thermal stress components. The first two are principal stress components along x- and y-axes, respectively and last one is shear stress component in the x-y plane, ρ is the density of the medium and *u*, *v* are the displacement components along x and y directions respectively, B is body force and its components along x and y axis are B_x and B_y respectively . ω is the rotational component i.e. $\omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$ and $P = s_{22} - s_{11}$.

The body forces along x and y axis under constant primary magnetic field H_0 parallel to z-axis are given by

$$B_x = \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} \right) \tag{12}$$

$$B_y = \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2} \right) \tag{13}$$

where, μ_e is permittivity of the medium.

Following Biot [13], the stress-strain relations with incremental isotropy are

$$s_{11} = (\lambda + 2\mu + P)e_{xx} + (\lambda + P)e_{yy} + 2\mu e_{xx} - \gamma\left(\mathrm{T} + \tau \frac{\partial \mathrm{T}}{\partial x}\right) \quad (14)$$

$$s_{22} = \lambda e_{xx} + (\lambda + 2\mu)e_{yy} - \gamma\left(\mathrm{T} + \tau \frac{\partial \mathrm{T}}{\partial x}\right) \quad (15)$$

$$s_{12} = 2\mu e_{xy} \quad (16)$$

where,

$$e_{xx} = \frac{\partial u}{\partial x}, \qquad e_{yy} = \frac{\partial v}{\partial x}, \qquad e_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \quad (17)$$

where, e_{xx} and e_{yy} are the principle strain components and e_{xy} is the shear strain component, $\gamma = (3\lambda + 2\mu)\alpha_t$, α_t is the coefficient of linear expansion of the material, λ μ are Lame's constants, T is the incremental change of temperature from the initial state and τ is second relaxation time.

3 Formulation of the problem

We consider a transversely isotropic, homogeneous elastic half space under constant magnetic field acting along z-axis and initial compressive stress P acting along x-axis at absolute temperature T_0 (Figure-1). A plane SV-wave is incident at an angle θ at $y = 0$, such that it get reflected and giving three waves namely reflected SV at an angle θ, thermal-waves at an angle θ_1 and P-wave at an angle θ_2 respectively as shown in the diagram.

4 Boundary conditions

The following boundary conditions are supplemented at $y = 0$:

i. $\nabla f_x = s_{12} - Pe_{xy} = 0,$

ii. $\nabla f_y = s_{22} = 0,$

iii. $\frac{\partial T}{\partial y} + hT = 0.$ (18)

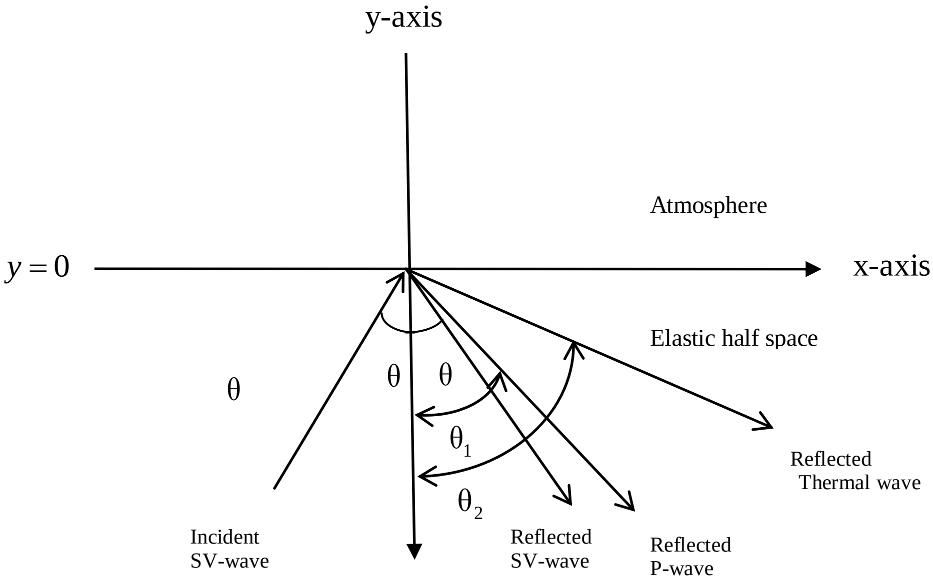

Figure 1 Reflection of magneto-thermoelastic plane waves

5 Solution of the problem

From Eq. (12), Eq. (13), Eq. (14), Eq. (15), Eq. (16) and Eq. (17), we get

$$(\lambda+2\mu+P)\frac{\partial^2 u}{\partial x^2}+\left(\lambda+\mu+\frac{P}{2}\right)\frac{\partial^2 v}{\partial x\partial y}+\left(\mu+\frac{P}{2}\right)\frac{\partial^2 u}{\partial^2 y}+\mu_e H_0^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}\right)=\rho\frac{\partial^2 u}{\partial t^2}+\gamma\left(\frac{\partial \mathrm{T}}{\partial x}+\tau\frac{\partial^2 \mathrm{T}}{\partial t\partial x}\right) \quad (19)$$

$$(\lambda+2\mu)\frac{\partial^2 v}{\partial y^2}+\left(\lambda+\mu+\frac{P}{2}\right)\frac{\partial^2 u}{\partial x\partial y}+\left(\mu-\frac{P}{2}\right)\frac{\partial^2 v}{\partial^2 x}+\mu_e H_0^2\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}\right)=\rho\frac{\partial^2 v}{\partial t^2}+\gamma\left(\frac{\partial \mathrm{T}}{\partial y}+\tau\frac{\partial^2 \mathrm{T}}{\partial t\partial y}\right) \quad (20)$$

Eq. (5) can be modified as

$$K\nabla^2\mathrm{T}=\rho c_p\left(\frac{\partial \mathrm{T}}{\partial t}+\tau\frac{\partial^2 \mathrm{T}}{\partial t^2}\right)+\gamma \mathrm{T}_0\left[\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\tau_0\delta_{ij}\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right] \quad (21)$$

Eq. (19) and Eq. (20) can be solved by choosing potential functions ϕ and ψ as

$$u=\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial y} \text{ and } v=\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial y} \quad (22)$$

From Eq. (19) and (22), we get

$$\nabla^2\phi = \frac{\rho}{(\lambda+2\mu+\mu_e H_0^2+P)}\frac{\partial^2\phi}{\partial t^2} + \frac{\gamma}{(\lambda+2\mu+\mu_e H_0^2+P)}\left(\mathrm{T}+\tau\frac{\partial \mathrm{T}}{\partial t}\right) \tag{23}$$

$$\nabla^2\psi = \frac{\rho}{\left(\mu+\frac{P}{2}\right)}\frac{\partial^2\psi}{\partial t^2} \tag{24}$$

From Eq. (20) and Eq. (22), we get

$$\nabla^2\phi = \frac{\rho}{(\lambda+2\mu+\mu_e H_0^2)}\frac{\partial^2\phi}{\partial t^2} + \frac{\gamma}{(\lambda+2\mu+\mu_e H_0^2)}\left(\mathrm{T}+\tau\frac{\partial \mathrm{T}}{\partial t}\right) \tag{25}$$

$$\nabla^2\psi = \frac{\rho}{\left(\mu-\frac{P}{2}\right)}\frac{\partial^2\psi}{\partial t^2} \tag{26}$$

Eq. (23) and Eq. (25) represent magneto- thermo compression waves along x- axis and y- axis respectively, whereas Eq. (24) and Eq. (26) represent magneto- thermo distortional waves along x- axis and y- axis respectively. For initial stress along x-axis, the four equations (23)-(26) reduced to two equations as

$$\nabla^2\phi = \frac{1}{c_1^2}\frac{\partial^2\phi}{\partial t^2} + \frac{\gamma}{(\lambda+2\mu+\mu_e H_0^2+P)}\left(T+\tau\frac{\partial T}{\partial t}\right) \tag{27}$$

$$\nabla^2\psi = \frac{1}{c_2^2}\frac{\partial^2\psi}{\partial t^2} \tag{28}$$

where,

$$c_1^2 = \frac{(\lambda+2\mu+\mu_e H_0^2+P)}{\rho} \text{ and } c_2^2 = \frac{\left(\mu-\frac{P}{2}\right)}{\rho} \tag{29}$$

c_1 is known as P-wave velocity and c_2 is called SV-wave velocity. Also, for P-wave $v=0$ and for SV-wave $u=0$.

Now, from Eq. (20) and (22), we get

$$K\nabla^2 \mathrm{T} = \rho c_p \left(\frac{\partial \mathrm{T}}{\partial t} + \tau \frac{\partial^2 \mathrm{T}}{\partial t^2} \right) + \gamma \mathrm{T}_0 \left[\frac{\partial}{\partial t}\left(\nabla^2 \phi\right) + \tau_0 \delta_{ij} \frac{\partial^2}{\partial t^2}\left(\nabla^2 \phi\right) \right] \tag{30}$$

where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

From equations (27), (28) and (30), we conclude that P-wave depends on the presence of magnetic and thermal field whereas SV-wave remains unaffected which is in accordance with the GL-theory.

The solution of Eq. (27), Eq. (28) and Eq. (30) is plane harmonic waves travelling perpendicular to the x-y plane, which is given as

$$\phi = \phi_1 \exp[i\{k(x\sin\theta + y\cos\theta) - \omega t\}] \tag{31}$$

$$\psi = \psi_1 \exp[i\{l(x\sin\theta + y\cos\theta) - \omega t\}] \tag{32}$$

$$\mathrm{T} = \mathrm{T}_1 \exp[i\{k(x\sin\theta + y\cos\theta) - \omega t\}] \tag{33}$$

where, k and l are compression and rotational wave numbers, ω is angular frequency.

From Eq. (27), Eq. (29) and Eq. (33), we get

$$c_1^2 \left(\frac{\omega^2}{c_1^2} - k^2 \right) \phi_1 - \frac{\gamma}{\rho}\left(1 - i\omega\tau\right) \mathrm{T}_1 = 0 \tag{34}$$

From Eq. (30), Eq. (31) and Eq. (33), we get

$$-i\mathrm{T}_0 \gamma \omega \left(1 - i\omega\tau_0\delta_{ij}\right) k^2 \phi_1 + \left(i\rho c_p \omega\left(1 - i\omega\tau_0\right) - \mathrm{K}k^2\right) \mathrm{T}_1 = 0 \tag{35}$$

In order to satisfy Eq. (34) and Eq. (35), the determinant of the coefficients of both Eq. (34) and Eq. (35) will be zero, therefore

$$\begin{vmatrix} c_1^2 \left(\frac{\omega^2}{c_1^2} - k^2 \right) & -\frac{\gamma}{\rho}\left(1 - i\omega\tau\right) \\ -i\mathrm{T}_0 \gamma \omega \left(1 - i\omega\tau_0\delta_{ij}\right) k^2 & \left(i\rho c_p \omega\left(1 - i\omega\tau_0\right) - \mathrm{K}k^2\right) \end{vmatrix} = 0 \tag{36}$$

Expanding Eq. (36), we get

$$\Lambda^4 - \left(1+\bar{\tau}_{\mathrm{T}} - i\bar{\varphi}\right)\Lambda^2 - i\bar{\varphi} = 0 \tag{37}$$

where,

$$\Lambda = \frac{\omega}{kc_1}, \varphi = \frac{\omega \mathrm{K}}{\rho c_p c_1^2}, \bar{\varphi} = \frac{\varphi}{\left(1-i\omega\tau_0\right)}, \tau_{\mathrm{T}} = \frac{\gamma^2 \mathrm{T}_0}{\rho^2 c_p c_1^2}$$ and

$$\bar{\tau}_{\mathrm{T}} = \frac{\tau_{\mathrm{T}}\left(1-i\omega\tau_0\delta_{ij}\right)\left(1-i\omega\tau\right)}{\left(1-i\omega\tau_0\right)} \tag{38}$$

Eq. (37) is biquadratic in Λ, it means that P-wave and thermal wave travel with different velocities. Therefore, on striking the SV-wave at y = 0 making an angle θ in the solid half space it will have one reflected SV-wave making an angle θ, P-wave and thermal wave at an angle θ_1 and θ_2 (fig. 1). Therefore from the above discussion we can take displacement potential and perturbation temperature in the following form

$$\phi = \alpha_1 \exp[i\{k_1(x\sin\theta_1 + y\cos\theta_1) - \omega t\}] + \alpha_2 \exp[i\{k_2(x\sin\theta_2 + y\cos\theta_2) - \omega t\}] \tag{39}$$

$$\psi = \beta_1 \exp[i\{l(x\sin\theta + y\cos\theta) - \omega t\}] + \beta_2 \exp[i\{l(x\sin\theta - y\cos\theta) - \omega t\}] \tag{40}$$

$$\mathrm{T} = \delta_1 \exp[i\{k_1(x\sin\theta + y\cos\theta) - \omega t\}] + \delta_2 \exp[i\{k_2(x\sin\theta_2 + y\cos\theta_2) - \omega t\}] \tag{41}$$

where, α_1, α_2 represent amplitudes of the P-wave and thermal wave, β_1 represents amplitude of incident SV wave and β_2 is the amplitude of reflected SV-waves respectively.

Also, θ_1, θ_2 and θ are related to respective wave numbers as

$$k_1 \sin\theta_1 = k_2 \sin\theta_2 = l \sin\theta \tag{42}$$

Eq. (42) can be written in terms to Snell's law as

$$\frac{\sin\theta_1}{\Delta_1}=\frac{\sin\theta_2}{\Delta_2}=\frac{\sin\theta}{\sqrt{\Delta}} \tag{43}$$

where, $\Delta=\frac{c_2^2}{c_1^2}$ and Δ_1,Δ_2 are the roots of equation $\Lambda^4-\left(1+\overline{\tau}_{\mathrm{T}}-i\overline{\varphi}\right)\Lambda^2-i\overline{\varphi}=0,$ and are given as

$$\Delta_1=\frac{\omega}{k_1c_1},\Delta_2=\frac{\omega}{k_2c_2}\ \text{ and }\ \Delta=\left(\frac{c_2}{c_1}\right)^2 \tag{44}$$

Introducing Eq. (31) and Eq. (33) in into Eq. (30), we get

$$\delta_1=\frac{\omega^2\rho}{\gamma\left(1-i\omega\tau_0\right)}\left(\frac{\tau_{\mathrm{T}}\left(1-i\omega\tau_0\delta_{ij}\right)\left(1-i\omega\tau\right)}{\left(1-i\omega\tau_0\right)\left(\Delta_1^2+i\left[\frac{\varphi}{\left(1-i\omega\tau_0\right)}\right]\right)}\right)$$

and

$$\delta_2=\frac{\omega^2\rho}{\gamma\left(1-i\omega\tau_0\right)}\left(\frac{\tau_{\mathrm{T}}\left(1-i\omega\tau_0\delta_{ij}\right)\left(1-i\omega\tau\right)}{\left(1-i\omega\tau_0\right)\left(\Delta_2^2+i\left[\frac{\varphi}{\left(1-i\omega\tau_0\right)}\right]\right)}\right) \tag{45}$$

$$\mathrm{T}=\frac{\omega^2\rho}{\gamma\left(1-i\omega\tau_0\right)}\left(\frac{\tau_{\mathrm{T}}\left(1-i\omega\tau_0\delta_{ij}\right)\left(1-i\omega\tau\right)}{\left(1-i\omega\tau_0\right)\left(\Delta_1^2+i\left[\frac{\varphi}{\left(1-i\omega\tau_0\right)}\right]\right)}\right)\alpha_1\exp[i\{k_1(x\sin\theta+y\cos\theta)-\omega t\}$$

$$+\frac{\omega^2\rho}{\gamma\left(1-i\omega\tau_0\right)}\left(\frac{\tau_{\mathrm{T}}\left(1-i\omega\tau_0\delta_{ij}\right)\left(1-i\omega\tau\right)}{\left(1-i\omega\tau_0\right)\left(\Delta_2^2+i\left[\frac{\varphi}{\left(1-i\omega\tau_0\right)}\right]\right)}\right)\alpha_2\exp[i\{k_2(x\sin\theta_2+y\cos\theta_2)-\omega t\}] \tag{46}$$

Introducing Eq. (14), Eq. (15), Eq. (16) and Eq. (22) in the first boundary condition of Eq. (18), we get

$$\left(\mu+\frac{P}{2}\right)\left[2\frac{\partial^2\phi}{\partial x\partial y}+\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial^2 y}\right]\phi=0 \tag{47}$$

Introducing Eq. (14), Eq. (15), Eq. (16) and Eq. (22) in the second boundary condition of Eq. (18), we get

$$\lambda\frac{\partial^2\phi}{\partial y^2}+\left(\lambda+2\mu+\mu_e H_0^2\right)\frac{\partial^2\phi}{\partial y^2}+\left(2\mu+\mu_e H_0^2\right)\left(\frac{\partial^2\psi}{\partial x\partial y}\right)-\gamma\left(\frac{\partial \mathrm{T}}{\partial y}+\tau\frac{\partial^2\mathrm{T}}{\partial t\partial y}\right)=0 \tag{48}$$

Since we have taken the upper layer is thermally insulated, therefore from the third boundary condition of Eq. (18), we get

$$\frac{\partial T}{\partial y}=0 \tag{49}$$

Substituting Eq. (39) and Eq. (40) in Eq. (47) and with the help of Eq. (41) and Eq. (42), we get

$$\left(\frac{\beta_2}{\beta_1}\right)\cos 2\theta+\left(\frac{\alpha_1}{\beta_1}\right)\left(\frac{\Delta}{\Delta_1^2}\right)\sin 2\theta_1+\left(\frac{\alpha_2}{\beta_1}\right)\left(\frac{\Delta}{\Delta_2^2}\right)\sin 2\theta_2+\cos 2\theta=0 \tag{50}$$

Similarly, substituting Eq. (39), Eq. (40) and Eq. (41) in Eq. (48) and with the help of Eq. (29), Eq. (30), Eq. (42) and Eq. (43), we get

$$\begin{aligned}&\left(\frac{\beta_2}{\beta_1}\right)(1+\eta')\sin 2\theta+\left(\frac{\alpha_1}{\beta_1}\right)\left(\frac{1}{\Delta_1^2}\right)\left[1-2\Delta\sin^2\theta_1-\eta(1+\sin^2\theta_1)+\frac{\Delta_1^2\overline{\tau}_{\mathrm{T}}}{\Delta_1^2+\overline{\varphi}i}\right]\\&+\left(\frac{\alpha_2}{\beta_1}\right)\left(\frac{1}{\Delta_2^2}\right)\left[1-2\Delta\sin^2\theta_2-\eta(1+\sin^2\theta_2)+\frac{\Delta_2^2\overline{\tau}_{\mathrm{T}}}{\Delta_2^2+\overline{\varphi}i}\right]+\left(1+\eta'\right)\sin 2\theta=0\end{aligned} \tag{51}$$

where, $\eta=\dfrac{P}{c_1^2\rho}$ and $\eta'=\dfrac{P}{c_2^2\rho}$

Substituting Eq. (46) in Eq. (49) and with the help of Eq. (44), we get

$$\left(\frac{\alpha_1}{\beta_1}\right)\left(\frac{1}{\Delta_1}\right)\left[\frac{\cos\theta_1}{\Delta_1^2+\overline{\varphi}i}\right]+\left(\frac{\alpha_2}{\beta_1}\right)\left(\frac{1}{\Delta_2}\right)\left[\frac{\cos\theta_2}{\Delta_2^2+\overline{\varphi}i}\right]=0 \tag{52}$$

Eliminating $\frac{\alpha_1}{\beta_1}$ from Eq. (50) and Eq. (52), we get

$$\frac{\beta_2}{\beta_1} = -1 - \left(\frac{\alpha_1}{\beta_1}\right)\Delta\left[\frac{\Delta_2 \cos\theta_2 \sin 2\theta_1 \left(\Delta_1^2 + \bar{\varphi} i\right) - \Delta_1 \cos\theta_1 \sin 2\theta_2 \left(\Delta_2^2 + \bar{\varphi} i\right)}{\Delta_1^2 \Delta_2 \cos\theta_2 \left(\Delta_1^2 + \bar{\varphi} i\right) \cos 2\theta}\right] \quad (53)$$

Eliminating $\frac{\alpha_2}{\beta_1}$ from Eq. (51) and Eq. (52), we get

$$\frac{\beta_2}{\beta_1} = 1 + \left(\frac{\alpha_1}{\beta_1}\right)\left\{\frac{\begin{array}{l}\Delta_2 \cos\theta_2 \left(\Delta_1^2 + \bar{\varphi} i\right)\left\{1 - 2\Delta \sin^2\theta_1 - \eta\left(1 + \sin^2\theta_1\right)\right\} \\ + \bar{\tau}_T \Delta_1^2 \Delta_2 \cos\theta_2 - \Delta_1 \cos\theta_1 \left(\Delta_1^2 + \bar{\varphi} i\right)\left\{\begin{array}{l} 1 - 2\Delta \sin^2\theta_2 \\ -\eta\left(1 + \sin^2\theta_2\right)\end{array}\right\} - \bar{\tau}_T \Delta_2^2 \Delta_1 \cos\theta_1\end{array}}{\Delta_1^2 \Delta_2 \cos\theta_2 \left(\Delta_1^2 + \bar{\varphi} i\right) \sin 2\theta \left(1 + \eta'\right)}\right\} \quad (54)$$

Equating Eq. (53) and Eq. (54), we get

$$R_{P_1} = \frac{\alpha_1}{\beta_1} = \frac{\mathrm{A}}{\mathrm{H}} \quad (55)$$

Equating Eq. (54) and Eq. (55), we get

$$R_{SV} = \frac{\beta_2}{\beta_1} = -\frac{\mathrm{B}}{\mathrm{H}} \quad (56)$$

Equating Eq. (52) and Eq. (55), we get

$$R_{P2} = \frac{\alpha_2}{\beta_1} = -\frac{C}{\mathrm{H}} \quad (57)$$

where, A, B, C and H are given as

$$\begin{aligned} A = {} & \Delta_2 \cos\theta_2[\left(\Delta_1^2+\bar{\varphi}i\right)\{\Delta\cos 2(\theta+\theta_1)+(1-\Delta)\cos 2\theta \\ & -\eta\cos 2\theta\left(1+\sin^2\theta_1\right)-\eta'\Delta\sin 2\theta\sin 2\theta_1\}+\bar{\tau}_T\Delta_1^2\cos 2\theta] \\ & -\Delta_1\cos\theta_1[\left(\Delta_2^2+\bar{\varphi}i\right)\{\Delta\cos 2(\theta+\theta_2)+(1-\Delta)\cos 2\theta \\ & -\eta\cos 2\theta\left(1+\sin^2\theta_2\right)-\eta'\Delta\sin 2\theta\sin 2\theta_2\}+\bar{\tau}_T\Delta_2^2\cos 2\theta] \end{aligned} \tag{58}$$

$$B = -2\Delta_1^2\Delta_2\cos\theta_2\left(\Delta_1^2+\bar{\varphi}i\right)\cos 2\theta\sin 2\theta\left(1+\eta'\right) \tag{59}$$

$$C = -2\Delta_2^2\Delta_1\cos\theta_1\left(\Delta_2^2+\bar{\varphi}i\right)\cos 2\theta\sin 2\theta\left(1+\eta'\right) \tag{60}$$

$$\begin{aligned} H = {} & \Delta_2 \cos\theta_2[\left(\Delta_1^2+\bar{\varphi}i\right)\{\Delta\cos 2(\theta-\theta_1)+(1-\Delta)\cos 2\theta \\ & -\eta\cos 2\theta\left(1+\sin^2\theta_1\right)+\eta'\Delta\sin 2\theta\sin 2\theta_1\}+\bar{\tau}_T\Delta_1^2\cos 2\theta] \\ & -\Delta_1\cos\theta_1[\left(\Delta_2^2+\bar{\varphi}i\right)\{\Delta\cos 2(\theta-\theta_2)+(1-\Delta)\cos 2\theta \\ & -\eta\cos 2\theta\left(1+\sin^2\theta_2\right)+\eta'\Delta\sin 2\theta\sin 2\theta_2\}+\bar{\tau}_T\Delta_2^2\cos 2\theta] \end{aligned} \tag{61}$$

$R_{SV} = \dfrac{\beta_2}{\beta_1} =$ reflection coefficients of plane SV-wave, $R_{P_1(c_1)} = \dfrac{\alpha_1}{\beta_1} =$ represents reflection coefficients of thermal wave and $R_{P_2(c_2)} = \dfrac{\alpha_2}{\beta_1} =$ reflection coefficients of P- wave.

6 Numerical analysis and calculations

Approximate expression for reflection coefficients is obtained by assuming practical values of $\bar{\tau}_T \ll 1$ and $\bar{\varphi} \ll 1$ for elastic materials. Solving Eq. (38) and retaining only first degree terms of $\bar{\tau}_T$ and $\bar{\varphi}$, we get

$$\Delta_1 = 1+\frac{1}{2}\bar{\tau}_T \quad \text{and} \quad \Delta_2 = i^{\frac{3}{2}}\bar{\varphi}^{\frac{1}{2}} \tag{62}$$

Eq. (43) can be written as

$$\sin\theta_1 = \frac{\Delta_1}{\sqrt{\Delta}}\sin\theta, \sin\theta_2 = \frac{\Delta_2}{\sqrt{\Delta}}\sin\theta, \tag{63}$$

$$\cos\theta_1 = \sqrt{1-\sin^2\theta_1} = \left(1-\frac{1+\bar{\tau}_T}{\Delta}\sin^2\theta\right)^{\frac{1}{2}}, \cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \left(1+\frac{i\bar{\varphi}}{\Delta}\sin^2\theta\right)^{\frac{1}{2}}.$$

The equations (58-61) can be written by using Eq. (63) as

$$R_{P_1} = \frac{\alpha_1}{\beta_1} = \frac{\mathrm{A}_0}{\mathrm{H}^0}, R_{SV} = \frac{\beta_2}{\beta_1} = -\frac{\mathrm{B}_0}{\mathrm{H}^0} \text{ and } R_{P_2} = 0 \tag{64}$$

where,

$$\mathrm{B}_0 = pq(1+\eta') - r + \eta s\cos 2\theta, \quad \mathrm{A}_0 = 2\cos 2\theta\sin 2\theta(1+\eta')(1+\bar{\tau}_T),$$

$$q = \left(1-\frac{1+\bar{\tau}_T}{\Delta}\sin^2\theta\right)^{\frac{1}{2}}, \quad \mathrm{H}^0 = pq(1+\eta') + r - \eta s\cos 2\theta,$$

$$\mathrm{p} = 4\sqrt{\Delta}\left(1+\frac{1}{2}\bar{\tau}_T\right)\sin^2\theta\cos\theta, \quad s = \left(1+\frac{1+\bar{\tau}_T}{\Delta}\sin^2\theta\right)^{\frac{1}{2}},$$

$$r = \cos^2\theta - 2\bar{\tau}_T\cos 2\theta\sin^2\theta + \bar{\tau}_T\cos 2\theta\left(1-i^{\frac{3}{2}}\bar{\varphi}^{\frac{1}{2}}\left(1-\frac{1+\bar{\tau}_T}{\Delta}\sin^2\theta\right)^{\frac{1}{2}}\right). \tag{65}$$

From Eq. (64), we conclude that there is no P-wave in the reflection when practical values $\bar{\tau}_{\mathrm{T}} \ll 1$ and $\bar{\varphi} \ll 1$ for elastic materials are assumed.

Various graphs are plotted between R_{SV}, $R_{P_1(c_1)}$, $R_{P_2(c_2)}$ and θ for taking $S = 0.1, 0.3, 0.5, 0.7, 0.9$ for tensile stress and $S = -0.1, -0.3, -0.5, -0.7. -0.9$ for compressional stress. R_{SV}, $R_{P_1(c_1)}$ and $R_{P_2(c_2)}$ are calculated by taking parameters for copper alloy (Table 1). The results are compared with purposed model and standard model from approximation and are illustrated graphically with the help of MATLAB software. The results are closed to the standard model. The various curves are plotted by approximating the Eq. (55), Eq. (56) and (57) by considering relaxation factor and coupling factor very small, we observed that no thermal P-wave is reflected while both reflected SV-wave and reflected P-wave is seen. Figure 2 to Figure 6 are plotted without using approximation, Figure 7 to Figure 11 are plotted after using approximation for Eq. (64) at various incident angles. A graphical view is taken for variation of angle of incidence θ from 0^0 to 40^0 and from 0^0 to 50^0; two series are taken while plotting the graphs i.e. taking $S = 0.1, 0.3, 0.5, 0.7, 0.9$ for tensile stresses and $S = -0.1, -0.3, -0.5, -0.7. -0.9$ for compressional stresses. Here $S = \frac{P}{2\mu}$ is known as stress parameter. It is observed that SV-wave is greatly affected by the presence of magnetic field and temperature of the solid half space. Figure 2 shows that the maxima and minima of reflection coefficients of SV-wave are in the range $10^0 \le \theta \le 30^0$ for $S = 0.1, 0.3, 0.5, 0.7, 0.9$. The variation of reflection coefficients for SV reflected wave in magneto-thermal medium is same for both compressional and tensile stress; the only difference is the reversal of the stress (Fig. 3). Figure 4 is plotted for the reflection coefficients of P-wave, the reflection coefficient is minimum at 20^0 and in figure 5, the maxima and minima of reflection coefficients of SV-wave are in the range $10^0 \le \theta \le 30^0$. Figure 6 is plotted for reflection coefficient of thermal wave for various values of stress $S = 0.1, 0.3, 0.5, 0.7, 0.9$, from this figure it is clear that the maximum value of reflection coefficients of thermal waves occur at an angle of incident 20^0 without using approximation. However in figure 7, the range of maxima and minima for reflection coefficients of thermal waves is $20^0 \le \theta \le 40^0$ when approximation is used. Figure 8 shows that the maximum of reflection coefficients of SV-wave is at an angle 32^0 for $S = -0.1, -0.3, -0.5, -0.7. -0.9$ after using approximation.

Table 1

Parameter	Numerical Value
$\bar{\tau}_T$	0.005
ρ	$7.14\times10^3 kg/m^3$,
$\mu_e H_0^2$	$1.24\times10^9 N/m^2$
λ	$9.5\times10^{10} N/m^2$
$\bar{\varphi}$	0.005
α_t	$16.6\times10^{-6} K^{-1}$,
K	$401\ W/(m.K)$
c_p	$0.39\ KJ/Kg\ K$
μ	$4.5\times10^{10}\ N/m^2$

Similarly, Figure 9 to Figure 11 is plotted for reflected SV, reflected P and reflected thermal wave after using approximation. Figure 12 to Figure 14 are plotted for reflected SV, reflected P and reflected thermal wave at various values of magnetic field keeping initial stress at 0.5. It is observed that in the presence of magnetic field there is remarkable variation in these curves. Figures (15–17) are plotted between the reflection coefficients of P, T and SV waves against angle of incidence θ for relaxation times $\tau_0 = 0s, 0.04 \times 10^{-12}s, 0.8 \times 10^{-12}s$ at constant $H = 0$ and $S = 0.5$, it is clearly observed that the effect of relaxation time is prominent for $\tau_0 = 0.04 \times 10^{-12}s$ and the effect is more for increase in relaxation time.

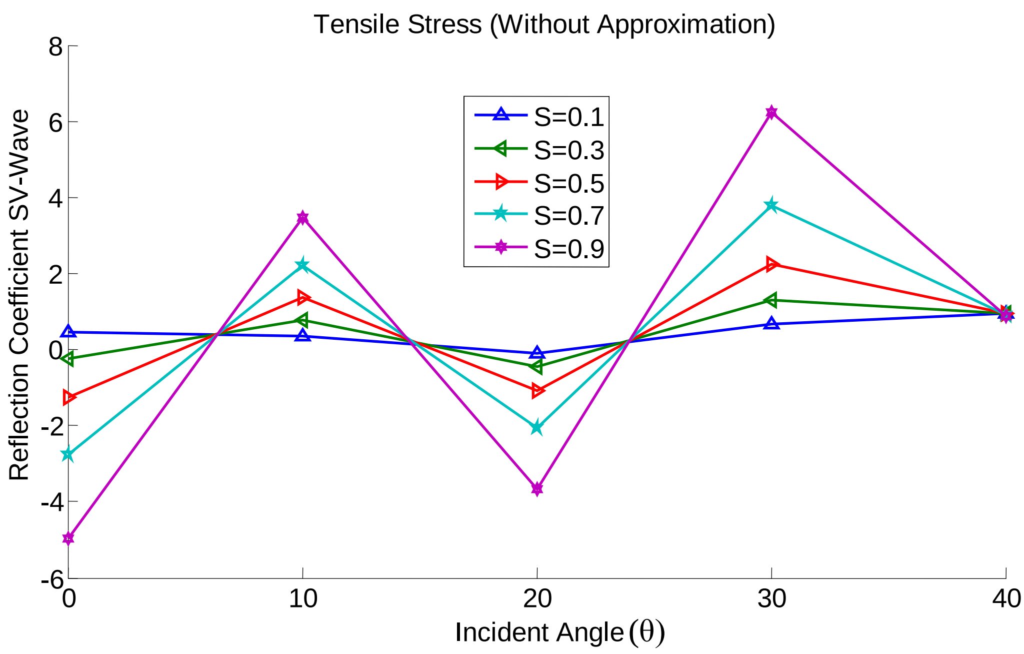

Figure 2

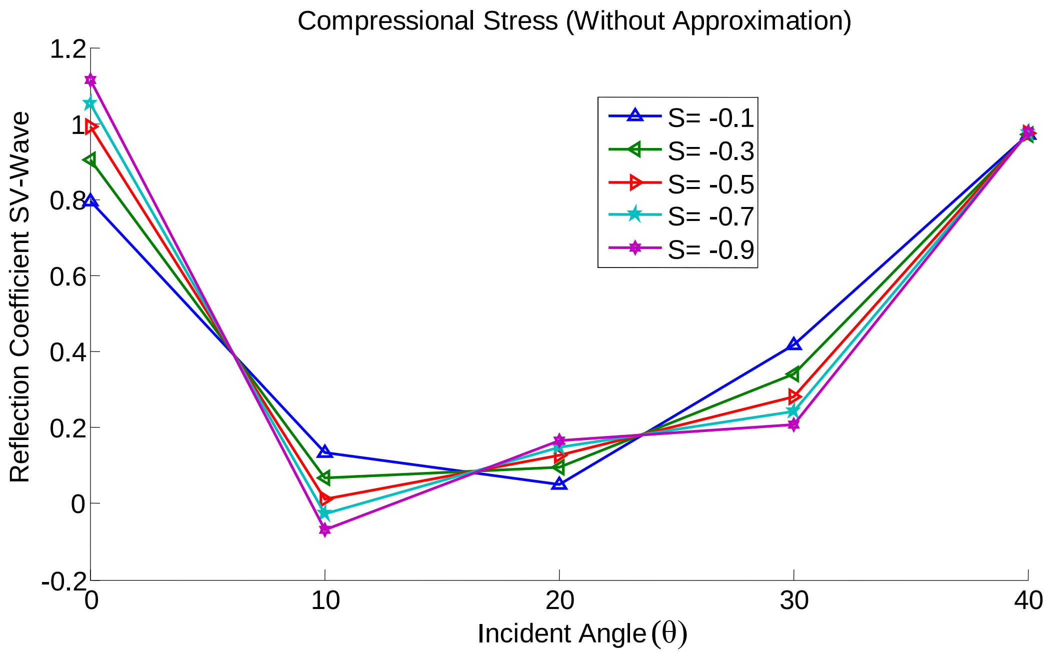

Figure 3

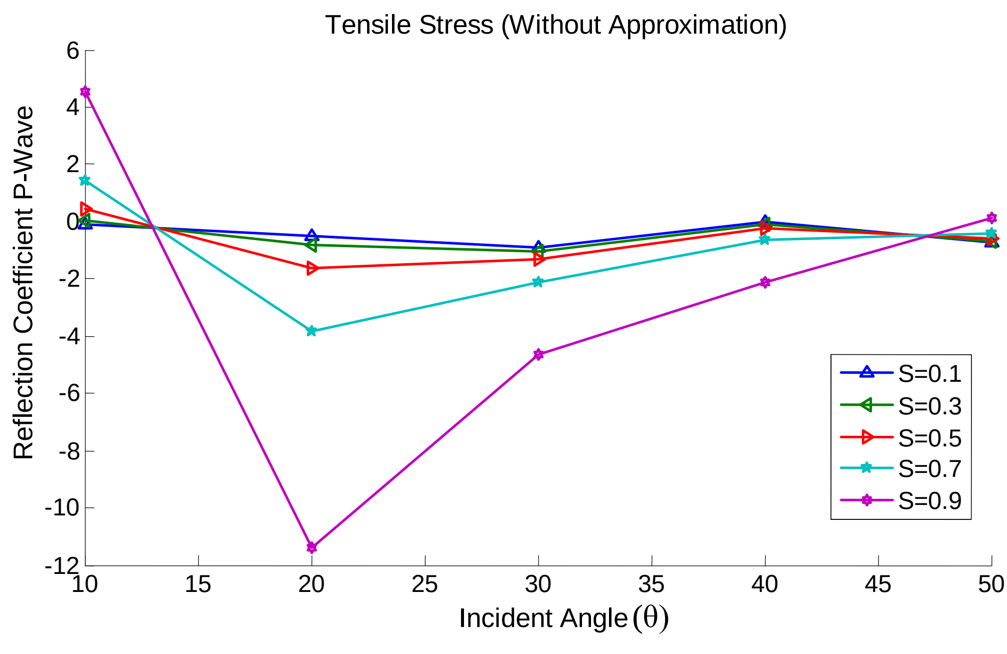

Figure 4

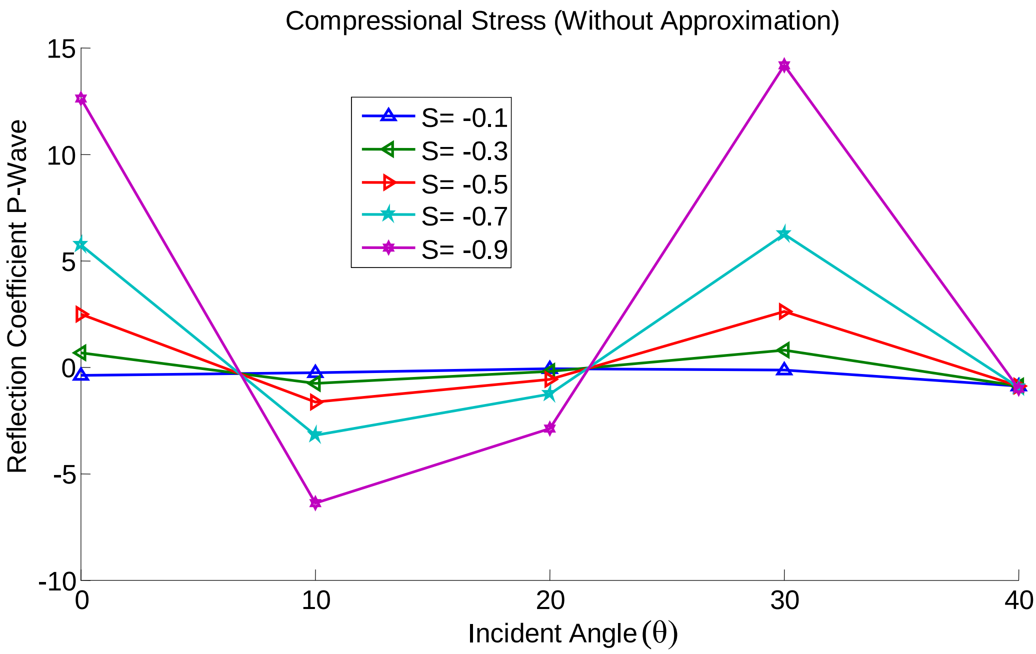

Figure 5

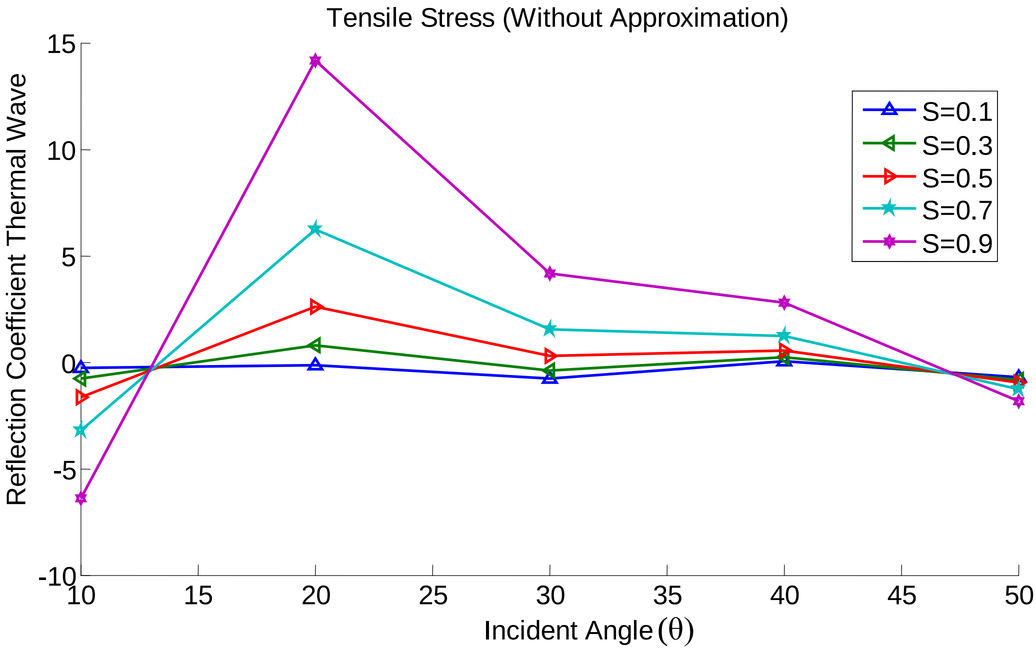

Figure 6

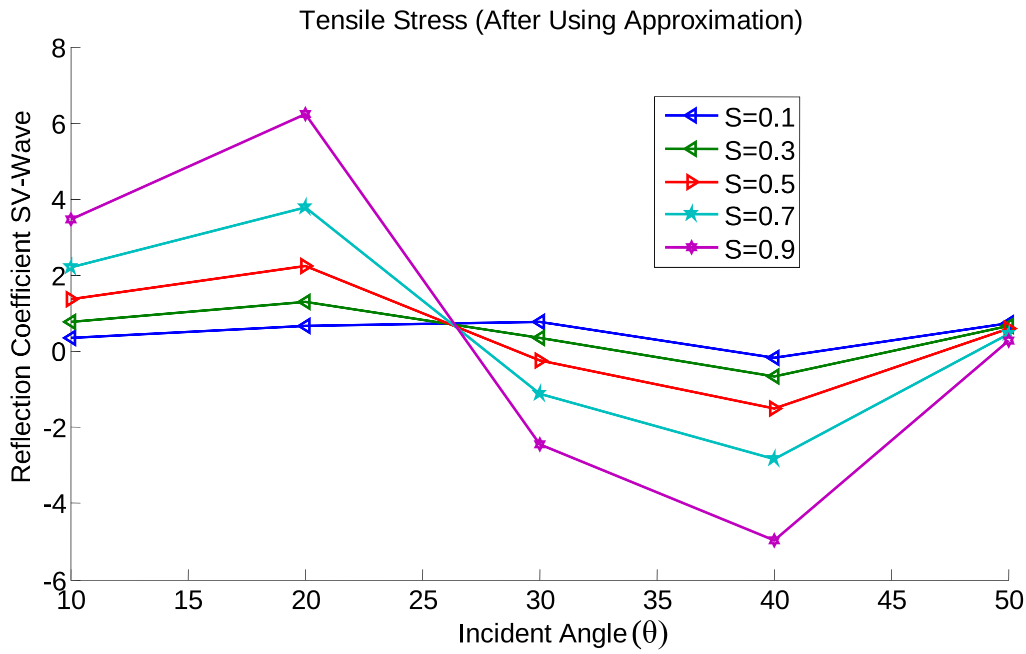

Figure7

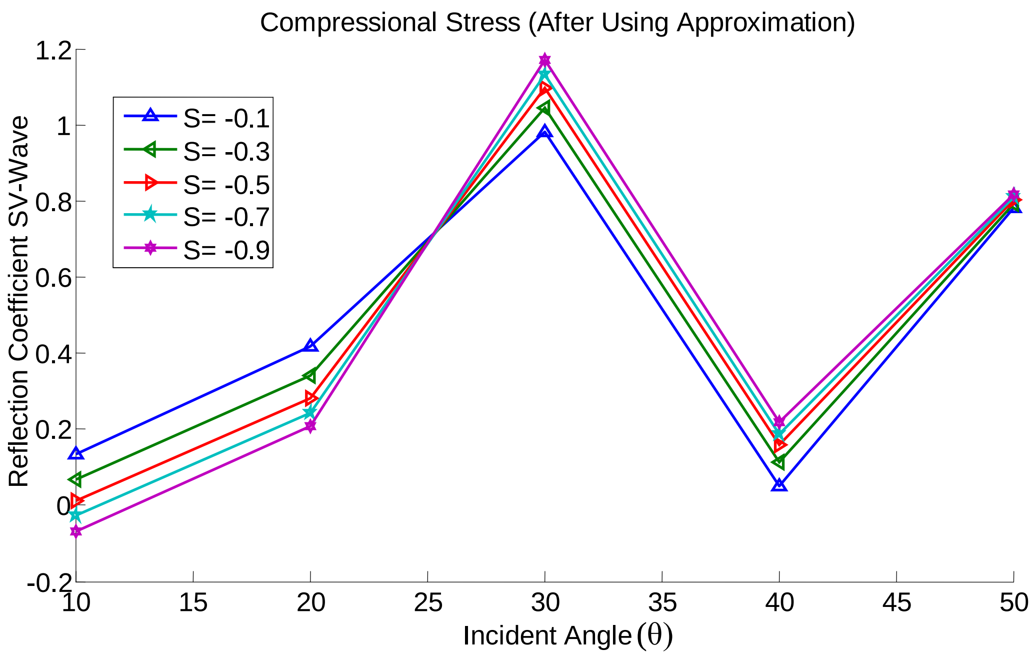

Figure 8

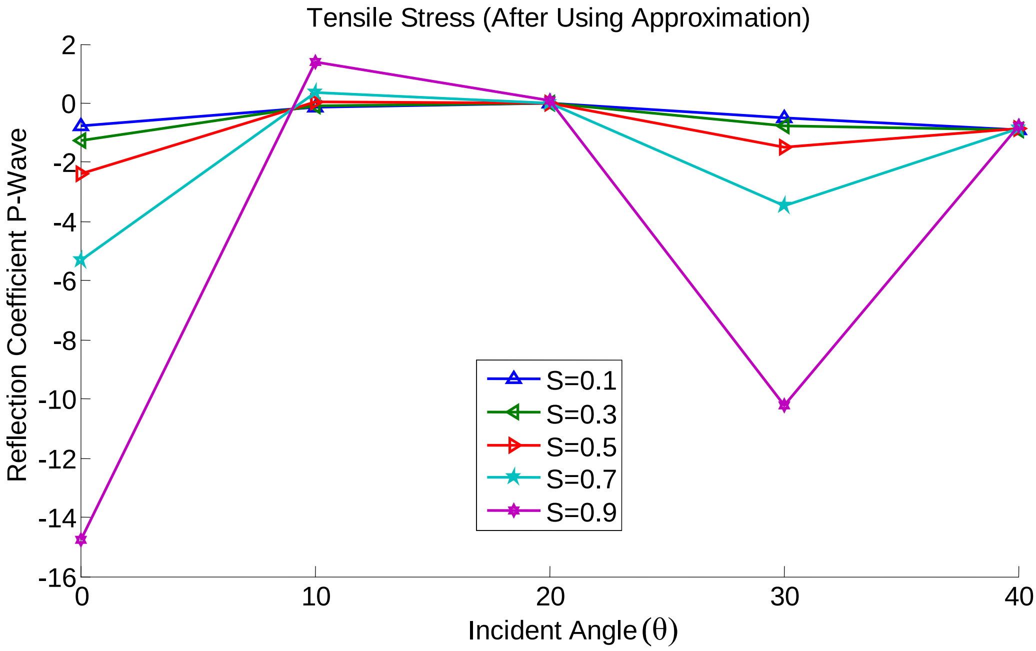

Figure 9

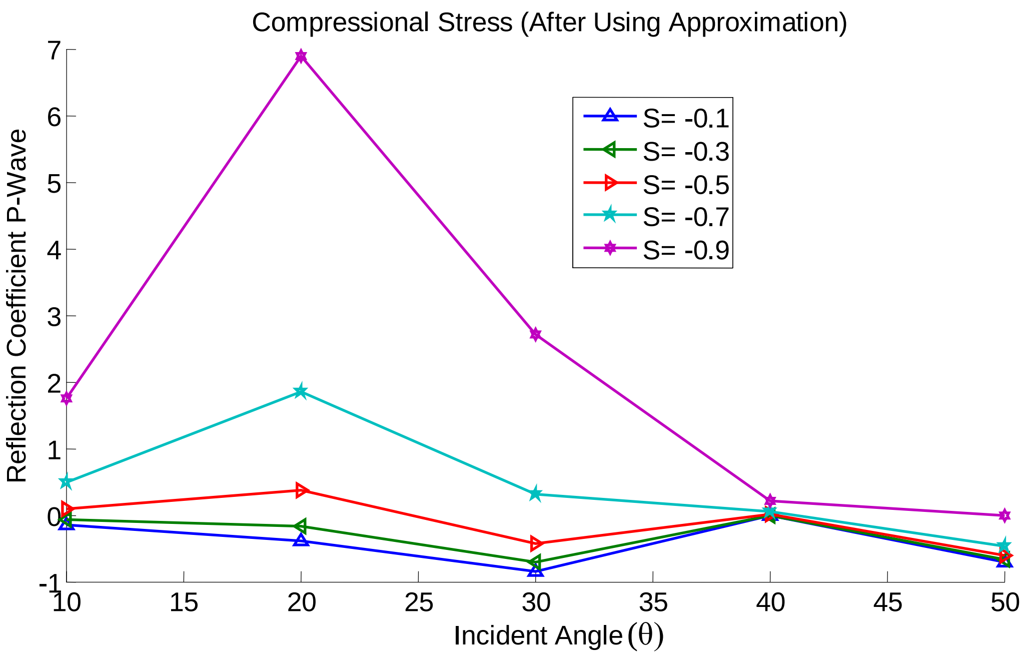

Figure 10

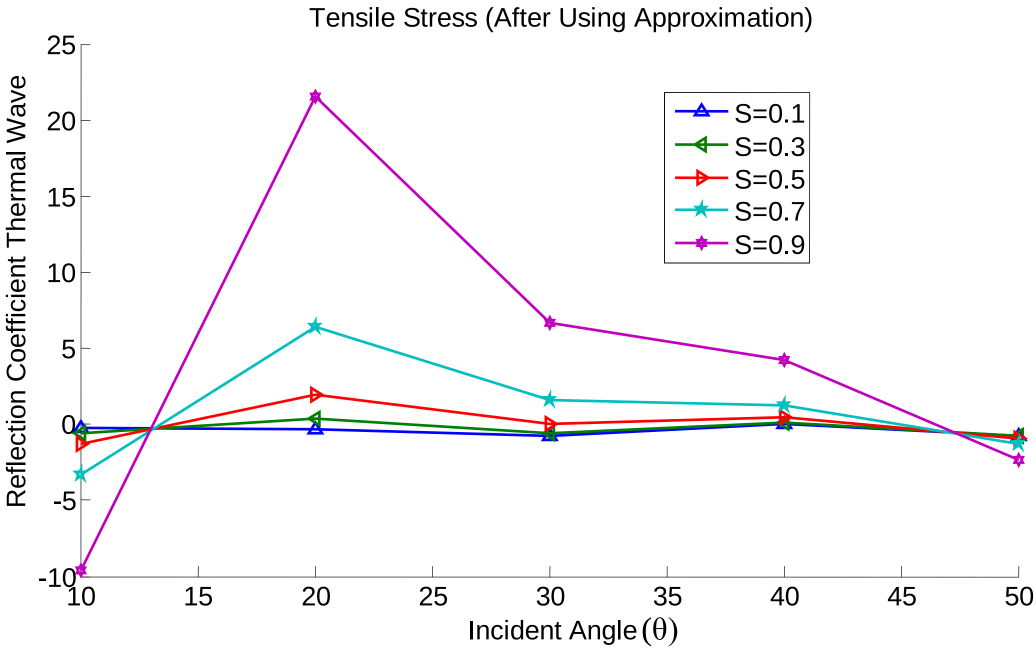

Figure 11

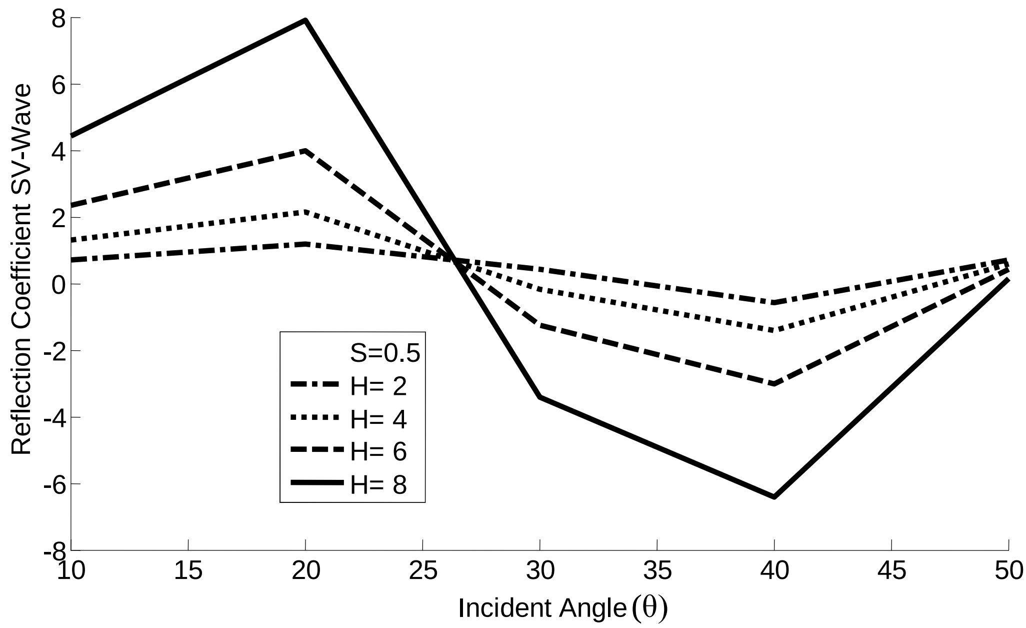

Figure 12

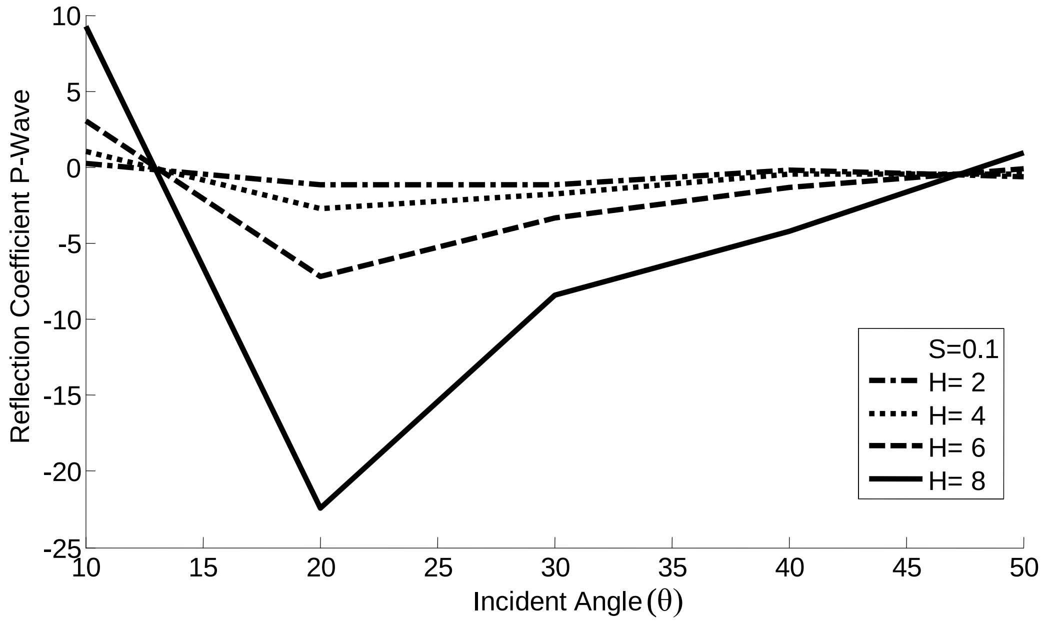

Figure 13

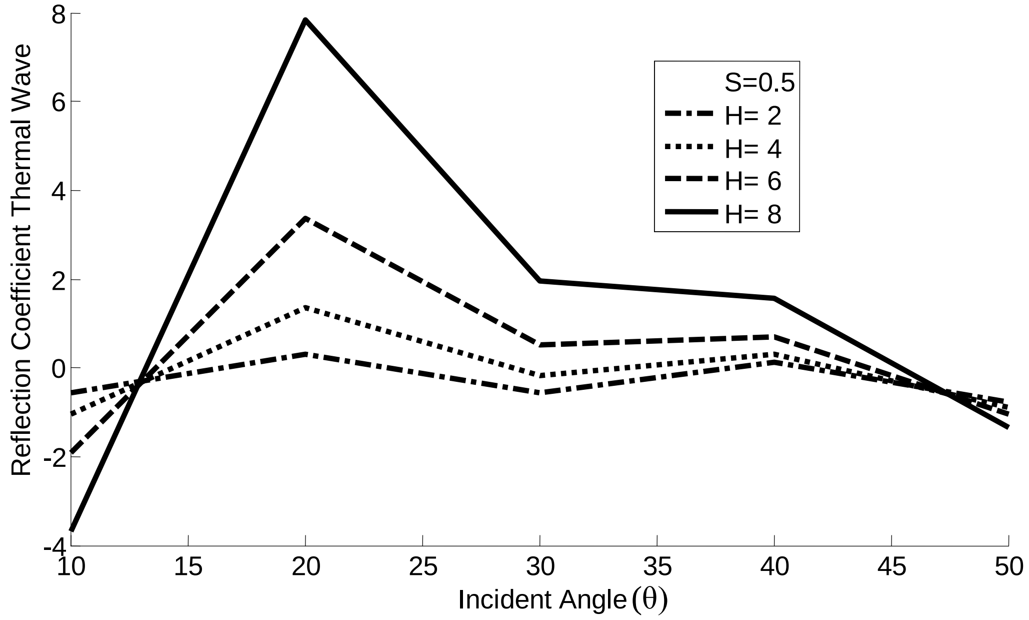

Figure 14

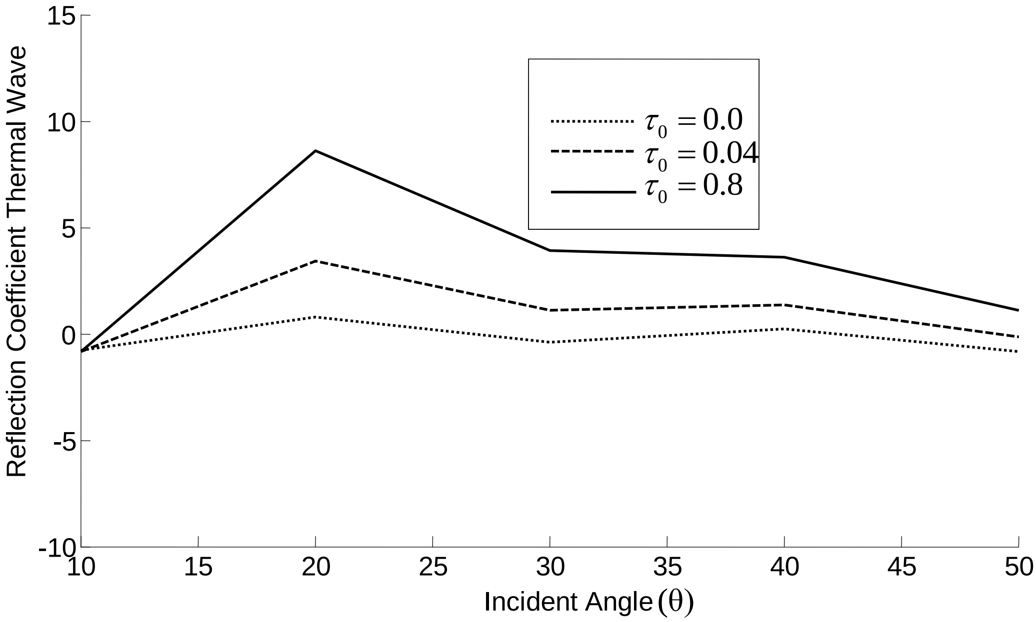

Figure 15

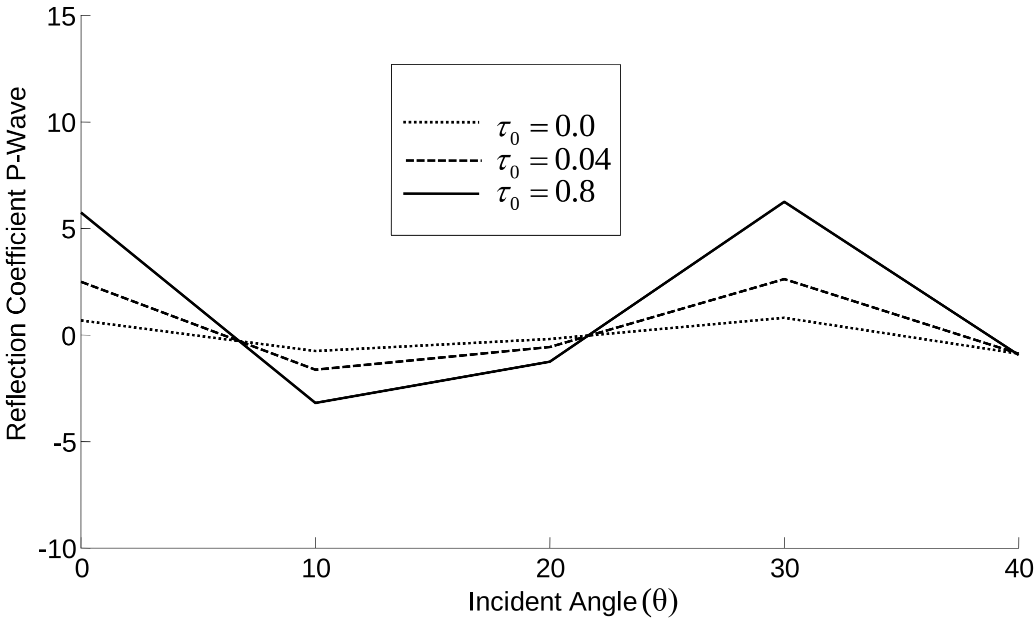

Figure 16

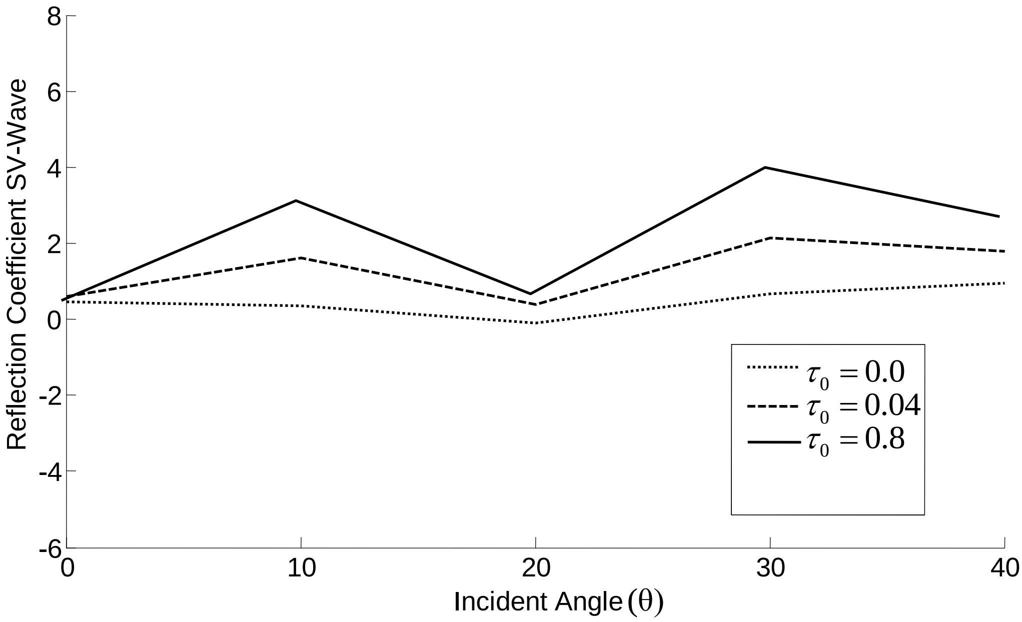

Figure 17

7 Conclusion

The purpose of this study is to show the combined effect of temperature, magnetic field, relaxation time and stress on the propagation of elastic plane SV-waves through a solid isotropic elastic half space. It has been observed that in case of free space, very small energy is reflected, however in case of magneto-thermal medium, the SV incident wave is greatly modifies in the presence of stress as well as magnetic field of the medium. The results are compared with purposed model and standard model using from approximation. It is clearly observed that the effect of relaxation time on reflection coefficients of P, T and SV waves is prominent for $\tau_0 = 0.04 \times 10^{-12}\mathrm{s}$ and the effect is more for increase in relaxation time. The results are closed to the standard model. This model is useful to study the problems involving heat change, magnetic field, mechanical stress applied at the boundary of the surface. The results presented in this paper may be useful for geophysicists to analyze material structures and rocks through nondestructive testing. The solution of such problems also affects different geomagnetic cases.

Acknowledgments

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