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Action of Fuzzy Translation Operators in Semigroups

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Abstract

In this paper the concept of fuzzy translation operators in a non-empty set has been introduced and some important properties have been investigated. It is observed that with suitable restrictions, fuzzy subsemigroups and fuzzy bi-ideals remains invariant under action of these two fuzzy translation operators.

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1 Introduction

Uncertainty is an attribute of information and uncertain data are presented in various domains. The most appropriate theory for dealing with uncertainties was introduced by Zadeh[6] in 1965 by defining fuzzy set which has opened up keen insights and applications in vast range of scientific fields. Rosenfeld[5] pioneered the study of fuzzy algebraic structures by introducing the notions of fuzzy groups and showed that many results in groups can be extended in an elementary manner to develop algebraic concepts. After that Kuroki[2, 3, 4] started the study of fuzzy ideal theory in semigroups.

In this paper as an abstraction of the geometric notion of translation, we introduce two operators T_{α^+} and T_{α^-} and call them fuzzy translation operators. Then the action of these operators on semigroups has been investigated. It is also observed that with suitable restrictions, fuzzy subsemigroups and fuzzy bi-ideals remain invariant under action of fuzzy translation operators T_{α^+} and T_{α^-}.

2 Preliminaries

Definition 2.1. [6] A fuzzy subset of a non-empty set X is a function $\mu : X \to [0,1]$.

Definition 2.2. [2] Let μ be a fuzzy subset of a set X. Then for $t \in [0,1]$ the set $\mu_t = \{x \in X : \mu(x) \geq t\}$ is called level subset of μ.

Definition 2.3. [1] A non-empty subset A of a semigroup S is called a subsemigroup of S if $AA \subseteq A$.

Definition 2.4. [1] A subsemigroup A of a semigroup S is called a bi-ideal of S if $ASA \subseteq A$.

Definition 2.5. [2] A non-empty fuzzy subset μ of a semigroup S is called a fuzzy subsemigroup of S if $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in S$.

Definition 2.6. [2] A fuzzy subsemigroup μ of a semigroup S is called a fuzzy bi-ideal of S if $\mu(xyz) \geq \min\{\mu(x), \mu(z)\}$ for all $x, y, z \in S$.

3 Fuzzy Translation Operators

Unless or otherwise stated throughout this paper S stands for a semigroup.

Definition 3.1. Let X be a non-empty set, α be a non-negative real number and μ be a fuzzy subset of X. We define

$$T_{\alpha^+}(\mu)(x) = \min\{\mu(x) + \alpha, 1\}$$

and

$$T_{\alpha^-}(\mu)(x) = \max\{\mu(x) - \alpha, 0\}$$

for all $x \in X$ where $T_{\alpha^+}(\mu)$ and $T_{\alpha^-}(\mu)$ are respectively called the α-up fuzzy translation operator and α-down fuzzy translation operator of μ.

Remark 1. In the Definition 3.1 when $\alpha > 1$, $T_{\alpha^+}(\mu)(x) = 1$ and $T_{\alpha^-}(\mu)(x) = 0$ for all $x \in X$ and for all $\mu \in F(X)(F(X)$ denotes the collection of all fuzzy subsets of $X)$. Now we define two fuzzy operators $1, 0 : F(X) \to F(X)$ by $1(\mu) = \lambda_X$ and $0(\mu) = 0_X$ for all $\mu \in F(X)$ where $\lambda_X(x) = 1$ and $0_X(x) = 0$ for all $x \in X$. Then we see that $\{T_{\alpha^+} : \alpha \in \mathbb{R}^+ \cup \{0\}\} = \{T_{\alpha^+} : \alpha \in [0, 1]\} \cup 1$ and $\{T_{\alpha^-} : \alpha \in \mathbb{R}^+ \cup \{0\}\} = \{T_{\alpha^-} : \alpha \in [0, 1]\} \cup 0$.

Proposition 3.2. *Let μ be a fuzzy subset of a non-empty set X and α be a non-negative real number. Then*

(1) $T_{\alpha^+}(\overline{\mu}) = \overline{T_{\alpha^-}(\mu)}$,

(2) $T_{\alpha^-}(\overline{\mu}) = \overline{T_{\alpha^+}(\mu)}$ *where* $\overline{\mu} = 1 - \mu$.

Proof. (1) Let $x \in X$. Then

$$\begin{aligned} T_{\alpha^+}(\overline{\mu}) &= \min\{\overline{\mu}(x) + \alpha, 1\} \\ &= \min\{1 - \mu(x) + \alpha, 1\} \\ &= 1 - \max\{\mu(x) - \alpha, 0\} \\ &= 1 - T_{\alpha^-}(\mu)(x) \\ &= \overline{T_{\alpha^-}(\mu)}(x). \end{aligned}$$

Hence $T_{\alpha^+}(\overline{\mu}) = \overline{T_{\alpha^-}(\mu)}$.

(2) Similarly we can have the proof by routine calculation. □

Remark 2. In general for a given non-negative real number α, T_{α^+} and T_{α^-} are not inverse fuzzy translation operators. That is $T_{\alpha^+}(T_{\alpha^-}(\mu))$ and $T_{\alpha^-}(T_{\alpha^+}(\mu))$ are not equal with μ which is clear from the following example.

Example 1. Let $X = \{a, b, c\}$ and $\mu : X \to [0, 1]$ be a fuzzy subset of X defined as

$$\mu(x) = \begin{cases} 1 \text{ if } x = a \\ \frac{1}{2} \text{ if } x = b \\ 0 \text{ if } x = c \end{cases}$$

Let $\alpha = \frac{1}{4}$. Then $T_{\alpha^+}(\mu)$ and $T_{\alpha^-}(\mu)$ are given by

$$T_{\alpha^+}(\mu)(x) = \begin{cases} 1 \text{ if } x = a \\ \frac{3}{4} \text{ if } x = b \\ \frac{1}{4} \text{ if } x = c \end{cases}$$

and

$$T_{\alpha^-}(\mu)(x) = \begin{cases} \frac{3}{4} \text{ if } x = a \\ \frac{1}{4} \text{ if } x = b \\ 0 \text{ if } x = c \end{cases}$$

Now $T_{\alpha^+}(T_{\alpha^-}(\mu))$ and $T_{\alpha^-}(T_{\alpha^+}(\mu))$ are given by

$$T_{\alpha^+}(T_{\alpha^-}(\mu))(x) = \begin{cases} 1 \text{ if } x = a \\ \frac{1}{2} \text{ if } x = b \\ \frac{1}{4} \text{ if } x = c \end{cases}$$

and

$$T_{\alpha^-}(T_{\alpha^+}(\mu))(x) = \begin{cases} \frac{3}{4} \text{ if } x = a \\ \frac{1}{2} \text{ if } x = b \\ 0 \text{ if } x = c \end{cases}$$

Hence $T_{\alpha^+}(T_{\alpha^-}(\mu)) \neq \mu$ and $T_{\alpha^-}(T_{\alpha^+}(\mu)) \neq \mu$. Also we observe that $T_{\alpha^+}(T_{\alpha^-}(\mu)) \neq T_{\alpha^-}(T_{\alpha^+}(\mu))$

Example 2. In Example 1, if we take $\beta = \frac{1}{5}$, then by routine verification we can show $T_{\alpha^+}(T_{\beta^+}(\mu)) = T_{\beta^+}(T_{\alpha^+}(\mu)) = T_{(\alpha+\beta)^+}(\mu))$ and $T_{\alpha^-}(T_{\beta^-}(\mu)) = T_{\beta^-}(T_{\alpha^-}(\mu)) = T_{(\alpha+\beta)^-}(\mu))$.

Remark 3. The set of all α-up(down) fuzzy translation operators forms a commutative semigroup.

Proposition 3.3. *Let S be a semigroup and μ be a fuzzy subsemigroup of S. Then $T_{\alpha^+}(\mu)$ is a fuzzy subsemigroup of S for any non-negative real number α.*

Proof. Let α be a non-negative real number and $x, y \in S$. Then

$$\begin{aligned} T_{\alpha^+}(\mu)(xy) &= \min\{\mu(xy) + \alpha, 1\} \\ &\geq \min[\min\{\mu(x), \mu(y)\} + \alpha, 1] \\ &= \min[\min\{\mu(x) + \alpha, 1\}, \min\{\mu(y) + \alpha, 1\}] \\ &= \min\{T_{\alpha^+}(\mu)(x), T_{\alpha^+}(\mu)(y)\}. \end{aligned}$$

Hence $T_{\alpha^+}(\mu)$ is a fuzzy subsemigroup of S. □

Proposition 3.4. *Let S be a semigroup and μ be a fuzzy subsemigroup of S. Then $T_{\alpha^-}(\mu)$ is a fuzzy subsemigroup of S for any non-negative real number α.*

Proof. Let α be a non-negative real number and $x, y \in S$. Then

$$\begin{aligned} T_{\alpha^-}(\mu)(xy) &= \max\{\mu(xy) - \alpha, 0\} \\ &\geq \max[\min\{\mu(x), \mu(y)\} - \alpha, 0] \\ &= \max[\min\{\mu(x) - \alpha, \mu(y) - \alpha\}, 0] \\ &= \min[\max\{\mu(x) - \alpha, 0\}, \max\{\mu(y) - \alpha, 0\}] \\ &= \min\{T_{\alpha^-}(\mu)(x), T_{\alpha^-}(\mu)(y)\}. \end{aligned}$$

Hence $T_{\alpha^-}(\mu)$ is a fuzzy subsemigroup of S. □

Remark 4. The converse of the Proposition 3.3 is not always true which is clear from the following example.

Example 3. Let $S = \{a, b, c\}$ with the following cayley table:

$.$	a	b	c
a	a	b	c
b	b	a	c
c	c	c	a

Then $(S, .)$ is a semigroup. We define a fuzzy subset $\mu : S \to [0,1]$ as

$$\mu(x) = \begin{cases} \frac{1}{2} \text{ if } x = a \\ \frac{3}{4} \text{ if } x = b \\ \frac{1}{4} \text{ if } x = c. \end{cases}$$

Since $\mu(bb) = \mu(a) = \frac{1}{2} \not\geq \min\{\mu(b), \mu(b)\} = \frac{3}{4}$, so μ is not a fuzzy subsemigroup of S.

Now let $\alpha = \frac{3}{5}$. Then $T_{\alpha^+}(\mu)$ is given by

$$T_{\alpha^+}(\mu)(x) = \begin{cases} 1 \text{ if } x = a, b \\ \frac{17}{20} \text{ if } x = c \end{cases}$$

which is a fuzzy subsemigroup of S.

Definition 3.5. Let S be a semigroup and μ be a fuzzy subsemigroup of S. Then we define $S_\mu := \{x \in S : \mu(x) = 1\}$.

Remark 5. Clearly S_μ is a subsemigroup of S.

Proposition 3.6. *Let S be a semigroup and μ be a fuzzy subset of S and $T_{\alpha^+}(\mu)$ be a fuzzy subsemigroup of S for any non-negative real number α with $\alpha < 1 - \sup\{\mu(x) : x \in S - S_\mu\}$. Then μ is a fuzzy subsemigroup of S.*

Proof. Let $x, y \in S$. Then

$$T_{\alpha^+}(\mu)(xy) \geq \min\{T_{\alpha^+}(\mu)(x), T_{\alpha^+}(\mu)(y)\} \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

CASE (1) : Let $T_{\alpha^+}(\mu)(x) = T_{\alpha^+}(\mu)(y) = 1$. Then $\mu(x) + \alpha \geq 1$ and $\mu(y) + \alpha \geq 1$. So $T_{\alpha^+}(\mu)(xy) = 1$ which means $\mu(xy) + \alpha \geq 1$. Suppose if possible $\mu(xy) < 1$. Then $xy \in S - S_\mu$ which means $\mu(xy) \leq \sup\{\mu(x) : x \in S - S_\mu\}$. So $1 - \mu(xy) \geq 1 - \sup\{\mu(x) : x \in S - S_\mu\} > \alpha$ whence $\mu(xy) + \alpha < 1$ which is a contradiction. Hence $\mu(xy) = 1$. So $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$. Hence μ is a fuzzy subsemigroup of S.

CASE (2) : Let $T_{\alpha^+}(\mu)(x) = 1$ and $T_{\alpha^+}(\mu)(y) < 1$. Then $\mu(x) + \alpha \geq 1$ and $\mu(y) + \alpha < 1$. Then from (1), we have $\min\{\mu(xy) + \alpha, 1\} \geq \min\{1, \mu(y) + \alpha\} = \mu(y) + \alpha$, *i.e.*, $\mu(xy) + \alpha \geq \mu(y) + \alpha$, *i.e.*, $\mu(xy) \geq \mu(y) \geq \min\{\mu(x), \mu(y)\}$.

Hence μ is a fuzzy subsemigroup of S.

CASE (3) : Let $T_{\alpha^+}(\mu)(x) < 1$ and $T_{\alpha^+}(\mu)(y) < 1$. Then from (1), we have $\min\{\mu(xy)+\alpha, 1\} \geq \min\{\mu(x)+\alpha, \mu(y)+\alpha\}$, *i.e.*, $\mu(xy)+\alpha \geq \min\{\mu(x), \mu(y)\}+\alpha$, *i.e.*, $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$. Hence μ is a fuzzy subsemigroup of S. □

In a similar way we can prove the following proposition.

Proposition 3.7. *Let S be a semigroup, μ be a fuzzy subset of S and $T_{\alpha^-}(\mu)$ is a fuzzy subsemigroup of S for any non-negative real number α with $\alpha < \sup\{\mu(x) : x \in S - S_\mu\}$. Then μ is a fuzzy subsemigroup of S.*

Proposition 3.8. *Let S be a semigroup and μ be a fuzzy bi-ideal of S. Then $T_{\alpha^+}(\mu)(T_{\alpha^-}(\mu))$ is a fuzzy bi-ideal of S for any non-negative real number α.*

Proof. Let α be a non-negative real number. Clearly μ is a fuzzy subsemigroup of S. So by Proposition 3.3, $T_{\alpha^+}(\mu)$ is a fuzzy subsemigroup of S. Let $x, y, z \in S$. Then

$$\begin{aligned} T_{\alpha^+}(\mu)(xyz) &= \min\{\mu(xyz) + \alpha, 1\} \\ &\geq \min[\min\{\mu(x), \mu(z)\} + \alpha, 1] \\ &= \min[\min\{\mu(x) + \alpha, 1\}, \min\{\mu(z) + \alpha, 1\}] \\ &= \min\{T_{\alpha^+}(\mu)(x), T_{\alpha^+}(\mu)(z)\}. \end{aligned}$$

Hence $T_{\alpha^+}(\mu)$ is a fuzzy bi-ideal of S. Similarly we can show that $T_{\alpha^-}(\mu)$ is a fuzzy bi-ideal of S. □

Remark 6. It is clear from Example 3 that the converse of the above proposition is not always true.

Proposition 3.9. *Let S be a semigroup, μ be a fuzzy subset of S and $T_{\alpha^+}(\mu)$ be a fuzzy bi-ideal of S for any non-negative real number α with $\alpha < 1-\sup\{\mu(x) : x \in S - S_\mu\}$. Then μ is a fuzzy bi-ideal of S.*

Proof. Similar as Proposition 3.6. □

Proposition 3.10. *Let S be a semigroup and μ be a fuzzy ideal of S. Then $T_{\alpha^+}(\mu)(T_{\alpha^-}(\mu))$ is a fuzzy ideal of S for any non-negative real number α.*

Proof. Let α be a non-negative real number. Let $x, y \in S$. Then

$$\begin{aligned} T_{\alpha^+}(\mu)(xy) &= \min\{\mu(xy) + \alpha, 1\} \\ &\geq \min\{\mu(x) + \alpha, 1\} \text{ (as } \mu \text{ is a fuzzy right ideal)} \\ &= T_{\alpha^+}(\mu)(x). \end{aligned}$$

So $T_{\alpha^+}(\mu)$ is a fuzzy right ideal of S. Similarly we can show that $T_{\alpha^+}(\mu)$ is a fuzzy left ideal of S. Hence $T_{\alpha^+}(\mu)$ is a fuzzy ideal of S. Similarly we can show that $T_{\alpha^-}(\mu)$ is a fuzzy ideal of S. □

Proposition 3.11. *Let S be a semigroup, μ be a fuzzy subset of S and $T_{\alpha^+}(\mu)$ be a fuzzy ideal of S for any non-negative real number α with $\alpha < 1-\sup\{\mu(x) : x \in S - S_\mu\}$. Then μ is a fuzzy ideal of S.*

Proof. Let $x, y \in S$. Then

$$T_{\alpha^+}(\mu)(xy) \geq \min\{T_{\alpha^+}(\mu)(x), T_{\alpha^+}(\mu)(y)\}\text{.................................}(1)$$

CASE (1) : Let $T_{\alpha^+}(\mu)(x) = T_{\alpha^+}(\mu)(y) = 1$. Then $\mu(x) + \alpha \geq 1$ and $\mu(y)+\alpha \geq 1$. So $T_{\alpha^+}(\mu)(xy) = 1$ which means $\mu(xy)+\alpha \geq 1$. Suppose if possible $\mu(xy) < 1$. Then $xy \in S-S_\mu$ which means $\mu(xy) \leq \sup\{\mu(x) : x \in S-S_\mu\}$. So $1 - \mu(xy) \geq 1 - \sup\{\mu(x) : x \in S - S_\mu\} > \alpha$ whence $\mu(xy) + \alpha < 1$ which is a contradiction. Hence $\mu(xy) = 1$. So $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$. Hence μ is a fuzzy subsemigroup of S.

CASE (2) : Let $T_{\alpha^+}(\mu)(x) = 1$ and $T_{\alpha^+}(\mu)(y) < 1$. Then $\mu(x) + \alpha \geq 1$ and $\mu(y)+\alpha < 1$. Then from (1), we have $\min\{\mu(xy)+\alpha, 1\} \geq \min\{1, \mu(y)+\alpha\} = \mu(y) + \alpha$, *i.e.*, $\mu(xy) + \alpha \geq \mu(y) + \alpha$, *i.e.*, $\mu(xy) \geq \mu(y) \geq \min\{\mu(x), \mu(y)\}$. Hence μ is a fuzzy subsemigroup of S.

CASE (3) : Let $T_{\alpha^+}(\mu)(x) < 1$ and $T_{\alpha^+}(\mu)(y) < 1$. Then from (1), we have $\min\{\mu(xy)+\alpha, 1\} \geq \min\{\mu(x)+\alpha, \mu(y)+\alpha\}$, *i.e.*, $\mu(xy)+\alpha \geq \min\{\mu(x), \mu(y)\}+\alpha$, *i.e.*, $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$. Hence μ is a fuzzy subsemigroup of S. □

Lemma 3.12. *Let S be a semigroup, μ be a non-empty fuzzy subset of S and α be a non-negative real number. Then $(T_{\alpha^+}(\mu))_t = \mu_{\max\{t-\alpha,0\}}$ for all $t \in [0, 1]$.*

Proof. Let $t \in [0, 1]$. Then $x \in (T_{\alpha^+}(\mu))_t \Leftrightarrow T_{\alpha^+}(\mu)(x) \geq t \Leftrightarrow \min\{\mu(x) + \alpha, 1\} \geq t \Leftrightarrow \mu(x) + \alpha \geq t \Leftrightarrow \mu(x) \geq t - \alpha$ and $\mu(x) \geq 0 \Leftrightarrow x \in \mu_{\max\{t-\alpha,0\}}$. Hence $(T_{\alpha^+}(\mu))_t = \mu_{\max\{t-\alpha,0\}}$. □

Lemma 3.13. *Let S be a semigroup, μ be a non-empty fuzzy subset of S and α be a non-negative real number. Then for all $t \in (0, 1]$ with $t + \alpha < 1$, $(T_{\alpha^-}(\mu))_t = \mu_{t+\alpha}$.*

Proof. Let $t \in (0, 1]$ such that $t+\alpha < 1$. Let $x \in (T_{\alpha^-}(\mu))_t$. Then $\mu(x)-\alpha \geq 0$ otherwise $T_{\alpha^-}(\mu)(x) \geq t \Rightarrow \max\{\mu(x)-\alpha, 0\} \geq t \Rightarrow 0 \geq t$ which is absurd since $t \in (0, 1]$. Again as $T_{\alpha^-}(\mu)(x) \geq t$ we have $\mu(x) - \alpha = \max\{\mu(x) - \alpha, 0\} \geq t$ which implies that $x \in \mu_{t+\alpha}$.

Conversely, let $x \in \mu_{t+\alpha}$ where $t \in (0, 1]$. Then since $\mu(x) \geq (t + \alpha)$, $T_{\alpha^-}(\mu)(x) = \max\{\mu(x) - \alpha, 0\} \geq \max\{t, 0\} = t$.

□

Theorem 3.14. *Let S be a semigroup, α be a non-negative real number and μ be a non-empty fuzzy subset of S. Then $T_{\alpha^+}(\mu)$ is a fuzzy subsemigroup of S if and only if $\mu_{\max\{t-\alpha,0\}}$ is a subsemigroup of S, for all $t \in [0,1]$, provided $\mu_{\max\{t-\alpha,0\}} \neq \emptyset$.*

Proof. Let $T_{\alpha^+}(\mu)$ be a fuzzy subsemigroup of S. Let $t \in [0,1]$ be such that $\mu_{\max\{t-\alpha,0\}} \neq \emptyset$. Then $(T_{\alpha^+}(\mu))_t \neq \emptyset$ as $(T_{\alpha^+}(\mu))_t = \mu_{\max\{t-\alpha,0\}}$ (*cf.* Lemma 3.12). Let $x, y \in \mu_{\max\{t-\alpha,0\}} = (T_{\alpha^+}(\mu))_t$. Then $T_{\alpha^+}(\mu)(x) \geq t$ and $T_{\alpha^+}(\mu)(y) \geq t$. Since $T_{\alpha^+}(\mu)$ is a fuzzy subsemigroup of S,

$$\begin{aligned} T_{\alpha^+}(\mu)(xy) &= \min\{T_{\alpha^+}(\mu)(x), T_{\alpha^+}(\mu)(y)\} \\ &\geq \min\{t, t\} \\ &= t \end{aligned}$$

So $xy \in T_{\alpha^+}(\mu) = \mu_{\max\{t-\alpha,0\}}$. Hence $\mu_{\max\{t-\alpha,0\}}$ is a subsemigroup of S.

Conversely, let $\mu_{\max\{t-\alpha,0\}}$ be a subsemigroup of S for all $t \in [0,1]$ provided $\mu_{\max\{t-\alpha,0\}} \neq \emptyset$. Let $x, y \in S$. Let $T_{\alpha^+}(\mu)(x) = t_1$ and $T_{\alpha^+}(\mu)(y) = t_2$. Without loss of generality let $t_1 \geq t_2$. So $x, y \in (T_{\alpha^+}(\mu))_{t_2} = \mu_{\max\{t_2-\alpha,0\}}$(*cf.* Lemma 3.12). Then by hypothesis, $xy \in (T_{\alpha^+}(\mu))_{t_2} = \mu_{\max\{t_2-\alpha,0\}}$. Then $T_{\alpha^+}(\mu)(xy) \geq t_2 = \min\{t_1, t_2\} = \min\{T_{\alpha^+}(\mu)(x), T_{\alpha^+}(\mu)(y)\}$ Hence $T_{\alpha^+}(\mu)$ is a fuzzy subsemigroup of S. □

Theorem 3.15. *Let S be a semigroup, α be a non-negative real number and μ be a non-empty fuzzy subset of S. Then $T_{\alpha^-}(\mu)$ is a fuzzy subsemigroup of S if and only if $\mu_{t+\alpha}$ is a subsemigroup of S, for all $t \in (0,1]$ with $t + \alpha < 1$, provided $\mu_{t+\alpha} \neq \emptyset$.*

Proof. By using Lemma 3.13 and following the proof of Theorem 3.14 we can prove this theorem. □

In a similar manner we can have the following theorems by routine verification.

Theorem 3.16. *Let S be a semigroup, α be a non-negative real number and μ be a non-empty fuzzy subset of S. Then $T_{\alpha^+}(\mu)$ is a fuzzy bi-ideal of S if and only if $\mu_{\max\{t-\alpha,0\}}$ is a bi-ideal of S, for all $t \in [0,1]$, provided $\mu_{\max\{t-\alpha,0\}} \neq \emptyset$.*

Theorem 3.17. *Let S be a semigroup, α be a non-negative real number and μ be a non-empty fuzzy subset of S. Then $T_{\alpha^-}(\mu)$ is a fuzzy bi-ideal of S if and only if $\mu_{t+\alpha}$ is a bi-ideal of S, for all $t \in (0,1]$ with $t + \alpha < 1$, provided $\mu_{t+\alpha} \neq \emptyset$.*

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