

Hybridization of One's Assignment and Stepping Stone Method

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Abstract

To solve Assignment Problem many techniques and algorithms available, in which most famous algorithm is Hungarian Method. In this article we proposed hybrid method to solve Assignment Problem which is combination of One's Assignment Method and Stepping Stone Method. This gives optimal solution within few steps.

Mathematics Subject Classification: 90-08, 90B06, 90C05, 90C90.

Keywords:Assignment Problem, One's Assignment Method and Stepping Stone Method.

1 Introduction

We all are familiar with Assignment Problem which is one of the fundamental combinatorial optimization problems in the branch of Operation Research. An Assignment Problem is a particular case of Transportation Problem, which is also known as degenerated form of Transportation Problem. The main objective of Assignment Problem is to minimize cost or maximize the profit, in which one requirement is assigned to one and only one source.

To solve Assignment Problem – Visual Method, Enumeration Method, Transportation Method and Hungarian Method are available, but generally we use Hungarian Method [1] which developed by H. W. Kuhn in 1955. In Hungarian Method the optimal solution is obtained by cost reduction i.e. by subtracting each minimum row and column element from each corresponding row and column elements, we get at least one zero in each row and column, and by using

optimality we can find out optimal solution. Similarly to solve Assignment Problem One's Assignment Method [2, 3] can be used which gives optimal solution by dividing each minimum row and column element from each corresponding row and column elements, we get at least one ones in each row and column, and using optimality we can find optimal solution.

Moreover in Transportation Problem basic feasible solution is obtained by Using North - West Corner Method, Least - Cost Cell Method, Vogel's Approximation Method, and to find optimality -Stepping Stone Method, and Modified Distribution (MODI) Method is used.

2 Hybrid Method

In this article we proposed a hybrid method using One's Assignment and Stepping Stone Method to solve Assignment Problem, which gives optimal solution within few steps.

Hybrid Algorithm:

1. In a minimization case, find the minimum element of each row in the assignment matrix (say a_i) and write it on the right hand side of the matrix. Then divide each element of i^{th} row of the matrix by a_i . These operations create at least one ones in each rows. In term of ones for each row and column do assignment. Otherwise go to step 2.
2. Find the minimum (maximum) element of each column in assignment matrix (say b_j), and write it below j^{th} column. Then divide each element of j^{th} column of the matrix by b_j . These operations create at least one ones in each columns. Make assignment in terms of ones. If no feasible assignment can be achieved from step (1) and (2) then go to step 3.
3. In cost matrix number of one's should be $\geq n$. If this condition fails then find approximate value close to one's (i.e.1 and between < 1.5) and consider them as 1, otherwise go to step 4.
4. Now, first select cells having value 1 (occupied cell) in corresponding row and column (i.e. close path will not form), and mark as '*', fixed that cell as assignment cell. If tie is occurs for value 1 in corresponding row and column, then go to step 5.
5. Select cell that occur tie for value 1 and unoccupied cell to be evaluated.
6. Beginning at this cell, trace a closed path back to the original cell via cells that are currently being used (only horizontal or vertical moves allowed). You can only change directions at occupied cells (here occupied cell contain value 1).
7. Beginning with a plus (+) sign at cell to be evaluated, place alternative minus (-) signs and plus signs on each corner cell of the closed path just traced.
8. Calculate an improvement index, by adding together the unit cost figures found in each cell containing a plus sign and then subtracting the unit costs

in each cell containing a minus sign.

9. Repeat steps 5 to 8 until an improvement index has been calculated for all required cells.

Allocation of Assignment Cells:

1. First allocate that cells having value 1 in corresponding row and column (i.e. closed path will not form), and marked as '*', delete the corresponding row and column.
2. If tie is occur for one and '*', then choose minimum value in corresponding column, and delete corresponding row and column.
3. Choose minimum value from an improvement index.

3 Mathematical Concepts[4]

Primal Method: As is well known, the $n \times n$ assignment problem is equivalent to dual linear programs.

$$\text{Minimize } Z(X) = \sum_{i=1}^n c_{ij}x_{ij} \quad (1)$$

Subject to constraints,

$$\sum_{j=1}^n x_{ij} = 1, \text{ for } i = 1..n$$

$$\sum_{i=1}^n x_{ij} = 1, \text{ for } j = 1..n$$

$$x_{ij} \geq 0, \text{ for all } i, j.$$

$$\text{and } x_{ij} = 0 \text{ or } 1, \text{ for all } i, j.$$

or **Dual Method:**

$$\text{Maximize } Z(U, V) = \sum_{i=1}^n u_i \sum_{j=1}^n v_j \quad (2)$$

Subject to constraints,

$$u_i + v_j \leq c_{ij}, \text{ for } i, j = 1..n$$

$$1 \leq \frac{c_{ij}}{u_i + v_j}, \text{ for } i, j = 1..n$$

The criterion for optimality for equation (1) and (2) is the normality condition:

$$\frac{c_{ij}x_{ij}}{(u_i + v_j)x_{ij}} = 1, \text{ all}(i, j) \quad (3)$$

This means that X is a feasible assignment and (U, V) satisfies condition (3).

4 Definition

for any feasible primal solution X, the transformed objective function is,

$$\begin{aligned} \sum_{i=1}^n \sum_{j=1}^n \frac{c_{ij}x_{ij}}{(u_i + v_j)x_{ij}} &= \frac{\sum_{i=1}^n \sum_{j=1}^n c_{ij}x_{ij}}{\sum_{i=1}^n u_i \sum_{j=1}^n x_{ij} + \sum_{j=1}^n v_j \sum_{i=1}^n x_{ij}} \\ &= \frac{\sum_{i=1}^n \sum_{j=1}^n c_{ij}x_{ij}}{\sum_{i=1}^n u_i + \sum_{j=1}^n v_j} \end{aligned}$$

5 Testing for an optimal assignment

To test for an optimal assignment we find one's $\geq n$ in cost matrix, and try to find out cycle from occupied and unoccupied cells.

6 Illustrations

Example : A departmental head has four subordinates, and four tasks to be performed. The subordinates differ in efficiency, and the tasks differ in their intrinsic difficulty. His estimate, of the time each man would take to perform each task, is given in the matrix below:

Task	Men			
	A	B	C	D
S1	18	26	17	11
S2	13	28	14	26
S3	38	19	18	15
S4	19	26	24	10

How should the tasks be allocated, one to a man, so as to minimize the total man-hours?

Task	Men				
	A	B	C	D	Mfn
S1	18	26	17	11	11
S2	13	28	14	26	13
S3	38	19	18	15	15
S4	19	26	24	10	10

Row reduction by dividing corresponding minimum row value,

	A	B	C	D
S1	1.6	2.36	1.54	1
S2	1	2.15	1.07	2
S3	2.53	1.26	1.20	1
S4	1.9	2.6	2.4	1
Mn	1	1.26	1.07	1

Column reduction by dividing corresponding minimum column value,

	A	B	C	D
S1	1.6	1.87	1.43	1
S2	1	1.70	1	2
S3	2.53	1	1.12	1
S4	1.9	2.06	2.24	1

	A	B	C	D
S1	18	26	17	11
	1.6	1.87	1	1
S2	13	28	14	26
	1	1.70	1	2
S3	38	19	18	15
	2.53	1	1	1
S4	19	26	24	10
	1.9	2.06	2.24	1

Now, here by optimality condition in cost matrix cells having 1 should be $\geq n$. To overcome this problem we find values < 1.5 , i.e. cell (1, 3) and (3, 3) satisfies this condition, so we assume this values as 1, and try to find out closed path for occupied cells and unoccupied cells.

To find improvement index compare with original table value.

Here for cells (2, 1), (2, 3), (3, 2) and (4, 4) having value 1, and does not form a path (cycle), so we mark this cells as '*', and assign as assignment cells.

So, tie occurs in second row between (2, 1) and (2, 3) so find minimum value in corresponding column, whichever is most minimum give that cell priority.

Unoccupied cells:

$(1, 1) = (A, S1) - (C, S1) + (C, S2) - (A, S2) = 18 - 17 + 14 - 13 = 2$, similarly for all others i.e.

$(1, 1) = 2, (1, 2) = 8, (1, 3) = 3, (1, 4) = -3$.

$(2, 1) = *, (2, 2) = 13, (2, 3) = *, (2, 4) = 15$.

$(3, 1) = 21, (3, 2) = *, (3, 3) = -3, (3, 4) = 3$.

$(4, 1) = -, (4, 2) = 12, (4, 3) = 11, (4, 4) = *$.

Allocation of Assignment cells,

First we assign an assignment cells those who are having value 1 in respective cells, and then we assign cells that contains 1 and * (no path), i.e. cells (3, 2) and (4, 4) as an assignment cells.

Here tie occurs in second row, between cells (2, 1) and (2, 3), by observation in 1st and 3rd column, 3rd column having minimum value, (2, 3) will shift to other cell, and (2, 1) will assign as assignment cell, but in third row already assignment cell exist so this last priority goes to cell (1, 3) which next minimum value in third column.

So, optimal solution for this problem is,

$$z = (1, 3) + (2, 1) + (3, 2) + (4, 4) = 17+13+19+10 = 59.$$

7 Conclusion

In this article a new alternative method is proposed to solve assignment problem, which gives same optimal result as given by Hungarian method. So we conclude that the Hungarian method and Hybrid method give same optimal solution. However the technique for solving an assignment problem using Hybrid method is simple and it gives optimal solution within few steps.

References

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Received: January, 2015