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Rank, Mode, Divergence and Spread on Generalized Triangular Fuzzy Numbers

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Abstract

This paper proposes a In this paper, we want to propose a method for ranking of generalized triangular fuzzy numbers based on rank, mode, divergence and spread.

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Keywords: Divergence, Left and Right spread, Ranking fuzzy numbers, Generalized fuzzy numbers.

1 Introduction

In1965, Zadeh [19] introduced the concept of fuzzy set theory to meet those problems.Fuzzy numbers allow us to make the mathematical model of linguistic variable or fuzzy environment. In 1978, Dubois and Prade defined any of the fuzzy numbers as a fuzzy subset of the real line [6]. Ranking fuzzy numbers were first proposed by Jain [7] for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Bortolan and Degani [2] reviewed some of these ranking methods [3,9-11] for ranking fuzzy subsets. Chen [3] presented ranking fuzzy numbers with maximizing set and minimizing set. [8] and Wang and Lee [18] also used the centroid concept in developing their ranking index. Chen and Chen [4] presented a method for ranking generalized trapezoidal fuzzy numbers. Abbasbandy and Hajjari [1] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some -levels of trapezoidal fuzzy numbers. Chen and Chen [5] presented a method for fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads. Rezvani [10-16] evaluated the system of ranking fuzzy numbers and proposed a ranking trapezoidal fuzzy numbers. Moreover, Pushpinder Singh [17] proposed a method for ranking of generalized trapezoidal fuzzy numbers based on rank, mode, divergence and spread.

2 Preliminaries

Generalized fuzzy number $\widetilde{N}$ is described as any fuzzy subset of the real line R, whose membership function $\mu_{\widetilde{N}}(u)$ satisfies the following conditions,

(i) $\mu_{\widetilde{N}}(u)$ is a continuous mapping from R to the closed interval $[0, 1]$,

(ii) $\mu_{\widetilde{N}}(u) = 0$, $-\infty < u \leq \underline{a}$,

(iii) $\mu_L(u) = L(u)$ is strictly increasing on $[\underline{a}, a]$,

(iv) $\mu_{\widetilde{N}}(u) = w$, $u = a$,

(v) $\mu_R(u) = R(u)$ is strictly decreasing on $[a, \overline{a}]$,

(vi) $\mu_{\widetilde{N}}(u) = 0$, $\overline{a} \leq u < \infty$,

where $0 < w \leq 1$ and $\underline{a}$, a and $\overline{a}$ are real numbers.

We call this type of generalized fuzzy number a triangular fuzzy number, and it is denoted by

$$\widetilde{N} = (\underline{a}, a, \overline{a}; w)_{LR} \; . \tag{1}$$

When $w = 1$, this type of generalized fuzzy number is called normal fuzzy number and is represented by

$$\widetilde{N} = (\underline{a}, a, \overline{a})_{LR} \; . \tag{2}$$

Let L^{-1} and R^{-1} be the inverse function of functions L and R respectively, then the graded mean h-level value of $\widetilde{N} = (\underline{a}, a, \overline{a})_{LR}$ is

$$h[L^{-1}(h) + R^{-1}(h)]/2 \; . \tag{3}$$

Therefore, the graded mean integration representation of generalized triangular fuzzy number $\widetilde{N}$ with grade w is

$$G(\widetilde{N}) = \int_0^w h(\frac{L^{-1}(h) + R^{-1}(h)}{2}) \; dh / \int_0^w h \; dh \; , \tag{4}$$

where h lies between 0 and w, $0 < w \leq 1$. So membership function is given by

$$\mu_{\widetilde{N}}(x) = \begin{cases} w(\frac{u-\underline{a}}{a-\underline{a}}) & \underline{a} \leq u \leq a, \\ w & w = a, \\ w(\frac{u-\overline{a}}{a-\overline{a}}) & a \leq u \leq \overline{a}. \end{cases} \tag{5}$$

Here,

$$L(u) = w(\frac{u-\underline{a}}{a-\underline{a}}), \qquad \underline{a} \leq u \leq a, \tag{6}$$

and

$$R(u) = w(\frac{u-\overline{a}}{a-\overline{a}}), \qquad a \leq u \leq \overline{a}, \tag{7}$$

then

$$L^{-1}(h) = \underline{a} + \frac{(a-\underline{a})h}{w}, \qquad 0 \leq h \leq w, \tag{8}$$

$$R^{-1}(h) = \overline{a} - \frac{(\overline{a}-a)h}{w}, \qquad 0 \leq h \leq w, \tag{9}$$

Theorem 1. Let $\widetilde{N} = (\underline{a}, a, \overline{a}; w)_{LR}$, be generalized triangular fuzzy number with $0 < w \leq 1$ and $\underline{a}$, a and $\overline{a}$ are real numbers. Then the graded mean integration representation of $\widetilde{N}$ is

$$G(\widetilde{N}) = \frac{\underline{a} + 4a + \overline{a}}{6} \ . \tag{10}$$

Proof:

$$P(N) = \int_0^w \frac{h}{2}[\underline{a} + \frac{(a-\underline{a})h}{w} + \overline{a} - \frac{(\overline{a}-a)h}{w}]dh / \int_0^w h\ dh$$

$$= \int_0^w \frac{h}{2}[(\underline{a}+\overline{a}) + \frac{(2a-\underline{a}-\overline{a})h}{w}]dh / \int_0^w h\ dh$$

$$= w^2\frac{\underline{a}+\overline{a}}{4} + w^2\frac{2a-\underline{a}-\overline{a}}{6} / \frac{w^2}{2} = \frac{\underline{a}+\overline{a}}{4} + \frac{2a-\underline{a}-\overline{a}}{6} / \frac{1}{2}$$

$$= \frac{\underline{a}+4a+\overline{a}}{12} / \frac{1}{2} = \frac{\underline{a}+4a+\overline{a}}{6} \ .$$

3 Find $R(\widetilde{N})$, $mode(\widetilde{N})$, $divergence(\widetilde{N})$, Left and Right spread$(\widetilde{N})$

Let $\widetilde{N} = (\underline{a}, a, \overline{a}; w)_{LR}$ be generalized triangular fuzzy number, then we find the values of $R(\widetilde{N})$, $mode(\widetilde{N})$, $divergence(\widetilde{N})$, Left and Right spread.

$$R(\widetilde{N}) = \frac{1}{2}\int_0^w [L^{-1}(h) + R^{-1}(h)]\ dh$$

$$= \frac{1}{2}\int_0^w [\underline{a} + \frac{(a-\underline{a})h}{w} + \overline{a} - \frac{(\overline{a}-a)h}{w}]\ dh = \frac{1}{2}\int_0^w [(\underline{a}+\overline{a}) + \frac{(2a-\underline{a}-\overline{a})h}{w}]\ dh$$

$$= \frac{w(\underline{a}+\overline{a})}{2} + \frac{w(2a-\underline{a}-\overline{a})}{4} = \frac{w(\underline{a}+2a+\overline{a})}{4}$$

$$\Rightarrow R(\widetilde{N}) = \frac{w(\underline{a}+2a+\overline{a})}{4}\ . \tag{11}$$

Now, for $mode(\widetilde{N})$

$$mode(\widetilde{N}) = \frac{1}{2}\int_0^w a\ dh = \frac{aw}{2}\ , \tag{12}$$

For $divergence(\widetilde{N})$

$$divergence(\widetilde{N}) = w(\overline{a} - \underline{a})\ , \tag{13}$$

For Left and right spread$(\widetilde{N})$

$$Left\ spread(\widetilde{N}) = w(a - \underline{a})\ , \tag{14}$$

$$Right\ spread(\widetilde{N}) = w(\overline{a} - a)\ . \tag{15}$$

4 Some property for generalized triangular fuzzy numbers

Theorem 2. Let $\widetilde{N} = (\underline{a}, a, \overline{a}; w_a)_{LR}$ and $\widetilde{M} = (\underline{b}, b, \overline{b}; w_b)_{LR}$ be two generalized triangular fuzzy numbers, then

(i) *If* $R(\widetilde{N}) = R(\widetilde{M})$ *then* $w_a(\underline{a}+2a+\overline{a}) = w_b(\underline{b}+2b+\overline{b})$, (16)

(ii) *If* $mode(\widetilde{N}) = mode(\widetilde{M})$ *then* $aw_a = bw_b$, (17)

(iii) *If* $divergence(\widetilde{N}) = divergence(\widetilde{M})$ *then* $w_a(\overline{a}-\underline{a}) = w_b(\overline{b}-\underline{b})$. (18)

Proof:

$$\textbf{(i)}\ We\ have\ R(\widetilde{N}) = \frac{w_a(\underline{a}+2a+\overline{a})}{4} = \frac{w_b(\underline{b}+2b+\overline{b})}{4}$$

$$\Rightarrow w_a(\underline{a}+2a+\overline{a}) = w_b(\underline{b}+2b+\overline{b}) = R(\widetilde{M})\ .$$

$$\textbf{(ii)}\ We\ have\ mode(\widetilde{N}) = \frac{aw_a}{2} = \frac{bw_b}{2} \Rightarrow aw_a = bw_b = mode(\widetilde{M})\ .$$

(iii) $We\ have\ divergence(\widetilde{N}) = w_a(\overline{a}-\underline{a}) = w_b(\overline{b}-\underline{b}) = divergence(\widetilde{M})\ .$

Also of theorem 2. we can say

$$w_a\ \underline{a} = w_b\ \underline{b}\ , \tag{19}$$

$$w_a\ \overline{a} = w_b\ \overline{b}\ , \tag{20}$$

$$w_a\ a = w_b\ b\ . \tag{21}$$

Theorem 3. Let $\widetilde{N} = (\underline{a}, a, \overline{a}; w_a)_{LR}$ and $\widetilde{M} = (\underline{b}, b, \overline{b}; w_b)_{LR}$ be two generalized triangular fuzzy numbers, then

(i) $Left\ spread(\widetilde{N}) > Left\ spread(\widetilde{M})\ iff\ w_a a > w_b b,$

(ii) $Left\ spread(\widetilde{N}) < Left\ spread(\widetilde{M})\ iff\ w_a a < w_b b,$

(iii) $Left\ spread(\widetilde{N}) = Left\ spread(\widetilde{M})\ iff\ w_a a = w_b b.$

Proof:

(i) Let $Left\ spread(\widetilde{N}) > Left\ spread(\widetilde{M}) \Rightarrow w_a(a-\underline{a}) > w_b(b-\underline{b})$ with use equation (19), we have

$w_a a > w_b b$.

Contrary

Let $w_a a > w_b b$, with use equation (19), we have

$w_a(a-\underline{a}) > w_b(b-\underline{b}) \Rightarrow Left\ spread(\widetilde{N}) > Left\ spread(\widetilde{M}).$

(ii) Let $Left\ spread(\widetilde{N}) < Left\ spread(\widetilde{M}) \Rightarrow w_a(a-\underline{a}) < w_b(b-\underline{b})$ with use equation (19), we have

$w_a a < w_b b$.

Contrary

Let $w_a a < w_b b$, with use equation (19), we have

$w_a(a - \underline{a}) < w_b(b - \underline{b}) \Rightarrow Left\ spread(\widetilde{N}) < Left\ spread(\widetilde{M})$.

(iii) Let $Left\ spread(\widetilde{N}) = Left\ spread(\widetilde{M}) \Rightarrow w_a(a - \underline{a}) = w_b(b - \underline{b})$ with use equation (19), we have

$w_a a = w_b b$.

Contrary

Let $w_a a = w_b b$, with use equation (19), we have

$w_a(a - \underline{a}) = w_b(b - \underline{b}) \Rightarrow Left\ spread(\widetilde{N}) = Left\ spread(\widetilde{M})$.

Theorem 4. Let $\widetilde{N} = (\underline{a}, a, \overline{a}; w_a)_{LR}$ and $\widetilde{M} = (\underline{b}, b, \overline{b}; w_b)_{LR}$ be two generalized triangular fuzzy number, then

(i) $Right\ spread(\widetilde{N}) > Right\ spread(\widetilde{M})\ iff\ -w_a a > -w_b b$,

(ii) $Right\ spread(\widetilde{N}) < Right\ spread(\widetilde{M})\ iff\ -w_a a < -w_b b$,

(iii) $Right\ spread(\widetilde{N}) = Right\ spread(\widetilde{M})\ iff\ -w_a a = -w_b b$.

Proof:

(i) Let $Right\ spread(\widetilde{N}) > Right\ spread(\widetilde{M}) \Rightarrow w_a(\overline{a} - a) > w_b(\overline{b} - b)$ with use equation (20), we have

$-w_a a > -w_b b$.

Contrary

Let $-w_a a > -w_b b$, with use equation (20), we have

$w_a(\overline{a}-a) > w_b(\overline{b}-b) \Rightarrow Right\ spread(\widetilde{N}) > Right\ spread(\widetilde{M})$.

(ii) Let $Right\ spread(\widetilde{N}) < Right\ spread(\widetilde{M}) \Rightarrow w_a(\overline{a}-a) < w_b(\overline{b}-b)$ with use equation (20), we have

$-w_a a < -w_b b$.

Contrary

Let $-w_a a < -w_b b$, with use equation (20), we have

$w_a(\overline{a}-a) < w_b(\overline{b}-b) \Rightarrow Right\ spread(\widetilde{N}) < Right\ spread(\widetilde{M})$.

(iii) Let $Right\ spread(\widetilde{N}) = Right\ spread(\widetilde{M}) \Rightarrow w_a(\overline{a}-a) = w_b(\overline{b}-b)$ with use equation (20), we have

$-w_a a = -w_b b$.

Contrary

Let $-w_a a = -w_b b$, with use equation (20), we have

$w_a(\overline{a}-a) = w_b(\overline{b}-b) \Rightarrow Right\ spread(\widetilde{N}) = Right\ spread(\widetilde{M})$.

Theorem 5. Let $\widetilde{N} = (\underline{a}, a, \overline{a}; w_a)_{LR}$ and $\widetilde{M} = (\underline{b}, b, \overline{b}; w_b)_{LR}$ be two generalized triangular fuzzy number, such that

$$\textbf{(a)}\ R(\widetilde{N}) = R(\widetilde{M})\ ,$$

$$\textbf{(b)}\ mode(\widetilde{N}) = mode(\widetilde{M})\ ,$$

$$\textbf{(c)}\ divergence(\widetilde{N}) = divergence(\widetilde{M})\ ,$$

then

(i) $Left\ spread(\widetilde{N}) > Left\ spread(\widetilde{M})\ iff\ Right\ spread(\widetilde{N}) > Right\ spread(\widetilde{M})$,

(ii) $Left\ spread(\widetilde{N}) < Left\ spread(\widetilde{M})\ iff\ Right\ spread(\widetilde{N}) < Right\ spread(\widetilde{M})$,

(iii) $Left\ spread(\widetilde{N}) = Left\ spread(\widetilde{M})\ iff\ Right\ spread(\widetilde{N}) = Right\ spread(\widetilde{M})$.

Proof:

(i) Let $a = a_1 + a_2$ and $b = b_1 + b_2$, so $Left\ spread(\widetilde{N}) > Left\ spread(\widetilde{M}) \Leftrightarrow w_a(a_1 - \underline{a}) > w_b(b_1 - \underline{b})$, with use (19) iff $w_a\ a_1 > w_b\ b_1$. Now of (21)($w_a\ (a_1 + a_2) = w_b\ (b_1 + b_2)$), we have iff $w_a\ a_2 < w_b\ b_2 \Leftrightarrow -w_a\ a_2 > -w_b\ b_2$. Then with (20) iff $w_a(\overline{a} - a_2) > w_b(\overline{b} - b_2) \Leftrightarrow Right\ spread(\widetilde{N}) > Right\ spread(\widetilde{M})$,

(ii) Let $a = a_1 + a_2$ and $b = b_1 + b_2$, so $Left\ spread(\widetilde{N}) < Left\ spread(\widetilde{M}) \Leftrightarrow w_a(a_1 - \underline{a}) < w_b(b_1 - \underline{b})$, with use (19) iff $w_a\ a_1 < w_b\ b_1$. Now of (21)($w_a\ (a_1 + a_2) = w_b\ (b_1 + b_2)$), we have iff $w_a\ a_2 > w_b\ b_2 \Leftrightarrow -w_a\ a_2 < -w_b\ b_2$. Then with (20) iff $w_a(\overline{a} - a_2) < w_b(\overline{b} - b_2) \Leftrightarrow Right\ spread(\widetilde{N}) < Right\ spread(\widetilde{M})$,

(iii) Let $a = a_1 + a_2$ and $b = b_1 + b_2$, so $Left\ spread(\widetilde{N}) = Left\ spread(\widetilde{M}) \Leftrightarrow w_a(a_1 - \underline{a}) = w_b(b_1 - \underline{b})$, with use (19) iff $w_a\ a_1 = w_b\ b_1$. Now of (21)($w_a\ (a_1 + a_2) = w_b\ (b_1 + b_2)$), we have iff $w_a\ a_2 = w_b\ b_2 \Leftrightarrow -w_a\ a_2 = -w_b\ b_2$. Then with (20) iff $w_a(\overline{a} - a_2) = w_b(\overline{b} - b_2) \Leftrightarrow Right\ spread(\widetilde{N}) = Right\ spread(\widetilde{M})$.

5 Proposed approach for ranking of generalized triangular fuzzy numbers

Let $\widetilde{N} = (\underline{a}, a, \overline{a}; w_a)_{LR}$ and $\widetilde{M} = (\underline{b}, b, \overline{b}; w_b)_{LR}$ be two generalized triangular fuzzy number, then use the following steps to compare $\widetilde{N}, \widetilde{M}$

$*$ step 1: Find $R(\widetilde{N})$ and $R(\widetilde{M})$

Case (i) If $R(\widetilde{N}) > R(\widetilde{M})$ then $\widetilde{N} > \widetilde{M}$

Case (ii) If $R(\widetilde{N}) < R(\widetilde{M})$ then $\widetilde{N} < \widetilde{M}$

Case (iii) If $R(\widetilde{N}) = R(\widetilde{M})$ then go to step 2.

$*$ step 2: Find $mode(\widetilde{N})$ and $mode(\widetilde{M})$

Case (i) If $mode(\widetilde{N}) > mode(\widetilde{M})$ then $\widetilde{N} > \widetilde{M}$

Case (ii) If $mode(\widetilde{N}) < mode(\widetilde{M})$ then $\widetilde{N} < \widetilde{M}$

Case (iii) If $mode(\widetilde{N}) = mode(\widetilde{M})$ then go to step 3.

$*$ step 3: Find $divergence(\widetilde{N})$ and $divergence(\widetilde{M})$

Case (i) If $divergence(\widetilde{N}) > divergence(\widetilde{M})$ then $\widetilde{N} > \widetilde{M}$

Case (ii) If $divergence(\widetilde{N}) < divergence(\widetilde{M})$ then $\widetilde{N} < \widetilde{M}$

Case (iii) If $divergence(\widetilde{N}) = divergence(\widetilde{M})$ then go to step 4.

$*$ step 4: Find $Left\ spread(\widetilde{N})$ and $Left\ spread(\widetilde{M})$

Case (i) If $Left\ spread(\widetilde{N}) > Left\ spread(\widetilde{M})$

i.e., $w_a\ a > w_b\ b$ then $\widetilde{N} > \widetilde{M}$ from theorem 3.

Case (ii) If $Left\ spread(\widetilde{N}) < Left\ spread(\widetilde{M})$

i.e., $w_a\ a < w_b\ b$ then $\widetilde{N} < \widetilde{M}$ from theorem 3.

Case (iii) If $Left\ spread(\widetilde{N}) = Left\ spread(\widetilde{M})$

i.e., $w_a\ a = w_b\ b$ then go to step 5 from theorem 3.

$*$ step 5: Find w_a and w_b Case (i) If $w_a > w_b$ then $\widetilde{N} > \widetilde{M}$

Case (ii) If $w_a < w_b$ then $\widetilde{N} < \widetilde{M}$

Case (iii) If $w_a = w_b$ then $\widetilde{N} = \widetilde{M}$

6 Examples

In this section, examples of fuzzy numbers are compared using the proposed approach.

Example 1. Let $\widetilde{N} = (0.2, 0.3, 0.5; 0.21)$ and $\widetilde{M} = (0.1, 0.15, 0.25; 0.42)$ be two generalized triangular fuzzy number

$*$ step 1

$R(\widetilde{N}) = 0.06825$ and $R(\widetilde{M}) = 0.06825$. Since $R(\widetilde{N}) = R(\widetilde{M})$, so go to step 2

$*$ step 2

$mode(\widetilde{N}) = 0.0315$ and $mode(\widetilde{M}) = 0.0315$ .Since $mode(\widetilde{N}) = mode(\widetilde{M})$, so go to step 3

$*$ step 3

$divergence(\widetilde{N}) = 0.063$ and $divergence(\widetilde{M}) = 0.063$. Since $divergence(\widetilde{N}) = divergence(\widetilde{M})$, so go to step 4

$*$ step 4

Left spread$(\widetilde{N}) = 0.021$ and Left spread$(\widetilde{M}) = 0.021$. Since Left spread$(\widetilde{N})$=Left spread$(\widetilde{M})$, so go to step 5

$*$ step 5

$w_a = 0.21$, $w_b = 0.42$ Since $w_a < w_b \Rightarrow \widetilde{N} < \widetilde{M}$.

Example 2. Let $\widetilde{N} = (0.4, 0.6, 0.8; 0.35)$ and $\widetilde{M} = (0.3, 0.5, 0.7; 0.42)$ be two generalized triangular fuzzy number

$*$ step 1

$R(\widetilde{N}) = 0.21$ and $R(\widetilde{M}) = 0.21$. Since $R(\widetilde{N}) = R(\widetilde{M})$, so go to step 2

$*$ step 2

$mode(\widetilde{N}) = 0.105$ and $mode(\widetilde{M}) = 0.105$ .Since $mode(\widetilde{N}) = mode(\widetilde{M})$, so go to step 3

$*$ step 3

$divergence(\widetilde{N}) = 0.14$ and $divergence(\widetilde{M}) = 0.168$.

So $divergence(\widetilde{N}) < divergence(\widetilde{M})$ then $N < M$.

Example 3. Let $\widetilde{N} = (0.1, 0.3, 0.5; 1)$ and $\widetilde{M} = (0.2, 0.2, 0.6; 1)$ be two generalized triangular fuzzy number

$*$ step 1

$R(\widetilde{N}) = 0.3$ and $R(\widetilde{M}) = 0.3$. Since $R(\widetilde{N}) = R(\widetilde{M})$, so go to step 2

$*$ step 2

$mode(\widetilde{N}) = 0.15$ and $mode(\widetilde{M}) = 0.1$ .So $mode(\widetilde{N}) > mode(\widetilde{M})$ then $N > M$.

Example 4. Let $\widetilde{N} = (0.2, 0.3, 0.5; 0.21)$ and $\widetilde{M} = (0.1, 0.4, 0.4; 0.21)$ be two generalized triangular fuzzy number

$*$ step 1

$R(\widetilde{N}) = 0.06825$ and $R(\widetilde{M}) = 0.06825$. Since $R(\widetilde{N}) = R(\widetilde{M})$, so go to step 2

$*$ step 2

$mode(\widetilde{N}) = 0.0315$ and $mode(\widetilde{M}) = 0.042$ .So $mode(\widetilde{N}) < mode(\widetilde{M})$, then $A < B$.

Example 5. Let $\widetilde{N} = (0.2, 0.3, 0.4; 0.26)$ and $\widetilde{M} = (0.1, 0.2, 0.3; 0.26)$ be two generalized triangular fuzzy number

$*$ step 1

$R(\widetilde{N}) = 0.078$ and $R(\widetilde{M}) = 0.052$. So $R(\widetilde{N}) > R(\widetilde{N})$ then $N > M$.

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