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Some Pproblems On Infinitesimal Holomorphically Projective Transformations In Kaehlerian Manifold With Recurrent Curvature Tensor

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ABSTRACT:

Aim of thispaper is to delineate theinfinitesimal holomorphically projective transformations in Kaehlerian manifold with recurrent curvature tensor. In section 2 and 3, we haveestablished few theorems.

KEY WORDS:

Riemannian manifold, Kaehlerian manifold, infinitesimal holomorphically projective curvature tensor, recurrent curvature tensor, infinitesimal holomorphically projective transformation.

1.INTRODUCTION:

Definition 1.1:

A n-dimensional Riemannian manifold M_n with the metric tensor $g_{\alpha\beta}$ and an affine structure F^{α}_{β} is called Kaehlerian manifold, if the following relations are holds:

(1.1) $F^{\alpha}_{\gamma} F^{\gamma}_{\beta} = - \delta^{\alpha}_{\beta}$,

(1.2) $\nabla_{\gamma} F^{\alpha}_{\beta} = 0$,

(1.3) $g_{\alpha[\beta}F^{\beta}_{\gamma]} = 0$,

(1.4) $g_{\alpha\beta} F^{\beta}_{\gamma}= F_{\alpha\gamma}$,

(1.5) $g^{\alpha\gamma} F^{\beta}_{\gamma}= F^{\alpha\beta}$

and

(1.6) $F_{\alpha\beta}= - F_{\beta\alpha}$.

Wherein δ^{α}_{β} is the Kronecker delta, ∇_{γ} denotes the covariant derivative in Kaehlerian manifolds M_n.

Further,$R^{\varepsilon}_{\alpha\beta\gamma}$ and $R_{\alpha\beta}$ are the Riemannian and Ricci tensors respectively then we have the following conditions[1]:

(1.7) $R^{\varepsilon}_{\alpha\beta\gamma} F^{\beta}_{\delta}F^{\gamma}_{\nu}= R^{\varepsilon}_{\alpha\delta\nu}$,

(1.8) $R_{\alpha\beta} F^{\alpha}_{\gamma}F^{\beta}_{\delta}= R_{\gamma\delta}$,

(1.9) $R^{\varepsilon}_{\alpha\beta\gamma} F^{\alpha}_{\delta} = R^{\alpha}_{\delta\beta\gamma} F^{\varepsilon}_{\alpha}$,

(1.10) $R_{\alpha\beta} F^{\beta}_{\gamma}= H_{\alpha\gamma}$,

(1.11) $R_{\alpha\beta\gamma\delta}F^{\gamma\delta} =2H_{\alpha\beta}$,

(1.12) $H_{\alpha\beta} F^{\beta}_{\gamma}= - R_{\alpha\gamma}$

(1.13) $H_{\alpha\beta}= - H_{\beta\alpha}$

and

(1.14) $H_{\alpha\gamma}g^{\alpha\beta} = R\, F^{\beta}_{\gamma}$.

2. INFINITESIMALHOLOMORPHICALLY PROJECTIVE TRANSFORMATIONS IN KAEHLERIAN MANIFOLD WITH CURVATURE TENSOR:

Definition 2.1:

A vector field v^{α} is calledholomorphically projective transformation briefly HPT, if it satisfies the condition

(2.1) $L_v\{_{\beta}{}^{\alpha}{}_{\gamma}\} = P_{\varepsilon}(\delta^{\varepsilon}_{\beta}\delta^{\alpha}_{\gamma} - \xi^{\varepsilon}_{\beta}\xi^{\alpha}_{\gamma}) + P_{\varepsilon}(\delta^{\varepsilon}_{\gamma}\delta^{\alpha}_{\beta} - \xi^{\varepsilon}_{\gamma}\xi^{\alpha}_{\beta})$.

Wherein L_v denotes the operator of Lie derivative with respect to v^{α}, $\{_{\beta}{}^{\alpha}{}_{\gamma}\}$is the Christoffel symbol of second kind, $P_{\varepsilon}$is certain vector and ξ^{α}_{β} is the complex structure.

Definition 2.2:

If a vector field v^{α} satisfies the condition

(2.2) $L_v\{_{\beta}{}^{\alpha}{}_{\gamma}\} = \nabla_{\beta}\nabla_{\gamma}v^{\alpha} + R^{\alpha}_{\beta\gamma\varepsilon}v^{\varepsilon} = 0$

is termed as infinitesimal affine transformation briefly IAT in Kaehlerian manifold with curvature tensor.

Definition 2.3:

If a vector field v^{α} satisfies the relation

(2.3) $L_v\{_{\beta}{}^{\alpha}{}_{\gamma}\} = P_{\gamma}\delta^{\alpha}_{\beta} + P_{\beta}\delta^{\alpha}_{\gamma} - P^{*}_{\gamma}\delta^{\alpha}_{\beta} - P^{*}_{\beta}\delta^{\alpha}_{\gamma}$

is called infinitesimal holomorphically projective transformation briefly IHPT in Kaehlerian manifold.

Wherein

(2.4) $P^{*}_{\beta} = \xi^{\alpha}_{\beta}P_{\alpha}$.

Definition 2.4:

If a vector field v^{α} satisfies the condition

(2.5) $R^{\alpha}_{\beta\gamma\varepsilon}v^{\varepsilon}= P_{\gamma}\delta^{\alpha}_{\beta} + P_{\beta}\delta^{\alpha}_{\gamma} - P^{*}_{\gamma}\delta^{\alpha}_{\beta} - P^{*}_{\beta}\delta^{\alpha}_{\gamma} - \nabla_{\beta}\nabla_{\gamma}v^{\alpha}$

istermed as infinitesimal holomorphically projective transformation briefly IHPT in Kaehlerian manifold with curvature tensor.

Definition 2.5:

If an infinitesimal holomorphically projective transformation reduces to an infinitesimal affine transformation then its satisfies the condition

(2.6) $P_{\alpha} = 0$.

Definition 2.6:

A tensor $P^{\varepsilon}_{\alpha\beta\gamma}$ is said to be infinitesimal holomorphically projective curvature tensorbriefly IHPC-tensor of infinitesimal holomorphically projective transformations in Kaehlerian manifold with curvature tensor if it satisfies the condition

(2.7) $P^{\varepsilon}_{\alpha\beta\gamma}=R^{\varepsilon}_{\alpha\beta\gamma}-[1/(n+2)][\delta^{\varepsilon}_{\gamma}R_{\alpha\beta}-\delta^{\varepsilon}_{\beta}R_{\alpha\gamma}+(F^{\varepsilon}_{\beta}R_{\nu\gamma}-F^{\varepsilon}_{\gamma}R_{\nu\beta})F^{\nu}_{\alpha}+2F^{\varepsilon}_{\alpha}R_{\nu\gamma}F^{\nu}_{\beta}$.

In this regard, we have the following theorems:

Theorem 2.1:

Aninfinitesimal holomorphically projective curvature tensor of an infinitesimal holomorphically projective transformations in Kaehlerian manifold with curvature tensor holds the relation $P_{\alpha\beta} = [(n+5)/(n+2)]R_{\alpha\beta}$.

Proof:

Contracting equation (2.7) and using equation (1.10), we get

(2.8) $P_{\alpha\beta} = R_{\alpha\beta} - [1/(n+2)][H_{\nu\beta}F^{\nu}{}_{\alpha} + 2H_{\nu\alpha}F^{\nu}{}_{\beta}]$

By virtue of equations (1.12) and (2.8), we obtain

(2.9) $P_{\alpha\beta} = [(n+5)/(n+2)]R_{\alpha\beta}$.

Theorem 2.2:

An infinitesimal holomorphically projective curvature tensor of an infinitesimal holomorphically projective transformations in Kaehlerian manifold with curvature tensor is symmetric with respect to the covariant indices.

Proof:

Interchanging the indices α and β in equation (2.9), we get

(2.10) $P_{\beta\alpha} = [(n+5)/(n+2)]R_{\beta\alpha}$

Since $R_{\alpha\beta}$ is symmetric with respect to α and β then

(2.11) $R_{\alpha\beta} = R_{\beta\alpha}$

From equations (2.10) and (2.11), we obtain

(2.12) $P_{\beta\alpha} = [(n+5)/(n+2)]R_{\alpha\beta}$

By virtue of equations (2.9) and (2.12), we get

(2.13) $P_{\beta\alpha} = P_{\alpha\beta}$.

3. INFINITESIMALHOLOMORPHICALLY PROJECTIVE TRANSFORMATIONS IN KAEHLERIAN MANIFOLD WITH RECURRENT CURVATURE TENSOR:

Definition 3.1:

A Kaehlerian manifold is called Kaehlerian manifoldwith recurrent curvature tensor if its curvature tensor $R^{\varepsilon}{}_{\alpha\beta\gamma}$ satisfies the condition

(3.1) $\nabla_a R^{\varepsilon}{}_{\alpha\beta\gamma} = \lambda_a R^{\varepsilon}{}_{\alpha\beta\gamma}$.

Wherein $\lambda_a$is a non-zero recurrent tensor field.

Definition 3.2:

A Kaehlerian manifold is called Kaehlerian manifold with Ricci-recurrent curvature tensor if its curvature tensor $R_{\alpha\beta}$ satisfies the condition

(3.2) $\nabla_a R_{\alpha\beta} = \lambda_a R_{\alpha\beta}$.

Wherein $\lambda_a$is a non-zero recurrent tensor field.

Definition 3.2:

If an infinitesimal holomorphically projective curvature tensorof an infinitesimal holomorphically projective transformationsin Kaehlerian manifold with curvature tensor holds the condition

(3.3) $\nabla_a P^{\varepsilon}_{\alpha\beta\gamma} = \lambda_a P^{\varepsilon}_{\alpha\beta\gamma}$,

is called an infinitesimal holomorphically projective recurrentcurvature tensor of an infinitesimal holomorphically projective transformations in Kaehlerian manifold with recurrent curvature tensor.

Wherein $\lambda_a$is a non-zero recurrent tensor field.

In this regard, we have the following theorem:

Theorem 3.1:

If Kaehlerian manifold with curvature tensor is Ricci-recurrent then an infinitesimal holomorphically projective curvature tensor of an infinitesimal holomorphically projective transformations in Kaehlerian manifoldadmits the condition $\nabla_a P_{\alpha\beta} = \lambda_a P_{\alpha\beta}$:

Proof:

Taking the covariant differentiation of equation (2.9), we get

(3.4) $\nabla_a P_{\alpha\beta} = [(n+5)/(n+2)](\nabla_a R_{\alpha\beta})$

Since Kaehlerian manifold with curvature tensor is Ricci-recurrent then from equations (3.2) and (3.4), we obtain

(3.5) $\nabla_a P_{\alpha\beta} = [(n+5)/(n+2)](\lambda_a R_{\alpha\beta})$

In view of equations (2.9) and (3.5), we get

(3.6) $\nabla_a P_{\alpha\beta} = \lambda_a P_{\alpha\beta}$.

REFERENCES

[1]. Adati, T. and Miyazawa, T.: On a Riemannian space with recurrent and conformal curvature, Tensor, N.S., 18, P. (348-354), (1967).

[2]. Chauhan, T.S.; Chauhan, I.S. and Singh, Rajeev Kumar: On Einstein Kaehlerian space with Recurrent Bochner curvature tensor, ActaCienciaIndica, Vol.XXXIV M, No. 1, P. (23-26), (2008).

[3]. Mikes, J.: On holomorphically projective mappings of Kahlerian spaces, Ukr. Geom. Sb., Kharkov, P.(90-98), 23(1980).

[4]. Mikes, J. and Pokorna, O.: On holomorphically projective mappings onto Kahlerian spaces, Suppl. Rend. Circ. Mat. Palermo, Ser. II, P.(181-186), 69(2002).

[5]. Negi, U.S. and Semwal, Ashish: On Kaehlerianconharmonic recurrent and symmetric manifold, International Journal of Mathematical Archive, P.(1405-1410), 3(4), (2012).

[7]. Otsuki, T. and Tashiro, Y.: On curves in Kaehlerian spaces, Math. J. Okayama Univ., P. (57-78), 4(1954).

[8]. Tachibana, S. and Ishihara, S.: On infinitesimal holomorphically projective transformations in Kahlerian manifolds, Tohoku Math. Jour., P.(77-101), 12(1960).

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