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On a compound Markov binomial risk model with time-correlated claims

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Abstract

In this paper we consider an extension to the compound Markov binomial risk model in which two kinds of dependent claims are introduced. For the proposed risk model, the generating functions of two conditional expected discounted penalty functions are obtained. Based on these results, we derive a recursive formula for the conditional expected discounted penalty function when there is no claim at time 0. The relationship between the two conditional expected discounted penalty functions is then investigated.

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1 Introduction

As a discrete analogue of classical compound Poisson risk model, the compound binomial risk model was first proposed by [1] and further studied by a fair amount of researchers, see [2] and the references therein for details. [3] extended the compound binomial model to the case involving two types of correlated risks. The occurrence of one claim may induce the occurrence of another claim with different distribution of severity. However, the time of occurrence of the induced claim may be delayed to the next period with a certain probability. They obtained recursive formula for the finite time ruin probabilities and explicit expressions for ultimate ruin probabilities in some special cases. [4] further investigated the joint distribution of the surplus prior to ruin and deficit at ruin. [5, 6] extended the compound binomial risk model to the compound Markov binomial model which introduces time-dependence in the aggregate claim amount increments governed by a Markov process.

Motivated by the above mentioned literature, in the present paper we consider the compound Markov binomial risk model with time correlated claims.

2 The Model

Let $\{I_t,\, t \in N^+\}$ be a stationary homogeneous Markov chain with state space $\{0, 1\}$ and transition probability matrix

$$P = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix} = \begin{pmatrix} 1-(1-\pi)q & (1-\pi)q \\ (1-q)(1-\pi) & \pi+(1-\pi)q \end{pmatrix},$$

where $p_{ij} = \Pr(I_{t+1} = j \mid I_t = i)$ for $t \in \mathbb{N}$ and $i, j \in \{0, 1\}$. Initial probabilities are denoted by $\Pr(I_0 = 1) = q = 1 - \Pr(I_0 = 0)$ for $0 < q < 1$, and π is the dependence parameter with $0 \le \pi < 1$. Note that $\{I_t,\, t \in \mathbb{N}\}$ is sometimes called a Markov-Bernoulli sequence.

Now we consider a risk model which involves two kinds of insurance claims, namely the main claims and the by-claims. The main claim amounts $\{X_i,\, i \in \mathbb{N}^+\}$ are i.i.d. positive and integer valued r.v.'s with common probability function (p.f.) f_X, cumulative distribution function (c.d.f.) F_X and mean μ_X. The total amount of main claims up to time $k \in \mathbb{N}$ is defined as

$$U_t^X = \sum_{i=1}^{t} X_i I_i,$$

with $U_0^X = 0$. The occurrence r.v. I_i and individual claim amount r.v. X_i are independent in each time period.

Suppose that each main claim may induce a by-claim. The by-claim and its associated main claim may occur simultaneously with probability θ ($0 \le \theta \le 1$), or the occurrence of the by-claim may be delayed to the next time period with probability $1-\theta$. The by-claim amounts $\{Y_i,\, i \in \mathbb{N}^+\}$ are assumed to be i.i.d. positive integer valued r.v.'s having common p.f. f_Y, c.d.f. F_Y and mean μ_Y. The independence between $\{X_i,\, i \in \mathbb{N}^+\}$ and $\{Y_i,\, i \in \mathbb{N}^+\}$ is also assumed.

The premium rate is assumed to be equal to 1. The surplus process of an insurance company is defined as

$$U_t = u + t - U_t^X - U_t^Y, \quad t \in \mathbb{N}, \tag{1}$$

where $U_0 = u \in \mathbb{N}$ corresponds to the initial surplus and U_t^Y is the total amount of by-claims up to time t with $U_0^Y = 0$.

Since the sequence $\{I_t,\, t \in \mathbb{N}\}$ is stationary, we have $\Pr(I_t = 1) = q$ for $t \in \mathbb{N}^+$. From [3] we know that $\mathrm{E}[U_{t+1}] = tq\,(\mu_X + \mu_Y) + q\mu_X + q\theta\mu_Y$. Therefore, we assume that

$$q\,(\mu_X + \mu_Y) < 1, \tag{2}$$

which ensures that the surplus process U_t goes almost surely to infinity as $t \to \infty$.

Let $T = \min\{t \in \mathbb{N}^+,\, U_t < 0\}$ be the time of ruin with $T = \infty$ if ruin does not occur. Note that if ruin occurs, $|U_T|$ is the deficit at ruin and U_{T-1} is the surplus one period prior to ruin. Denote by

$$m(u\,|\,i) = \mathrm{E}\big[v^T w(U_{T-1}, |U_T|)\mathbf{1}_{\{T<\infty\}}\,|\,U_0 = u, I_0 = i\big], \quad i = 0, 1,$$

the conditional Gerber-Shiu discounted penalty function, where $\mathbf{1}_{\{A\}}$ is the indicator function of event A and $w(i,j) : \mathbb{N} \times \mathbb{N}^+ \to \mathbb{N}$ is a bounded function. The unconditional Gerber-Shiu function is defined as

$$m(u) = \mathrm{E}\big[v^T w(U_{T-1}, |U_T|)\mathbf{1}_{\{T<\infty\}}\,|\,U_0 = u\big],$$

then it is clear that

$$m(u) = (1-q)m(u\,|\,0) + qm(u\,|\,1). \tag{3}$$

3 Main Results

In this section, we aim to derive recursive equations for the conditional expected discounted penalty function $m(u\,|\,i)$, $i = 0, 1$. Similar to the one in [3], we study the claim occurrences in two scenarios. The first is a main claim and its associated by-claim occur concurrently, then the surplus process gets renewed. The second case is that a main claim occurs but its associated by-claim will be delayed to the next time period. Conditional on the second scenario, we define a complementary surplus process

$$U_{1t} = u + t - U_t^X - U_t^Y - Y, \quad t \in \mathbb{N}^+, \tag{4}$$

with $U_{10} = u$. Denote the corresponding conditional penalty function of process (4) by $m_1(u\,|\,i)$, $i = 0, 1$.

3.1 The generating function of the conditional penalty function

We first show that explicit expressions for the generating functions of the conditional expected discounted penalty functions can be obtained. Considering what will happen at time 1, we have

$$\begin{aligned} m(u \mid 0) &= vp_{00} m(u+1 \mid 0) \\ &\quad + vp_{01}\big(\sigma(u) + (1-\theta) w_1(u+1) + \theta w_3(u+1)\big), \end{aligned} \tag{5}$$

$$\begin{aligned} m(u \mid 1) &= vp_{10} m(u+1 \mid 0) \\ &\quad + vp_{11}\big(\sigma(u) + (1-\theta) w_1(u+1) + \theta w_3(u+1)\big), \end{aligned} \tag{6}$$

$$\begin{aligned} m_1(u \mid 1) &= vp_{10}\big(\sigma_1(u) + w_2(u+1)\big) \\ &\quad + vp_{11}\big(\sigma_2(u) + (1-\theta) w_3(u+1) + \theta w_4(u+1)\big), \end{aligned} \tag{7}$$

where

$$\sigma(u) = (1-\theta)\sum_{k=1}^{u+1} m_1(u+1-k \mid 1) f_X(k) + \theta \sum_{k=2}^{u+1} m(u+1-k \mid 1) f_{X*Y}(k),$$

$$\sigma_1(u) = \sum_{k=1}^{u+1} m(u+1-k \mid 0) f_Y(k),$$

$$\sigma_2(u) = (1-\theta)\sum_{k=2}^{u+1} m_1(u+1-k \mid 1) f_{X*Y}(k) + \theta \sum_{k=3}^{u+1} m(u+1-k \mid 1) f_{X*Y*Y}(k),$$

and

$$w_1(i) = \sum_{k=i+1}^{\infty} w(i-1, k-i) f_X(k), \quad w_2(i) = \sum_{k=i+1}^{\infty} w(i-1, k-i) f_Y(k),$$

$$w_3(i) = \sum_{k=i+1}^{\infty} w(i-1, k-i) f_{X*Y}(k), \quad w_4(i) = \sum_{k=i+1}^{\infty} w(i-1, k-i) f_{X*Y*Y}(k).$$

Throughout the rest of the paper, we will denote the generating function of a function by adding a hat on the corresponding letter. Multiplying (5) by z^{u+1} and summing over u from 0 to ∞, we get

$$\begin{aligned} z\hat{m}(z \mid 0) &= vp_{00}\big(\hat{m}(z \mid 0) - m(0 \mid 0)\big) + vp_{01}(1-\theta)\big(\hat{m}_1(z \mid 1)\hat{f}_X(z) + \hat{w}_1(z)\big) \\ &\quad + vp_{01}\theta\big(\hat{m}(z \mid 1)\hat{f}_{X*Y}(z) + \hat{w}_3(z)\big), \end{aligned}$$

which is equivalent to

$$\begin{aligned} &\Big(\frac{z}{v} - p_{00}\Big)\hat{m}(z \mid 0) - p_{01}\theta \hat{f}_{X*Y}(z)\hat{m}(z \mid 1) \\ &= p_{01}(1-\theta)\hat{f}_X(z)\hat{m}_1(z \mid 1) + p_{01}\hat{w}_{13}(z) - p_{00} m(0 \mid 0), \end{aligned} \tag{8}$$

where $w_{ij}(k) = (1-\theta)w_i(k) + \theta w_j(k)$ for $k \in \mathbb{N}^+$ and $1 \le i < j \le 4$.

For the other two processes (6) and (7), we have analogously

$$\begin{aligned}&- p_{10}\hat{m}(z\,|\,0) + \Big(\frac{z}{v} - p_{11}\theta \hat{f}_{X*Y}(z)\Big)\hat{m}(z\,|\,1)\\ &= p_{11}(1-\theta)\hat{f}_X(z)\hat{m}_1(z\,|\,1) + p_{11}\hat{w}_{13}(z) - p_{10}m(0\,|\,0),\end{aligned} \tag{9}$$

and

$$\begin{aligned}&- p_{10}\hat{f}_Y(z)\hat{m}(z\,|\,0) - p_{11}\theta \hat{f}_{X*Y*Y}(z)\hat{m}(z\,|\,1)\\ &= \Big(\frac{z}{v} - p_{11}(1-\theta)\hat{f}_{X*Y}(z)\Big)\hat{m}_1(z\,|\,1) + p_{10}\hat{w}_2(z) + p_{11}\hat{w}_{34}(z).\end{aligned} \tag{10}$$

In what follows we denote the determinant of a matrix $\boldsymbol{A}$ by $|\boldsymbol{A}|$, denote its transposed matrix and adjoint matrix by $\boldsymbol{A}^{\mathrm{T}}$ and $\boldsymbol{A}^*$, respectively. Let

$$\boldsymbol{A}(z) = \begin{pmatrix} p_{00} & p_{01}\theta\hat{f}_{X*Y}(z) & p_{01}(1-\theta)\hat{f}_X(z)\\ p_{10} & p_{11}\theta\hat{f}_{X*Y}(z) & p_{11}(1-\theta)\hat{f}_X(z)\\ p_{10}\hat{f}_Y(z) & p_{11}\theta\hat{f}_{X*Y*Y}(z) & p_{11}(1-\theta)\hat{f}_{X*Y}(z)\end{pmatrix},$$

$$\boldsymbol{B}(z) = \begin{pmatrix} p_{01}\hat{w}_{13}(z) - p_{00}m(0\,|\,0)\\ p_{11}\hat{w}_{13}(z) - p_{10}m(0\,|\,0)\\ p_{10}\hat{w}_2(z) + p_{11}\hat{w}_{34}(z)\end{pmatrix},$$

and $\hat{\boldsymbol{m}}(z) = \big(\hat{m}(z\,|\,0), \hat{m}(z\,|\,1), \hat{m}_1(z\,|\,1)\big)^{\mathrm{T}}$, then we can rewrite the Eq.s (8)-(10) in the following matrix form

$$\boldsymbol{Q}(z)\hat{\boldsymbol{m}}(z) = v\boldsymbol{B}(z), \tag{11}$$

where $\boldsymbol{Q}(z) = z\boldsymbol{I} - v\boldsymbol{A}(z)$ and $\boldsymbol{I}$ is the identity matrix. Solving the linear system of equations (11) gives

$$\hat{\boldsymbol{m}}(z) = \frac{v}{|\boldsymbol{Q}(z)|}\boldsymbol{Q}^*(z)\boldsymbol{B}(z). \tag{12}$$

Lemma 3.1. *For $0 < v \le 1$, the generalized Lundberg's equation*

$$\frac{|\boldsymbol{Q}(z)|}{z^2} = 0, \tag{13}$$

has a real root in the interval $(vp_{00}, v]$, say $\rho = \rho(v)$.

Proof. After careful calculations we obtain

$$\frac{|\boldsymbol{Q}(z)|}{z^2} = z - vp_{00} + \frac{v}{z}\left(v\pi - zp_{11}\right)\hat{f}_{X*Y}(z). \tag{14}$$

By noting that

$$\frac{|\boldsymbol{Q}(vp_{00})|}{(vp_{00})^2} = -\frac{p_{01}p_{10}}{p_{00}} v\hat{f}_{X*Y}(vp_{00}) < 0,$$

$$\frac{|\boldsymbol{Q}(v)|}{v^2} = v\Big(p_{01} + (\pi - p_{11})\hat{f}_{X*Y}(v)\Big) = vp_{01}\Big(1 - \hat{f}_{X*Y}(v)\Big) \geq 0,$$

we conclude that there exists a real number $\rho = \rho(v)$ in $(vp_{00}, v]$ such that $\frac{1}{\rho^2}|\boldsymbol{Q}(\rho)| = 0$. □

To ultimately invert $\hat{\boldsymbol{m}}(z)$, we first need an explicit expression for $m(0\,|\,0)$. Since $|\boldsymbol{Q}(\rho)| = 0$, we know that there exists at least one nontrivial solution of the following equation

$$[\boldsymbol{Q}(\rho)]^{\mathrm{T}}\boldsymbol{X} = \boldsymbol{0}. \tag{15}$$

Solving the homogeneous linear system of equations (15), we get a non-zero particular solution $\boldsymbol{X}_0$ as

$$\boldsymbol{X}_0 = \big(vp_{10}\hat{f}_Y(\rho),\, \theta(\rho - vp_{00})\hat{f}_Y(\rho),\, (1-\theta)(\rho - vp_{00})\big)^{\mathrm{T}}.$$

Therefore, we obtain from (11) that $\boldsymbol{X}_0^{\mathrm{T}}\boldsymbol{B}(\rho) = 0$, which implies

$$m(0\,|\,0) = \frac{\left\{\begin{array}{c} (\theta p_{11}(\rho - vp_{00}) + vp_{01}p_{10})\hat{f}_Y(\rho)\hat{w}_{13}(\rho) \\ +(1-\theta)(\rho - vp_{00})\big(p_{10}\hat{w}_2(\rho) + p_{11}\hat{w}_{34}(\rho)\big) \end{array}\right\}}{p_{10}\big(\theta\rho + v(1-\theta)p_{00}\big)\hat{f}_Y(\rho)}. \tag{16}$$

Moreover, substituting

$$\rho - vp_{00} = \frac{v}{\rho}(p_{11}\rho - v\pi)\hat{f}_{X*Y}(\rho),$$

into (16), we have the following result for the conditional expected discounted penalty function $m(0\,|\,0)$.

Theorem 3.2. *For $u = 0$, it holds that*

$$\begin{aligned} m(0\,|\,0) = {} & \frac{1}{p_{10}\rho\big(\theta\rho + v(1-\theta)p_{00}\big)}\big\{\rho\big(\theta h(\rho) + v(1-\theta)p_{01}p_{10}\big)\hat{w}_{13}(\rho) \\ & + v(1-\theta)h(\rho)\hat{f}_X(\rho)\big(p_{10}\hat{w}_2(\rho) + p_{11}\hat{w}_{34}(\rho)\big)\big\}, \end{aligned} \tag{17}$$

where $h(\rho) = \rho p_{11} - v\pi$.

On the other hand, one can easily compute that the adjoint matrix $\boldsymbol{Q}^*(z)$ of $\boldsymbol{Q}(z)$ has the form

$$\boldsymbol{Q}^*(z) = z(z - vp_{00})\boldsymbol{I} + v\boldsymbol{C}(z), \tag{18}$$

where

$$\boldsymbol{C}(z)=\begin{pmatrix} z\big(p_{00}-p_{11}\hat{f}_{X*Y}(z)\big) & zp_{01}\theta\hat{f}_{X*Y}(z) & zp_{01}(1-\theta)\hat{f}_X(z)\\ zp_{10} & -(1-\theta)h(z)\hat{f}_{X*Y}(z) & (1-\theta)h(z)\hat{f}_X(z)\\ zp_{10}\hat{f}_Y(z) & \theta h(z)\hat{f}_{X*Y*Y}(z) & -\theta h(z)\hat{f}_{X*Y}(z)\end{pmatrix}.$$

Now we are ready to give explicit expressions for the generating functions of the conditional expected discounted penalty functions. Let

$$\begin{aligned}\gamma(j)&=w_{13}(j)-v(1-\theta)p_{11}\sum_{i=1}^{j}f_{X*Y}(j-i+1)\,w_{13}(i)\\&\quad+v(1-\theta)\sum_{i=1}^{j}f_X(j-i+1)\big(p_{10}w_2(i)+p_{11}w_{34}(i)\big),\quad j\in\mathbb{N}^+,\end{aligned}$$

then $\hat{\gamma}(z)=\sum_{j=1}^{\infty}z^j\gamma(j)$ has the form

$$\begin{aligned}\hat{\gamma}(z)=&\left(1-v(1-\theta)p_{11}\frac{\hat{f}_{X*Y}(z)}{z}\right)\hat{w}_{13}(z)\\&+v(1-\theta)\frac{\hat{f}_X(z)}{z}\Big(p_{10}\hat{w}_2(z)+p_{11}\hat{w}_{34}(z)\Big).\end{aligned}$$

Substituting (18) into (12) we get

$$\hat{m}(z\,|\,0)=\frac{vz^2}{|\boldsymbol{Q}(z)|}\left[p_{01}\hat{\gamma}(z)+m(0\,|\,0)\left(v(\pi+(1-\theta)p_{01}p_{10})\frac{\hat{f}_{X*Y}(z)}{z}-p_{00}\right)\right],\quad(19)$$

and

$$\hat{m}(z\,|\,1)=\frac{vz}{|\boldsymbol{Q}(z)|}\left[h(z)\hat{\gamma}(z)+p_{10}m(0\,|\,0)\left(v(1-\theta)\frac{h(z)\hat{f}_{X*Y}(z)}{z}-z\right)\right],$$

where the constant $m(0\,|\,0)$ is determined by (17).

3.2 Recursive equation for $m(u\,|\,0)$

In this subsection, we aim to derive a defective renewal equation for $m(u\,|\,0)$. Setting $z=\rho$ in (19) yields

$$p_{00}m(0\,|\,0)=p_{01}\hat{\gamma}(\rho)+v\big(\pi+(1-\theta)p_{01}p_{10}\big)m(0\,|\,0)\frac{\hat{f}_{X*Y}(\rho)}{\rho}.\qquad(20)$$

Thus, substituting (20) into (19) yieds

$$\left[z-\rho-vp_{11}\big(\hat{f}_{X*Y}(z)-\hat{f}_{X*Y}(\rho)\big)+v^2\pi\left(\frac{\hat{f}_{X*Y}(z)}{z}-\frac{\hat{f}_{X*Y}(\rho)}{\rho}\right)\right]\hat{m}(z\,|\,0)$$
$$=vp_{01}\big(\hat{\gamma}(z)-\hat{\gamma}(\rho)\big)+v^2\big(\pi+(1-\theta)p_{01}p_{10}\big)m(0\,|\,0)\left(\frac{\hat{f}_{X*Y}(z)}{z}-\frac{\hat{f}_{X*Y}(\rho)}{\rho}\right),$$

which is equivalent to

$$z\hat{m}(z\,|\,0)=\frac{v\big(\rho h(z)\hat{f}_{X*Y}(z)-zh(\rho)\hat{f}_{X*Y}(\rho)\big)}{\rho(z-\rho)}\hat{m}(z\,|\,0)+vp_{01}\frac{z\big(\hat{\gamma}(z)-\hat{\gamma}(\rho)\big)}{z-\rho}$$
$$+v^2\big(\pi+(1-\theta)p_{01}p_{10}\big)m(0\,|\,0)\frac{\rho\hat{f}_{X*Y}(z)-z\hat{f}_{X*Y}(\rho)}{\rho(z-\rho)}. \tag{21}$$

For $j\in\mathbb{N}^+$, we define

$$\xi(j)=\frac{1}{\alpha_\rho}\left(\frac{v^2\pi}{\rho}f_{X*Y}(j)+v\frac{1-\hat{f}_{X*Y}(\rho)}{\rho(1-\rho)}h(\rho)\beta_\rho(j)\right),$$

where

$$\alpha_\rho=\frac{v^2\pi(1-\rho)+vh(\rho)\Big(1-\hat{f}_{X*Y}(\rho)\Big)}{\rho(1-\rho)},$$
$$\beta_\rho(j)=\left(\sum_{i=0}^{\infty}\rho^i f_{X*Y}(i+j)\right)\Bigg/\left(\sum_{i=0}^{\infty}\rho^i\bar{F}_{X*Y}(i)\right),$$

with $\bar{F}_{X*Y}(i)=\sum_{k=i+1}^{\infty}f_{X*Y}(k)$. Then it is easily seen that $\beta_\rho(j)$ is a p.f., and hence $\xi(j)$ is also a p.f. Moreover, direct calculations show that

$$\frac{v\big(\rho h(z)\hat{f}_{X*Y}(z)-zh(\rho)\hat{f}_{X*Y}(\rho)\big)}{\rho(z-\rho)}=\alpha_\rho\hat{\xi}(z).$$

Therefore, we can rewrite (21) as

$$z\hat{m}(z\,|\,0)=\alpha_\rho\hat{\xi}(z)\hat{m}(z\,|\,0)+vp_{01}\frac{z\big(\hat{\gamma}(z)-\hat{\gamma}(\rho)\big)}{z-\rho}$$
$$+v^2\big(\pi+(1-\theta)p_{01}p_{10}\big)m(0\,|\,0)\frac{\rho\hat{f}_{X*Y}(z)-z\hat{f}_{X*Y}(\rho)}{\rho(z-\rho)}. \tag{22}$$

The following theorem shows that $m(u\,|\,0)$ satisfies a recursive equation.

Theorem 3.3. *For $u \in \mathbb{N}$, we have*

$$m(u \,|\, 0) = \alpha_\rho \sum_{i=0}^{u} m(u-i \,|\, 0)\xi(i+1) + vp_{01} \sum_{i=u+1}^{\infty} \rho^{i-u-1}\gamma(i) + v^2\big(\pi + (1-\theta)p_{01}p_{10}\big)m(0 \,|\, 0) \sum_{i=u+2}^{\infty} \rho^{i-u-2} f_{X*Y}(i), \qquad (23)$$

where the constant $m(0 \,|\, 0)$ is determined by (17).

Proof. It is not different to see that

$$\hat{\xi}(z)\hat{m}(z \,|\, 0) = \sum_{j=1}^{\infty} z^j \sum_{i=1}^{j} m(j-i \,|\, 0)\xi(i),$$

$$\frac{z}{z-\rho}\big(\hat{\gamma}(z) - \hat{\gamma}(\rho)\big) = \sum_{j=1}^{\infty} z^j \sum_{i=0}^{\infty} \rho^i \gamma(i+j),$$

and

$$\frac{\rho \hat{f}_{X*Y}(z) - z\hat{f}_{X*Y}(\rho)}{z-\rho} = \frac{z}{z-\rho}\Big(\hat{f}_{X*Y}(z) - \hat{f}_{X*Y}(\rho)\Big) - \hat{f}_{X*Y}(z) = \sum_{j=1}^{\infty} z^j \sum_{i=1}^{\infty} \rho^i f_{X*Y}(i+j).$$

Therefore, we can rewrite (22) as

$$\sum_{j=0}^{\infty} z^{j+1} m(j \,|\, 0) = \alpha_\rho \sum_{j=0}^{\infty} z^{j+1} \sum_{i=0}^{j} m(j-i \,|\, 0)\xi(i+1) + vp_{01} \sum_{j=0}^{\infty} z^{j+1} \sum_{i=j+1}^{\infty} \rho^{i-j-1}\gamma(i) + \frac{v^2\big(\pi + (1-\theta)p_{01}p_{10}\big)}{\rho} m(0 \,|\, 0) \sum_{j=0}^{\infty} z^{j+1} \sum_{i=j+2}^{\infty} \rho^{i-j-1} f_{X*Y}(i),$$

which yields (23) by comparing the coefficients of z^{j+1}. □

3.3 The evaluation of $m(u \,|\, 1)$

It is known from (5) that

$$\frac{m(u \,|\, 0) - vp_{00}m(u+1 \,|\, 0)}{vp_{01}} = \sigma(u) + w_{13}(u+1). \qquad (24)$$

Substituting (24) into (6) yields

$$m(u \mid 1) = \frac{p_{11}}{p_{01}} m(u \mid 0) - \frac{v\pi}{p_{01}} m(u+1 \mid 0), \tag{25}$$

which is the relationship between the expected discounted penalty function in the ordinary renewal case and the delayed renewal case. We remark that in the classical compound Markov binomial risk model, the relationship between the expected discounted penalty function in the ordinary renewal case and the delayed renewal case has been discussed by [7].

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