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Study of chemostat model with delayed toxicant response and impulsive input

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Abstract

It this paper, a new Monod type chemostat model is considered. It is proved that microorganism-extinction periodic solution is globally attractive if the impulsive period satisfies some conditions. By introducing new study method, we prove that the system is permanent.

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1 Introduction

The chemostat is a simple and well-adopted laboratory apparatus used to culture microorganisms. It can be used to investigate microbial growth and has the advantage that parameters are easily measurable. It has began to occupy an increasing central role in ecological studies. Sterile growth medium enters the chemostat at a constant rate; the volume within the chemostat is preserved by allowing excess medium to flow out through a siphon. We inoculate this chemostat with a heterotrophic bacterium that finds, in the medium, a lot of all necessary nutrients but one. This last nutrient is the limiting substrate. The chemostat with impulsive input have been extensively investigated by many researcher([1, 2]). Much research, both theoretical and experimental, has been undertaken dealing with transient behavior of microbial population growth in the chemostat. While the Monod model [3] has some success in describing steady state growth rates (see [4]), it has been found inadequate to predict transients observed in chemostat experiments where the initial date is not at the globally attracting steady state. In recent years, many researchers pointed out that it was necessary and important to consider biological models with periodic perturbations, since these models might be quite naturally exposed in many real world phenomena. The chemostat models with impulsive input perturbation have been studied in ([5-6]). Many important and interesting results on the persistence, permanence and extinction of microorganisms, global stability, the existence of periodic oscillation are obtained.

While the most threatening problem is the change in both terrestrial and aquatic environment caused by the different kinds of stresses(toxicants, temperature, poflutants, etc.)affecting the long term survival of species, human life style and biodiversity of the habitat [7]. Presence of toxicant in the environments decreases the growth rate of species and its carrying capacity. In recent years,some investigations have been carried out to study the effect of toxicant on a single species population [8-9]. Most of the previous work assumed that input of toxicant was continuous. The toxicants, however, are often emitted to the environment with regular pulse [10]. A lot of data have indicated that the use of agriculture chemicals may cause potential harm to the health of both human beings and living beings. If the spraying of agriculture chemicals can be regarded as time pulse discharge, the continuous input of toxin can be regarded discharged and replaced by an impulsive perturbations. In this case, though the discharge of toxin is transient, the influence of the toxin will last long. Therefore, it is very important that how controls the pulse input cycle of toxin to protect the population persistent existence. The system approximates conditions for plankton growth in lakes are in a chemostat form, where the limiting nutrients such as silica and phosphate are supplied from streams draining the watershed.

In this paper we consider the following Monod type chemostat model with impulsive input and nutrient recycling in a polluted environment

$$\left\{\begin{array}{l}\left.\begin{array}{rcl}\dot{S}(t) &=& -DS(t)-\dfrac{\mu_1 S(t)x(t)}{\delta_1(K_1+S(t))}+brx(t)\\ \dot{x}(t) &=& -Dx(t)+\dfrac{\mu_1 S(t)x(t)}{K_1+S(t)}-\dfrac{\mu_2 c(t)x(t)}{\delta_2(K_2+x(t))}-rx(t)\\ \dot{c}(t) &=& -Dc(t)+\dfrac{\exp(-D\tau)\mu_2 x(t-\tau)c(t-\tau)}{K_2+x(t-\tau)}\end{array}\right\} t\neq nT,\\ \left.\begin{array}{rcl}S(t^+) &=& S(t)+p\\ x(t^+) &=& x(t)\\ c(t^+) &=& c(t)+q\end{array}\right\} t=nT,\end{array}\right. \tag{1.1}$$

where $t\in R_+=[0,\infty)$, $n\in Z_+$, Z_+ is the set of all positive integers; $S(t)$, $x(t)$ represent the concentration of limiting substrate, the microorganism at time t; $c(t)$ is the concentration of the toxicant in the chemostat; D is the dilution rate; δ_1 is the yield of the microorganism $x(t)$ per unit mass of substrate; δ_2 is the uptake constant of the toxicant $c(t)$ per unit mass of substrate; p, q is the amount of limiting substrate pulsed each T, T is the period of pulsing; μ_1, μ_2 is the maximum specific growth rate of the microorganism and toxicant; K_1, K_2 is the so-called half-saturation constant. Obviously, we have $0 \le \delta_1 \le 1$. $0 \le b \le 1$, D, μ_1, μ_2, K_1, K_2 are all positive constants.

Motivated by the application of the system (1.1) to population dynamics, we assume that solution of the system (1.1) satisfy the initial conditions:

$$\Big(\phi_1(s),\ \phi_2(s),\ \phi_3(s)\Big) \in C_+ = C\Big([-\tau, 0], R^3_+\Big), \phi_i(0) > 0 (i = 1, 2, 3). \quad (1.2)$$

Obviously, model(1.1) is the more general one of that in ([4]). This paper is organized as follows. In Section 2, we introduce some useful notations and lemmas. In Section 3, We will establish some new criteria on the permanence and global asymptotic stability for (1.1).

2 Preliminary results

In this section, we will give some notations and lemmas which will be used for our main results.

Let $R^3_+ = \{(x_1,\ x_2,\ x_3) \in R^3 : x_1 > 0, x_2 > 0, x_3 > 0\}$. $S(nT^+) = \lim\limits_{t\to nT^+} S(t)$, $x(nT^+) = \lim\limits_{t\to nT^+} x(t)$ and $c(nT^+) = \lim\limits_{t\to nT^+} c(t)$. $S(t)$, $c(t)$ is left continuous at $t = nT$ and $x(t)$ is continuous at $t = nT$.

We consider the following impulsive differential system

$$\begin{cases} \dot{u}(t) = -Du(t) & t \ne nT, \\ u(t^+) = u(t) + p & t = nT. \end{cases} \quad (2.1)$$

Using the similar way of proof of Lemma 2.2 and Lemma 2.3 in [4], we can prove the following Lemmas.

Lemma 2.1 *Assume that T is positive constant. Then System (2.1) has a positive periodic solution $u^*(t)$ for all $t \in (nT, (n+1)T]$ and $n \in Z_+$, which is globally uniformly attractive, where*

$$u^*(t) = \frac{p\exp\Big(-D(t-nT)\Big)}{1-\exp(-DT)},$$

Lemma 2.2 *For any positive solution $\Big(S(t),\ x(t),\ c(t)\Big)$ of system (1.1) with the initial value (1.2), then there exist constants $L > 0$, such that $S(t) \le L$, $x(t) \le L$ where $L = \frac{p\exp(-DT)}{\exp(DT)-1}$ and $0 < m < c(t) < M$ where $m = \frac{q\exp(-DT)}{1-\exp(-DT)}$ and $M = \delta_2\exp(-DT)L$.*

3 Main Results

The solution of system (1.1) corresponding to $x(t) = 0$ is called microorganism-free periodic solution. For system (1.1), if we choose $x(t) \equiv 0$, then system (1.1) becomes to the following system

$$\begin{cases} \dot{S}(t) = -DS(t), & t \neq nT, n \in Z_+, \\ \dot{c}(t) = -Dc(t), & t =\neq nT, n \in Z_+, \\ S(t^+) = S(t) + p, & t = nT, n \in Z_+, \\ c(t^+) = c(t) + q, & t = nT, n \in Z_+. \end{cases} \tag{3.1}$$

System (3.1) has a unique global uniformly attractive positive solution $\Big(S^*(t),\ c^*(t)\Big)$, where

$$S^*(t) = \frac{p\exp(-D(t-nT))}{1-\exp(-DT)}, c^*(t) = \frac{q\exp(-D(t-nT))}{1-\exp(-DT)}.$$

Hence, system (1.1) has a positive periodic solution $\Big(S^*(t),\ 0,\ c^*(t)\Big)$ at which microorganism culture fails. In the following, we will study the global asymptotical stability of the microorganism-free periodic solution $\Big(S^*(t),\ 0,\ c^*(t)\Big)$ as a solution of system (1.1).

Theorem 3.1 *Suppose*

$$\int_0^T \Big(\frac{\mu_1 S^*(t)}{K_1 + S^*(t)} - (D + r_1 + \frac{\mu_2 c^*(t)}{\delta_2(K_2+L)})\Big)dt \leq 0, \tag{3.2}$$

then periodic solution $\Big(S^*(t),\ 0,\ c^*(t)\Big)$ *of system (1.1) is globally attractive.*

Proof. Let $\Big(S(t),\ x(t),\ c(t)\Big)$ be any positive solution of system (1.1). Define a function as follows

$$W_1(t) = S(t) + \frac{1}{\delta}x(t),$$

similar to the proof of Lemma 2.3, we obtain $W_1(t) \leq u(t)$ for all $t \geq 0$, where $u(t)$ is the solution of system (2.1) and $u(t) \to u^*(t)$ as $t \to \infty$.

Hence, there exists a function $\alpha(t) : R_+ \to R$ satisfying $\alpha(t) \to 0$ as $t \to \infty$ such that

$$W_1(t) \leq u(t) = u^*(t) + \alpha(t)$$

for all $t \geq 0$. From the definition of $W_1(t)$ we have

$$S(t) \leq u^*(t) + \alpha(t) - \frac{1}{\delta}x(t) = S^*(t) + \alpha(t) - \frac{1}{\delta}x(t).$$

From condition (3.2), for any small enough $\varepsilon > 0$, we have

$$\int_0^T \Big(\frac{\mu_1(S^*(t) - \frac{\varepsilon}{\delta})}{K_1 + S^*(t) - \frac{\varepsilon}{\delta}} - (D + r_1 + \frac{\mu_2}{\delta_2(K_2+L)}(c^*(t) - \varepsilon))\Big)dt \leq 0. \tag{3.3}$$

It follows from the third equation of system (1.1) that

$$\begin{cases} \dot{c}(t) \geq -Dc(t), & t \neq nT, \\ c(t^+) = c(t) + q, & t = nT. \end{cases}$$

Using Lemma 2.2 and the comparison theorem of impulsive differential equation, for $\varepsilon > 0$ in (3.3), there exists a $T_1 > 0$ such that

$$c(t) \geq u(t) > c^*(t) - \varepsilon \quad \text{for all} \quad t \geq T_1,$$

where $u(t)$ is the solution of (2.1) with initial condition $u(0^+) = c(0^+)$ substituting p in (2.1) with q. Thus, from the second equation of system (1.1) we have

$$\dot{x}(t) \leq x(t)\Big[\frac{\mu(S^*(t) + \alpha(t) - \frac{1}{\delta}x(t))}{K + S^*(t) + \alpha(t) - \frac{1}{\delta}x(t)} - (D + r_1 + \frac{\mu_2}{\delta_2(K_2 + L)}(c^*(t) - \varepsilon))\Big] \quad (3.4)$$

for all $t \geq T_1$. Since $\lim_{t\to\infty} \alpha(t) = 0$, we obtain

$$\begin{aligned} &\lim_{t\to\infty} \int_t^{t+T} \Big[\frac{\mu(S^*(v) + \alpha(v) - \frac{\varepsilon}{\delta})}{K + S^*(v) + \alpha(v) - \frac{\varepsilon}{\delta}} - (D + r_1 + \frac{\mu_2}{\delta_2(K_2 + L)}(c^*(v) - \varepsilon))\Big]dt \\ &= \int_0^T \Big[\frac{\mu(S^*(t) - \frac{\varepsilon}{\delta})}{K + S^*(t) - \frac{\varepsilon}{\delta}} - (D + r_1 + \frac{\mu_2}{\delta_2(K_2 + L)}(c^*(t) - \varepsilon))\Big]dt \\ &\leq 0. \end{aligned}$$

Hence, there exist constants $\eta > 0$ and $T_2 > T_1$ such that

$$\int_t^{t+T} \Big[\frac{\mu(S^*(v) + \alpha(v) - \frac{\varepsilon}{\delta})}{K + S^*(v) + \alpha(v) - \frac{\varepsilon}{\delta}} - (D + r_1 + \frac{\mu_2}{\delta_2(K_2 + L)}(c^*(v) - \varepsilon))\Big]dt \leq -\eta \quad (3.5)$$

for all $t > T_2$ and $|\alpha(t)| < 1$.

If $x(t) \geq \varepsilon_0$ for all $t \geq T_2$, then from (3.4) we have

$$\dot{x}(t) \leq x(t)\Big[\frac{\mu(S^*(t) + \alpha(t) - \frac{\varepsilon}{\delta})}{K + S^*(t) + \alpha(t) - \frac{\varepsilon}{\delta}} - (D + r_1 + \frac{\mu_2}{\delta_2(K_2 + L)}(c^*(t) - \varepsilon))\Big]. \quad (3.6)$$

For all $t \geq T_2$, we can choose an integer $p \geq 0$ such that $t \in [T_2 + pT, T_2 + (p + 1)T)$, integrating (3.6) from T_0 to t, and from (3.5) we have

$$\begin{aligned} x(t) &\leq x(T_2)\exp\Big[\int_{T_2}^t \Big(\frac{\mu(S^*(v) + \alpha(v) - \frac{\varepsilon}{\delta})}{K + S^*(v) + \alpha(v) - \frac{\varepsilon}{\delta}} \\ &\qquad -(D + r_1 + \tfrac{\mu_2}{\delta_2(K_2+L)}(c^*(v) - \varepsilon))\Big)dv\Big] \\ &= x(T_2)\exp\Big(\int_{T_2}^{T_2+pT} + \int_{T_2+pT}^t\Big)\Big[\frac{\mu(S^*(v) + \alpha(v) - \frac{\varepsilon}{\delta})}{K + S^*(v) + \alpha(v) - \frac{\varepsilon}{\delta}} \\ &\qquad -(D + r_1 + \tfrac{\mu_2}{\delta_2(K_2+L)}(c^*(v) - \varepsilon))\Big)dv\Big] \\ &\leq x(T_2)\exp(-\eta p)\exp\{\int_{T_2+pT}^t \Big(\frac{\mu(S^*(v) + \alpha(v) - \frac{\varepsilon}{\delta})}{K + S^*(v) + \alpha(v) - \frac{\varepsilon}{\delta}} \\ &\qquad -(D + r_1 + \tfrac{\mu_2}{\delta_2(K_2+L)}(c^*(v) - \varepsilon))\Big)dv\} \\ &\leq x(T_2)\exp(-\eta p)\exp(\sigma_0 T), \end{aligned} \quad (3.7)$$

where $\sigma_0 = \frac{\mu(M+1-\frac{\varepsilon}{\delta})}{K+M+1-\frac{\varepsilon}{\delta}} + \varepsilon$. Since $p \to \infty$ as $t \to \infty$, we obtain $x(t) \to 0$ as $t \to \infty$ from (3.7), which leads to a contradiction. Hence, there exists a $t^* \geq T_2$ such that $x(t^*) < \varepsilon$.

Let $M_0 = \exp(\sigma_0 T)$, we claim that

$$x(t) \leq \varepsilon M_0 \quad \text{for all} \quad t \geq t^*.$$

In fact, if there exists a $t_1 \geq t^*$ such that $x(t_1) > \varepsilon M_0$, then there exists a $t_2 \in (t^*, t_1)$ such that $x(t_2) = \varepsilon$ and $x(t) > \varepsilon$ for $t \in (t_2, t_1)$. Choose an integer $p \geq 0$ such that $t_1 \in [t_2 + pT, t_2 + (p+1)T)$. Since for any $t \in (t_2, t_1)$

$$\dot{x}(t) \leq x(t)\Big(\frac{\mu(S^*(t) + \alpha(t) - \frac{\varepsilon}{\delta})}{K + S^*(t) + \alpha(t) - \frac{\varepsilon}{\delta}} - (D + r_1 + \frac{\mu_2}{\delta_2(K_2 + L)}(c^*(t) - \varepsilon))\Big),$$

integrating the above inequality from t_2 to t_1, from (3.5) we have

$$\begin{aligned} x(t_1) &\leq x(t_2)\exp\Big[\int_{t_2}^{t_1}\Big(\frac{\mu(S^*(t) + \alpha(t) - \varepsilon)}{K + S^*(t) + \alpha(t) - \varepsilon} \\ &\quad -(D + r_1 + \tfrac{\mu_2}{\delta_2(K_2+L)}(c^*(t) - \varepsilon))\Big)dt\Big] \\ &\leq x(t_2)\exp(-\eta p)\exp\Big(\int_{t_2+pT}^{t_1}(\frac{\mu(S^*(t) + \alpha(t) - \frac{\varepsilon}{\delta})}{K + S^*(t) + \alpha(t) - \frac{\varepsilon}{\delta}} \\ &\quad -(D + r_1 + \tfrac{\mu_2}{\delta_2(K_2+L)}(c^*(t) - \varepsilon)))dt\Big) \\ &\leq \varepsilon\exp(\sigma_0 T) = \varepsilon M_0. \end{aligned} \tag{3.8}$$

Obviously, from (3.8) we obtain a contradiction. Hence, $x(t) \leq \varepsilon M_0$ for all $t \geq t^*$. Since ε is arbitrary, we finally have $\lim\limits_{t\to\infty} x(t) = 0$. This completes the proof.

In what follows, we discuss the permanence of system (1.1), we have the following result.

Theorem 3.2 *System (1) is permanent, if*

$$\int_0^T\Big(\frac{\mu_1 S^*(t)}{K_1 + S^*(t)} - (D + r_1 + \frac{\mu_2 c^*(t)}{\delta_2(K_2 + L)})\Big)dt > 0. \tag{3.2}$$

Proof. Let $(S(t),\ x(t),\ y(t))$ be any solution of system (1.1) with initial value $(S(0^+),\ x(0),\ y(0)) \in R^3_+$. From Lemma 2.2, without lose of generality we can assume $S(t) < M$, $x(t) < M$, $y(t) < M$ for all $t \geq 0$. From first equation of system (1.1) we obtain

$$\begin{cases} \dot{S}(t) \geq -(D + \dfrac{\mu_1 M}{\delta_1 K_1})S(t), & t \neq nT, \\ S(t^+) = S(t) + p, & t = nT. \end{cases}$$

Consider the comparison system

$$\begin{cases} \dot{z}(t) = -(D + \dfrac{\mu_1 M}{\delta_1 K_1})z(t), & t \neq nT, \\ z(t^+) = z(t) + p, & t = nT \end{cases}$$

with initial value $z(0^+) = S(0^+)$. From Lemma 2.2 and the comparison theorem of impulsive differential equation, we obtain $S(t) \geq z(t)$ for all $t \geq 0$, where $z(t)$ is the solution of the comparison system. Then, we have $\lim\limits_{t\to+\infty} z(t) = z^*(t)$, where

$$z^*(t) = \frac{p\exp\Big(-(D + \dfrac{\mu_1 M}{\delta_1 K_1})(t - nT)\Big)}{1 - \exp\Big(-(D + \dfrac{\mu_1 M}{\delta_1 K_1})T\Big)}$$

for all $t \in (nT, (n+1)T]$ and $n \in Z_+$. Consequently,

$$\liminf_{t\to\infty} S(t) \geq \liminf_{t\to\infty} z(t) = z^*(t) \geq \frac{p\exp\Big(-(D + \dfrac{\mu_1 M}{\delta_1 K_1})T\Big)}{1 - \exp\Big(-(D + \dfrac{\mu_1 M}{\delta_1 K_1})T\Big)}.$$

This shows that $S(t)$ in system (1.1) is permanent.

From first equation of system (1.1) we obtain

$$\begin{cases} \dot{c}(t) = -Dc(t) & t \neq nT, \\ c(t^+) = c(t) + q & t = nT. \end{cases}$$

Using Lemma 2.2, we have

$$c(t) \geq c^*(t) = \frac{q\exp\Big(-D(t - nT)\Big)}{1 - \exp\Big(-(DT)\Big)} > \frac{q\exp\Big(-DT)\Big)}{1 - \exp\Big(-(DT)\Big)}$$

This shows that $c(t)$ in system (1.1) is permanent.

In the following, we prove that there exists a constant $m_2 > 0$ such that

$$\liminf_{t\to\infty} x(t) > m_2.$$

For any constant $m_3 > 0$, consider the following system

$$\begin{cases} \dot{y}(t) = -(D + \frac{\mu_1 m_3}{\delta_1 K_1})y(t), & t \neq nT, n \in Z_+, \\ y(t^+) = y(t) + p, & t = nT, n \in Z_+. \end{cases} \tag{3.8}$$

From Lemma 2.1, system (3.8) have a globally uniformly attractive positive T-periodic solution

$$y^*(t) = \frac{p\exp\Big(-(D + \frac{\mu_1 m_3}{\delta K_1})(t - nT)\Big)}{1 - \exp\Big(-(D + \frac{\mu_1 m_3}{\delta_1 K_1})T\Big)}, \quad t \in (nT, (n+1)T], \quad n \in Z_+.$$

Since $\lim_{m_3\to 0} y^*(t) = S^*(t)$, for above $\varepsilon_0 > 0$ there is a $m_3 > 0$ such that

$$y^*(t) \geq S^*(t) - \frac{\varepsilon_0}{2} \tag{3.9}$$

for all $t \geq 0$. Further, for above $\varepsilon_0 > 0$ and $M > 0$, there is a $T_0 = T_0(\varepsilon_0, M) > 0$ such that for any $t_0 \geq 0$ and $0 \leq y_0 \leq M$ we have

$$|y(t, t_0, y_0) - y^*(t)| < \frac{\varepsilon_0}{2} \tag{3.10}$$

for all $t \geq t_0 + T_0$, where $y(t, t_0, y_0)$ is the solution of system (3.10) with initial condition $y(t_0) = y_0$.

For any $t_0 \geq 0$, if $x(t) \leq m_3$ for all $t \geq t_0$, then from system (1.1) we have

$$\begin{cases} \dot{S}(t) \geq -(D + \frac{\mu_1 m_3}{\delta_1 K_1})S(t), & t \neq nT, n \in Z_+, \\ S(t^+) = S(t) + p, & t = nT, n \in Z_+. \end{cases}$$

for all $t \geq t_0$. By the comparison theorem of impulsive differential equation, we have $S(t) \geq y(t)$ for all $t \geq t_0$, where $y(t)$ is the solution of system (3.11) with initial condition $y(t_0^+) = S(t_0^+)$. From (3.9) and (3.10) we have

$$S(t) \geq S^*(t) - \varepsilon_0 \quad \text{for all} \quad t \geq t_0 + T_0. \tag{3.11}$$

From the second equation of system (1.1) we get

$$\dot{x}(t) \geq x(t)\Big(\frac{\mu_1(S^*(t) - \varepsilon_0)}{K_1 + S^*(t) - \varepsilon_0} - (D + \frac{\mu_2 M}{\delta_2(K_2 + m)})\Big) \tag{3.12}$$

for all $t \geq t_0 + T_1$. Let $n_0 \in N$ such that $n_0 T > t_0 + T$. Integrating (3.12) on $(nT, (n+1)T]$ for all $n \geq n_0$, we have

$$\begin{aligned} x((n+1)T) &\geq x(nT^+)\exp(\int_{nT}^{(n+1)T} \Big(\frac{\mu_1(S^*(t) - \varepsilon_0)}{K_1 + S^*(t) - \varepsilon_0} - (D + \frac{\mu_2 M}{\delta_2(K_2 + m)})\Big)dt) \\ &= x(nT)\exp(\sigma T). \end{aligned}$$

Hence, $x((n_0 + k)T) \geq x(n_0 T)\exp(k\sigma)$ for all $k \geq 0$. Consequently, we have $\lim_{t\to\infty} x((n_0 + k)T) = \infty$, which is a contradiction. Hence, there exists a $t_1 \geq t_0 + T$ such that $x(t_1) \geq m_3$.

If $x(t) \geq m_3$ for all $t \geq t_1$, then the conclusion of Theorem 3.2 is proved. Hence, we need only to consider those solution $(S(t),\ x(t),\ c(t))$ of system (1.1) such that $x(t)$ is oscillatory about m_3. Let t_1 and t_2 be two large enough times such that $x(t_1) = x(t_2) = m_3$ and $x(t) < m_3$ for all $t \in (t_1, t_2)$. When $t_2 - t_1 \leq T$, since

$$\dot{x}(t) \geq -\Big(D + \frac{\mu_2 M}{\delta_2(K_2 + m)})\Big)x(t)$$

for all $t \in (t_1, t_2)$, integrating this inequality for any $t \in [t_1, t_2]$, we have

$$x(t) \geq m_3 \exp\Big(-(D + \frac{\mu_2 M}{\delta_2(K_2+m)})\Big) \doteq m_2^*. \tag{3.13}$$

Let $t_2 - t_1 > T_1$. For any $t \in [t_1, t_2]$, if $t \leq t_1 + T$, then according to the above discussing on the case $t_2 - t_1 \leq T$, we also have inequality (3.16). Particularly, we obtain $x(t_1 + T) \geq m_2^*$. Since $x(t) \leq m_3$ for all $t \in [t_1, t_2]$, from system (1) we have

$$\begin{cases} \dot{S}(t) \geq -(D + \frac{\mu_1 m_3}{\delta_1 K_1})S(t), & t \neq nT, n \in Z_+, \\ S(t^+) = S(t) + p, & t = nT, n \in Z_+. \end{cases}$$

Hence, from the comparison theorem of impulsive differential equations we have $S(t) \geq y(t)$ for all $t \in [t_1, t_2]$, where $y(t)$ is the solution of system (3.11) with initial condition $y(t_1^+) = S(t_1^+)$. From (3.9) we have

$$y(t) \geq y^*(t) - \frac{\varepsilon_0}{2} \quad \text{for all} \quad t \in [t_1 + T_1, t_2].$$

From (3.9) we have

$$S(t) \geq S^*(t) - \varepsilon_0 \quad \text{for all} \quad t \in [t_1 + T_1, t_2].$$

From system (1.1) we have

$$\dot{x}(t) \geq x(t)\Big(\frac{\mu(S^*(t) - \varepsilon_0)}{K + S^*(t) - \varepsilon_0} - (D + \frac{\mu_2 M}{\delta_2(K_2+m)})\Big) \tag{3.14}$$

for all $t \in [t_1 + T, t_2]$. For any $t \in [t_1 + T, t_2]$, we choose an integer $p \geq 0$ such that $t \in [t_1 + pT, t_1 + (p+1)T)$. Integrating (2.16) from $t_1 + T$ to t, we have

$$\begin{aligned} x(t) &= x(t_1 + T_1)\exp(\int_{t_1+T_1}^{t} \Big(\frac{\mu_1(S^*(v) - \varepsilon_0)}{K_1 + S^*(v) - \varepsilon_0} - (D + \frac{\mu_2 M}{\delta_2(K_2+m)})\Big)dv) \\ &\geq m_2^* \exp(-hT) \\ &\doteq m_2, \end{aligned}$$

where $h = \sup\limits_{t \geq 0}\Big(|\frac{\mu_1(S^*(t) - \varepsilon_0)}{K_1 + S^*(t) - \varepsilon_0} - (D + \frac{\mu_2 M}{\delta_2 K_2})|\Big)$. From above discussion, we finally obtain $x(t) \geq m_2$ as $t \to \infty$, and m_2 is independent of any solution $(S(t), (t), c(t))$ of system (1.1). The proof of Theorem 3.2 is completed.

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