Mathematica Aeterna, Vol. 6, 2016, no. 3, 359 - 363

3-Lie bialgebras (L_c, C_b) and (L_c, C_c)

BAI Ruipu

College of Mathematics and Information Science,
Hebei University, Baoding, 071002, China
email: bairuipu@hbu.edu.cn

LIN Lixin

College of Mathematics and Information Science,
Hebei University, Baoding, 071002, China
email: llxhebeidaxue@163.com

Abstract

In this paper, we continue to study the structure of four dimensional 3-Lie bialgebras. we obtain that there exist seven classes 3-Lie bialgebras of types (L_{c_i}, C_{b_j}), for $1 \leq 3$, $1 \leq j \leq 2$ (Theorem 3.2). And thirteen classes 3-Lie bialgebras of types (L_{c_i}, C_{c_j}), for $1 \leq i, j \leq 3$ (Theorem 3.3).

2010 Mathematics Subject Classification: 17B05 17D30
Keywords: 3-Lie algebra, 3-Lie coalgebra, 3-Lie bialgebra.

1 Preliminaries

A 3-**Lie algebra** [1] is a vector space L endowed with a linear multiplication $\mu : L^{\wedge 3} \rightarrow L$ satisfying that, for all $x, y, z, u, v \in L$,

$\mu(u, v, \mu(x, y, z)) = \mu(x, y, \mu(u, v, z)) + \mu(y, z, \mu(u, v, x)) + \mu(z, x, \mu(u, v, y))$.

For defining 3-Lie coalgebras, we need to define following linear maps

$\omega_i : L \otimes L \otimes L \otimes L \otimes L \rightarrow L \otimes L \otimes L \otimes L \otimes L$, $1 \leq i \leq 3$, by

$\omega_1(x_1 \otimes x_2 \otimes x_3 \otimes x_4 \otimes x_5) = x_3 \otimes x_4 \otimes x_1 \otimes x_2 \otimes x_5$,

$\omega_2(x_1 \otimes x_2 \otimes x_3 \otimes x_4 \otimes x_5) = x_4 \otimes x_5 \otimes x_1 \otimes x_2 \otimes x_3$,

$\omega_3(x_1 \otimes x_2 \otimes x_3 \otimes x_4 \otimes x_5) = x_5 \otimes x_3 \otimes x_1 \otimes x_2 \otimes x_4$.

A 3-Lie coalgebra (L, Δ) [2] is a vector space L with a linear map $\Delta : L \rightarrow L \otimes L \otimes L$ satisfying

$Im(\Delta) \subset L \wedge L \wedge L$, and $(1 - \omega_1 - \omega_2 - \omega_3)(1 \otimes 1 \otimes \Delta)\Delta = 0$.

Let (L_1, Δ_1) and (L_2, Δ_2) be 3-Lie coalgebras. If there is a linear isomorphism $\varphi : L_1 \to L_2$ satisfying $(\varphi \otimes \varphi \otimes \varphi)(\Delta_1(e)) = \Delta_2(\varphi(e))$, for all $e \in L_1$, then (L_1, Δ_1) is isomorphic to (L_2, Δ_2), and φ is called *a 3-Lie coalgebra isomorphism*, where $(\varphi \otimes \varphi \otimes \varphi) \sum\limits_i (a_i \otimes b_i \otimes c_i) = \sum\limits_i \varphi(a_i) \otimes \varphi(b_i) \otimes \varphi(c_i)$.

A 3-Lie bialgebra[2] is a triple (L, μ, Δ) such that
(1) (L, μ) is a 3-Lie algebra with the multiplication $\mu : L \wedge L \wedge L \to L$,
(2) (L, Δ) is a 3-Lie coalgebra with $\Delta : L \to L \wedge L \wedge L$,
(3) Δ and μ satisfy the following identity, for $x, y, u, v, w \in L$,

$\Delta\mu(x, y, z) = ad^{(3)}_\mu(x, y)\Delta(z) + ad^{(3)}_\mu(y, z)\Delta(x) + ad^{(3)}_\mu(z, x)\Delta(y)$,

where $ad^{(3)}_\mu(x, y)$, $ad^{(3)}_\mu(z, x)$, $ad^{(3)}_\mu(y, z) : L \otimes L \otimes L \to L \otimes L \otimes L$ are linear maps defined by (similar for $ad^{(3)}_\mu(z, x)$ and $ad^{(3)}_\mu(y, z)$)

$ad^{(3)}_\mu(x, y)(u \otimes v \otimes w) = (ad_\mu(x, y) \otimes 1 \otimes 1)(u \otimes v \otimes w)$
$+(1 \otimes ad_\mu(x, y) \otimes 1)(u \otimes v \otimes w) + (1 \otimes 1 \otimes ad_\mu(x, y))(u \otimes v \otimes w)$
$= \mu(x, y, u) \otimes v \otimes w + u \otimes \mu(x, y, v) \otimes w + u \otimes v \otimes \mu(x, y, w)$.

Lemma 2.1[1] *Let (L, μ) be a 4-dimensional 3-Lie algebra with $\dim L^1 \neq 0, 2$, and e_1, e_2, e_3, e_4 be a basis of L. Then L is isomorphic to one and only one of the following*

$L_{b_1}.\mu(e_2, e_3, e_4) = e_1$. $L_{b_2}.\mu(e_1, e_2, e_3) = e_1$.
L_d. $\mu_d(e_2, e_3, e_4) = e_1, \mu_d(e_1, e_3, e_4) = e_2, \mu_d(e_1, e_2, e_4) = e_3$.
L_e. $\mu_e(e_2, e_3, e_4) = e_1, \mu_e(e_1, e_3, e_4) = e_2, \mu_e(e_1, e_2, e_4) = e_3, \mu_e(e_1, e_2, e_3) = e_4$.

2 3-Lie bialgebras of types (L_c, C_b) and (L_c, C_c)

First We give the classification of 3-Lie coalgebras of the types (L, C_b) and (L, C_c).

Lemma 3.1 [3][4] *Let (L, Δ) be a 4-dimensional 3-Lie coalgebra with m-dimensional derived algebra ($m \leq 2$), and e^1, e^2, e^3, e^4 be a basis of L. Then L isomorphic to one and only one of the $C_{b_1} = (L, \Delta_{b_1})$ and $C_{b_2} = (L, \Delta_{b_2})$, $C_{c_1} = (L, \Delta_{c_1})$,$C_{c_2} = (L, \Delta_{c_2})$, $C_{c_3} = (L, \Delta_{c_3})$,*

C_{b_1}. $\Delta_{b_1}(e^1) = e^2 \wedge e^3 \wedge e^4$; C_{b_2}. $\Delta_{b_2}(e^1) = e^1 \wedge e^2 \wedge e^3$;
C_{c_1}. $\Delta_{c_1}(e^1) = e^2 \wedge e^3 \wedge e^4$, $\Delta_{c_1}(e^2) = e^1 \wedge e^3 \wedge e^4$;
C_{c_2}. $\Delta_{c_2}(e^1) = \alpha e^2 \wedge e^3 \wedge e^4$, $\Delta_{c_2}(e^2) = e^2 \wedge e^3 \wedge e^4 + e^1 \wedge e^3 \wedge e^4$;
C_{c_3}. $\Delta_{c_3}(e^1) = e^1 \wedge e^3 \wedge e^4$, $\Delta_{c_3}(e^2) = e^2 \wedge e^3 \wedge e^4$; $\alpha \in F, \alpha \neq 0$.

For convenience, in the following, for a 3-Lie bialgebra (L, μ, Δ), if the 3-Lie algebra (L, μ) is the case (L, μ_{c_i}) in Lemma 2.1 and the 3-Lie coalgebra (L, Δ) is the case (L, Δ_b) and (L, Δ_c) in Lemma 3.1, then the 3-Lie bialgebra (L, μ_{c_i}, Δ_b) and (L, μ_{c_i}, Δ_c) are simply denoted by (L_{c_i}, C_b) and (L_{c_i}, C_c), which are called *the 3-Lie bialgebras of type (L_c, C_b), and (L_c, C_c)*, respectively.

For a given 3-Lie algebra L, in order to find all the 3-Lie bialgebra structures on L, we should find all the 3-Lie coalgebra structures on L which are compatible with the 3-Lie algebra L. Although a permutation of a basis of L gives isomorphic 3-Lie coalgebra, but it may lead to the non-equivalent 3-Lie bialgebra.

Theorem 3.2 *The only non-equivalent 3-Lie bialgebras of the types (L_{c_i}, C_{b_j}) for $i = 1, 2, 3$, $j = 1, 2$ are as follows:*

$(L_{c_1}, C_{b_1}, \Delta_1)$: $\Delta_1(e_3) = e_1 \wedge e_2 \wedge e_4$. $(L_{c_1}, C_{b_2}, \Delta_2)$: $\Delta_2(e_3) = e_3 \wedge e_1 \wedge e_2$.
$(L_{c_2}, C_{b_1}, \Delta_1)$: $\Delta_1(e_3) = e_1 \wedge e_2 \wedge e_4$. $(L_{c_2}, C_{b_2}, \Delta_2)$: $\Delta_2(e_3) = e_3 \wedge e_1 \wedge e_2$.
$(L_{c_3}, C_{b_1}, \Delta_1)$: $\Delta_1(e_3) = e_1 \wedge e_2 \wedge e_4$. $(L_{c_3}, C_{b_1}, \Delta_3)$: $\Delta_5(e_1) = e_2 \wedge e_3 \wedge e_4$.
$(L_{c_3}, C_{b_2}, \Delta_2)$: $\Delta_2(e_3) = e_3 \wedge e_1 \wedge e_2$.

Proof From Lemma 3.1, we need to verify that whether the following eight 3-Lie coalgebras of type C_{b_1}, which are obtained by permuting the basis e_1, e_2, e_3, e_4, are compatible with the 3-Lie algebra L_{c_i}, $i = 1, 2, 3$, respectively,

(1) $\Delta(e_1) = e_2 \wedge e_3 \wedge e_4$; (2) $\Delta(e_1) = e_2 \wedge e_4 \wedge e_3$;
(3) $\Delta(e_2) = e_1 \wedge e_4 \wedge e_3$; (4) $\Delta(e_2) = e_1 \wedge e_3 \wedge e_4$;
(5) $\Delta(e_3) = e_1 \wedge e_2 \wedge e_4$; (6) $\Delta(e_3) = e_1 \wedge e_4 \wedge e_2$;
(7) $\Delta(e_4) = e_1 \wedge e_2 \wedge e_3$; (8) $\Delta(e_4) = e_2 \wedge e_1 \wedge e_3$.

By a direct computation, only cases (5), (6), (7) and (8) of the type C_{b_1} are compatible with the 3-Lie algebra L_{c_1}, the cases (5), (6), (7) and (8) of the type C_{b_1} are compatible with the 3-Lie algebra L_{c_2}, and the eight cases of the type C_{b_1} are compatible with the 3-Lie algebra L_{c_3}.

Thanks to the following isomorphisms of the 3-Lie algebra L_{c_1}, $f: L \to L$, where
(L_{c_1}, C_{b_1}): $(5) \to (6), (7) \to (8) : f(e_1) = e_2, f(e_2) = e_1, f(e_3) = e_3, f(e_4) = e_4$;
$(5) \to (8) : f(e_1) = -e_1, f(e_2) = e_2, f(e_3) = e_4, f(e_4) = e_3$;
(L_{c_2}, C_{b_1}): $(5) \to (8), (6) \to (7) : f(e_1) = e_1, f(e_2) = e_2, f(e_3) = -e_4, f(e_4) = e_3$;
$(5) \to (6) : f(e_1) = \frac{1}{\sqrt{\alpha}} e_2, f(e_2) = \sqrt{\alpha} e_1 + \frac{1}{\sqrt{\alpha}} e_2, f(e_3) = -\sqrt{-1} e_3,$
$f(e_4) = \sqrt{-1} e_4$;
$(L_{c_3}, C_{b_1}) : (1) \to (2) : f(e_1) = e_1, f(e_2) = -e_2, f(e_3) = e_3, f(e_4) = e_4$;
$(1) \to (3), (2) \to (4), (5) \to (6) : f(e_1) = e_2, f(e_2) = e_1, f(e_3) = e_3,$
$f(e_4) = e_4$;
$(5) \to (8), (6) \to (7) : f(e_1) = e_1, f(e_2) = e_2, f(e_3) = e_4, f(e_4) = -e_3$;

we get the non-equivalent 3-Lie bialgebras $(L_{c_1}, C_{b_1}, \Delta_1)$, and the non-equivalent 3-Lie bialgebras $(L_{c_2}, C_{b_1}, \Delta_1)$. Since e_3 is not contained in the derived algebra of the 3-Lie algebra, we get the non-equivalent 3-Lie bialgebras $(L_{c_3}, C_{b_1}, \Delta_1)$, $(L_{c_3}, C_{b_1}, \Delta_3)$.

By Lemma 3.1, we need to verify that wether the twenty fourt isomorphic 3-Lie coalgebras of the type C_{b_1} are compatible with the 3-Lie algebra L_{c_i}, $i = 1, 2, 3$, respectively, (we omit the zero product)

(1) $\Delta e_1 = e_1 \wedge e_2 \wedge e_3$, (2) $\Delta e_1 = e_1 \wedge e_2 \wedge e_4$, (3) $\Delta e_1 = e_1 \wedge e_3 \wedge e_4$,
(4) $\Delta e_1 = e_1 \wedge e_3 \wedge e_2$, (5) $\Delta e_1 = e_1 \wedge e_4 \wedge e_2$, (6) $\Delta e_1 = e_1 \wedge e_4 \wedge e_3$,
(7) $\Delta e_2 = e_2 \wedge e_1 \wedge e_3$, (8) $\Delta e_2 = e_2 \wedge e_1 \wedge e_4$, (9) $\Delta e_2 = e_2 \wedge e_3 \wedge e_4$,
(10)$\Delta e_2 = e_2 \wedge e_3 \wedge e_1$, (11)$\Delta e_2 = e_2 \wedge e_4 \wedge e_1$, (12)$\Delta e_2 = e_2 \wedge e_4 \wedge e_3$,
(13)$\Delta e_3 = e_3 \wedge e_1 \wedge e_2$, (14)$\Delta e_3 = e_3 \wedge e_1 \wedge e_4$, (15)$\Delta e_3 = e_3 \wedge e_2 \wedge e_4$,
(16)$\Delta e_3 = e_3 \wedge e_2 \wedge e_1$, (17)$\Delta e_3 = e_3 \wedge e_4 \wedge e_1$, (18)$\Delta e_3 = e_3 \wedge e_4 \wedge e_2$,
(19)$\Delta e_4 = e_4 \wedge e_1 \wedge e_2$, (20)$\Delta e_4 = e_4 \wedge e_1 \wedge e_3$, (21)$\Delta e_4 = e_4 \wedge e_2 \wedge e_3$,
(22)$\Delta e_4 = e_4 \wedge e_2 \wedge e_1$, (23)$\Delta e_4 = e_4 \wedge e_3 \wedge e_1$, (24)$\Delta e_4 = e_4 \wedge e_3 \wedge e_2$.

By a direct computation, only cases (13), (16), (19) and (22) of the type C_{b_2} are compatible with the 3-Lie algebra L_{c_1}, L_{c_2}, and L_{c_3}, respectively.

Thanks to the following isomorphisms of the 3-Lie algebra
(L_{c_1}, C_{b_2}) $(13) \to (22), (16) \to (19) : f(e_1) = -e_1, f(e_2) = e_2, f(e_3) = e_4, f(e_4) = e_3$;
$(13) \to (16) : f(e_1) = -e_1, f(e_2) = e_2, f(e_3) = e_3, f(e_4) = -e_4$;
(L_{c_2}, C_{b_2}) $(13) \to (19), (16) \to (22) : f(e_3) = e_4, f(e_4) = -e_3, f(e_i) = e_i, i = 1, 2$;

(13) $\to$ (16) : $f(e_1)=\frac{1}{\sqrt{\alpha}}e_2, f(e_2)=\sqrt{\alpha}e_1+\frac{1}{\sqrt{\alpha}}e_2, f(e_i)=e_i, i=3,4;$
(L_{c_3}, C_{b_2}): (13) $\to$ (16), (19) $\to$ (22) : $f(e_1)=e_2, f(e_2)=e_1, f(e_i)=e_i, i=1,2;$
(13) $\to$ (19) : $f(e_1)=e_1, f(e_2)=e_2, f(e_3)=e_4, f(e_4)=-e_3;$
the non-equivalent 3-Lie bialgebras are $(L_{c_1}, C_{b_2}, \Delta_2)$, $(L_{c_2}, C_{b_2}, \Delta_2)$, $(L_{c_3}, C_{b_2}, \Delta_2)$. The proof is complete.

Theorem 3.3 *The non-equivalent* 3*-Lie bialgebras of the types* (L_{c_j}, C_{c_k}) *for* $1\le j,k\le 3$ *are as follows:*

$(L_{c_1}, C_{c_1}, \Delta_1)$ $\Delta_1(e_3)=e_4\wedge e_1\wedge e_2$, $\Delta_1(e_4)=e_3\wedge e_1\wedge e_2$;
$(L_{c_1}, C_{c_3}, \Delta_3)$ $\Delta_3(e_3)=e_3\wedge e_1\wedge e_2$, $\Delta_3(e_4)=e_4\wedge e_1\wedge e_2$;
$(L_{c_1}, C_{c_3}, \Delta_4)$ $\Delta_4(e_1)=e_1\wedge e_3\wedge e_4$, $\Delta_4(e_2)=e_2\wedge e_3\wedge e_4$;
$(L_{c_1}, C_{c_3}, \Delta_5)$ $\Delta_5(e_1)=e_1\wedge e_2\wedge e_4$, $\Delta_5(e_3)=e_3\wedge e_2\wedge e_4$;
$(L_{c_2}, C_{c_1}, \Delta_6)$ $\Delta_6(e_3)=e_4\wedge e_1\wedge e_2$, $\Delta_6(e_4)=e_3\wedge e_1\wedge e_2$;
$(L_{c_2}, C_{c_3}, \Delta_3)$ $\Delta_3(e_3)=e_3\wedge e_1\wedge e_2$, $\Delta_3(e_4)=e_4\wedge e_1\wedge e_2$;
$(L_{c_3}, C_{c_1}, \Delta_1)$ $\Delta_1(e_3)=e_4\wedge e_1\wedge e_2$, $\Delta_1(e_4)=e_3\wedge e_1\wedge e_2$;
$(L_{c_3}, C_{c_1}, \Delta_8)$ $\Delta_8(e_1)=e_2\wedge e_3\wedge e_4$, $\Delta_8(e_2)=e_1\wedge e_3\wedge e_4$;
$(L_{c_3}, C_{c_1}, \Delta_9)$ $\Delta_9(e_1)=e_3\wedge e_2\wedge e_4$, $\Delta_9(e_3)=e_1\wedge e_2\wedge e_4$;
$(L_{c_3}, C_{c_3}, \Delta_3)$ $\Delta_3(e_3)=e_3\wedge e_1\wedge e_2$, $\Delta_3(e_4)=e_4\wedge e_1\wedge e_2$;
$(L_{c_1}, C_{c_2}, \Delta_2)$ $\Delta_2(e_3)=\alpha e_4\wedge e_1\wedge e_2$, $\Delta_2(e_4)=e_4\wedge e_1\wedge e_2+e_3\wedge e_1\wedge e_2$;
$(L_{c_2}, C_{c_2}, \Delta_7)$ $\Delta_7(e_3)=\alpha e_4\wedge e_1\wedge e_2$, $\Delta_7(e_4)=e_4\wedge e_1\wedge e_2+e_3\wedge e_1\wedge e_2$;
$(L_{c_3}, C_{c_2}, \Delta_7)$ $\Delta_7(e_3)=\alpha e_4\wedge e_1\wedge e_2$, $\Delta_7(e_4)=e_4\wedge e_1\wedge e_2+e_3\wedge e_1\wedge e_2$, *where* $\alpha\in F, \alpha\neq 0$.

Proof From Lemma 3.1, and Theorem 2.3 in [4], we need to verify that whether the twelve 3-Lie coalgebras of type C_1, twenty four 3-Lie coalgebras of type C_2, and the twelve 3-Lie coalgebras of type C_3 which are obtained by permuting the basis e_1, e_2, e_3, e_4, are compatible with the 3-Lie algebra L_{c_i}, $i=1,2,3$ respectively.

By a direct computation, only the cases (11) and (12) of the type C_{c_1}, the cases (21), (22), (23) and (24) of the C_{c_2} and all the twelve cases of the type C_{c_3} are compatible with the 3-Lie algebra L_{c_1}, respectively. By the following isomorphisms of 3-Lie algebra $f: L_{c_1}\to L_{c_1}$:
(L_{c_1}, C_{c_1}) :(11) $\to$ (12) : $f(e_1)=e_2, f(e_2)=e_1, f(e_3)=e_3, f(e_4)=e_4;$
(L_{c_1}, C_{c_2}) : (21) $\to$ (22), (23) $\to$ (24) : $f(e_1)=e_2, f(e_2)=e_1, f(e_3)=e_3, f(e_4)=e_4;$
(21) $\to$ (24), (22) $\to$ (23) : $f(e_1)=e_2, f(e_2)=-e_1, f(e_3)=e_4, f(e_4)=e_3;$
(L_{c_1}, C_{c_3}) : (3) $\to$ (4) : $f(e_1)=e_1, f(e_2)=e_2, f(e_3)=-e_3, f(e_4)=-e_4;$
(1) $\to$ (2), (3) $\to$ (9), (4) $\to$ (10) : $f(e_1)=e_2, f(e_2)=-e_1, f(e_3)=e_4, f(e_4)=e_3;$
(3) $\to$ (8), (4) $\to$ (7), (5) $\to$ (9), (6) $\to$ (10), (11) $\to$ (12) : $f(e_1)=e_2, f(e_2)=e_1, f(e_3)=e_3, f(e_4)=e_4,$
and the derived algebra $L^1_{c_1}=Fe_1+Fe_2$, we get the non-equivalent 3-Lie bialgebras $(L_{c_1}, C_{c_1}, \Delta_1)$, $(L_{c_1}, C_{c_2}, \Delta_2)$ and $(L_{c_1}, C_{c_3}, \Delta_j)$, $j=3,4,5$.

Second, by the similar discussion to the above, only the cases (11) and (12) of the type C_{c_1}, the cases (21), (22), (23) and (24) of the type C_{c_2}, and the cases (11) and (12) of the type C_{c_3} in Theorem 3.5 are compatible with the 3-Lie algebra L_{c_2}, respectively. By the 3-Lie algebra isomorphisms $f: L_{C_2}\to L_{C_2}$:
(L_{c_2}, C_{c_1}): (11) $\to$ (12) : $f(e_1)=e_1, f(e_2)=e_2, f(e_3)=-e_4, f(e_4)=e_3;$
(L_{c_2}, C_{c_2}): (21) $\to$ (22), (23) $\to$ (24) : $f(e_1)=\frac{1}{\sqrt{\alpha}}e_2, f(e_2)=\sqrt{\alpha}e_1+\frac{1}{\sqrt{\alpha}}e_2, f(e_3)=e_3, f(e_4)=e_4;$

$(21) \to (23): f(e_1) = \frac{1}{\sqrt{\alpha}}e_2, f(e_2) = \sqrt{\alpha}e_1 + \frac{1}{\sqrt{\alpha}}e_2, f(e_3) = \sqrt{-1}e_4, f(e_4) = \sqrt{-1}e_3;$
(L_{c_2}, C_{c_3}): $(12) \to (11): f(e_1) = \frac{1}{\sqrt{\alpha}}e_2, f(e_2) = \sqrt{\alpha}e_1 + \frac{1}{\sqrt{\alpha}}e_2, f(e_3) = e_3, f(e_4) = e_4;$
we get the non-equivalent 3-Lie bialgebras $(L_{c_2}, C_{c_1}, \Delta_6)$, $(L_{c_2}, C_{c_2}, \Delta_7)$ and $(L_{c_2}, C_{c_3}, \Delta_3)$.

Lastly, we discuss the compatibility of 3-Lie coalgebras of the type C_{c_1}, C_{c_2} and C_{c_3} with 3-Lie algebra L_{c_3}. By a direct computation, the twelve cases of the 3-Lie coalgebras of the type C_{c_1}, the cases (21), (22), (23) and (24) of the type C_{c_2}, and the cases (11) and (12) of the type C_{c_3} in Theorem 2.3 [4] are compatible with 3-Lie algebra L_{c_3}, respectively. Thanks to the derived algebra $L_{c_3}^1 = Fe_1 + Fe_2$ and the following 3-Lie algebra isomorphisms $f : L_{c_3} \to L_{c_3}$:
(L_{c_3}, C_{c_1}): $(1) \to (2), (3) \to (4): \ f(e_1) = e_1, f(e_2) = -e_2, f(e_3) = e_3, f(e_4) = e_4;$
$(3) \to (8), \ (4) \to (7): f(e_1) = e_2, f(e_2) = e_1, f(e_3) = e_3, f(e_4) = e_4;$
$(3) \to (5), \ (4) \to (6), (7) \to (10), \ (8) \to (9), (11) \to (12):$
$f(e_1) = e_1, f(e_2) = e_2, f(e_3) = e_4, f(e_4) = -e_3;$
(L_{c_3}, C_{c_2}): $(21) \to (22), (23) \to (24):$
$f(e_1) = -e_1, f(e_2) = e_2, f(e_3) = e_3, f(e_4) = e_4;$
$(21) \to (24): f(e_1) = e_1, f(e_2) = e_2, f(e_3) = \sqrt{-1}e_4, f(e_4) = \sqrt{-1}e_3;$
(L_{c_3}, C_{c_3}) $(11) \to (12): \quad f(e_1) = e_2, f(e_2) = e_1, f(e_3) = e_3, f(e_4) = e_4;$
the non-equivalent 3-Lie bialgebras are $(L_{c_3}, C_{c_1}, \Delta_1)$, $(L_{c_3}, C_{c_1}, \Delta_8)$, $(L_{c_3}, C_{c_1}, \Delta_9)$, $(L_{c_3}, C_{c_2}, \Delta_7)$ and $(L_{c_3}, C_{c_3}, \Delta_3)$. The proof is complete.

Acknowledgements

The first author (R.-P. Bai) was supported in part by the Natural Science Foundation (11371245) and the Natural Science Foundation of Hebei Province (A2014 201006).

References

[1] V. Filippov, n-Lie algebras, *Sib. Mat. Zh.*, 26 (1985) 126-140.

[2] R. Bai, Y. Cheng, J. Li, W. Meng, 3-Lie bialgebras, *Acta Math. Scientia*, 2014, 34B(2):513-522.

[3] R. Bai, Y. Zhang, Classification of 3-Lie bialgebras of type (L_b, C_b), *Mathematica Aeterna*, 2016, 6(1): 25 - 29.

[4] R. Bai, Y. Zhang, 3-Lie bialgebras of type (L_b, C_c), *Mathematica Aeterna*, 2016, 6(2): 153 - 157.

Received: May, 2016