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A Modified Nonmonotone Method with 3-1 Piecewise NCP Function for Generalized Nonlinear Complementarity Problem

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Abstract

In this paper, based on the 3-1 piecewise NCP function, we proposed a modified nonmonotone method for generalized nonlinear complementarity problems. Two parameters are introduced to avoid compute function of $F(x)$ and $G(x)$ in the update algorithm. Combined with the nonmonotonic line search, only one system of equation is needed to solve at each iteration. In the end, under suitable conditions, the global convergent properties are proved and the numerical results are reported.

Mathematics Subject Classification: xxxxx

Keywords: generalized nonlinear complementarity problem, 3-1 piecewise NCP function, nonmonotone, global convergence

1 Introduction

The generalized nonlinear complementarity problem (GNCP): find a vector $x \in R^n$ such that

$$F(x) \geq 0, G(x) \geq 0, F(x)^\top G(x) = 0 \tag{1}$$

where $F, G : R^n \to R^n$ is continuously differentiable. When $G(x) = x$, GNCP reduces to the classic nonlinear complementarity problem(NCP). Moreover, when $F(x) = Mx + q$, with $M \in R^{n\times n}$ and $q \in R^n$, NCP becomes a linear complementarity problem(LCP). NCP has attracted much attention due to its various important applications to many fields in engineering and economics, mechanics, transportation, machine learning and management science [1-3]. GNCP also has been considered extensively by using special techniques in some literatures [4-9].

In the past few years, the GNCP also be devoted a growing attention. Liu et al [10] proposed a smoothing Newton-type algorithm for solving the GNCP based on a class of smoothing functions. Based on a new smoothing function, Zheng et al [11] solved it by a smoothing Newton-type method. They all introducing parameter μ to Fischer-Burmeister function, then got a new smoothing function. Chen and Ma [12] introduced a smoothing Broyden-like algorithm for the GNCP over a polyhedral cone based on the smoothing Fischer-Burmeister function. And it also need to introduce the smoothing parameter μ to Fischer-Burmeister function. However, it will increase the difficulty of the algorithm. Therefore, we will introduce a 3-1 piecewise NCP function in algorithm.

Motivated by the above idea, It is introduced for a convex combination of Newton-type method and Broyden-like method for the GNCP. Based on 3-1 piecewise NCP function, we transform the GNCP to smooth equations. Then the convex combination is adopted in nonmonotonic line search to update the iteration. Therefore, there is no need to introduce the parameter μ and only one smooth equation needs to be solved at each iteration. Furthermore, we let $s = F(x)$, $t = G(x)$ as the independent variable so that the algorithm is more simple and flexible. And any accumulation point that is generated by iteration sequence of x is a solution of GNCP. The global convergent properties are proved and the numerical results are reported.

This paper is organized as follows. In the next section, we introduce 3-1 piecewise NCP function and transform GNCP into a equivalent system of smooth equations. In Section 3, we introduce the algorithm in detail. In Section 4, we prove the algorithm to be well-defined and present the convergent properties of the presented algorithm. Numerical results are reported in Section 5.

2 Preliminary Notes

In order to make preparations for the algorithm, we give some definitions: P_0-function, P_0-matrix, NCP pair, NCP function.

Definition 2.1 ($\mathbf{P_0}$ **– function**) $F : R^n \longrightarrow R^n$ is a continuously differetiable P_0-function, i.e., for all $x, y \in R^n$ with $x \neq y$, there exists an index i_0 such that

$$x_{i_0} \neq y_{i_0}, (x_{i_0} - y_{i_0})[F_{i_0}(x) - F_{i_0}(y)] \geq 0 \tag{2}$$

Definition 2.2 ($\mathbf{P_0}$ **– matrix**) A matrix $M \in R^{n\times n}$ is said to be a P_0-matrix, if all its principal minors are non-negative.

Definition 2.3 (**NCP pair and NCP function**) We call a pair $(a, b) \in R^2$ to be an NCP pair if $a \geq 0, b \geq 0$ and $ab = 0$; a function $\Phi : R^2 \longrightarrow R$ is called an NCP function if $\Phi(a, b) = 0$ if and only if (a, b) is an NCP pair. The

3-1 piecewise linear NCP function is defined as:

$$\Phi(a,b)=\begin{cases} 3a-\dfrac{a^2}{b} & \text{if } b\geq a>0, \text{ or } 3b>-a\geq 0\ , \\ 3b-\dfrac{b^2}{a} & \text{if } a>b>0, \text{ or } 3a>-b\geq 0\ , \\ 9a+9b & \text{if } a\leq 0 \text{ and } -a\geq 3b, \text{ or } b\leq -3a\leq 0\ . \end{cases} \tag{3}$$

If $(a,b)\neq(0,0)$,then

$$\triangledown\Phi(a,b)=\begin{cases} \begin{pmatrix} 3-\dfrac{2a}{b} \\ \dfrac{a^2}{b^2} \end{pmatrix} & \text{if } b\geq a>0, \text{ or } 3b>-a\geq 0\ , \\ \begin{pmatrix} \dfrac{b^2}{a^2} \\ 3-\dfrac{2b}{a} \end{pmatrix} & \text{if } a>b>0, \text{ or } 3a>-b\geq 0\ , \\ \begin{pmatrix} 9 \\ 9 \end{pmatrix} & \text{if } 0\geq a \text{ and } -a\geq 3b, \text{ or } b\leq -3a\leq 0\ . \end{cases} \tag{4}$$

Denote $H: R^{3n}\longrightarrow R^{3n}$

$$H(s,t,x)=\begin{pmatrix} s-F(x) \\ t-G(x) \\ \Phi(s,t) \end{pmatrix}, \tag{5}$$

where s approaches $F(x)$, t approaches $G(x)$ and $s=F(x)$, $t=G(x)$ at the solution. $\Phi(s,t)$ is 3-1 piecewise NCP function.

Therefore, we can reform the GNCP(1) to the following minimization problem:

$$\min\ \ \Psi(s,t,x)=\|H(s,t,x)\| \tag{6}$$

3 Algorithm

For solving (6), we need to introduce the following symbols

$$(\xi_i^k,\eta_i^k)=\begin{cases} (1,1) & \text{(x,s)=(0,0)} \\ \nabla\Phi(x,s) & \text{otherwise} \end{cases} \tag{7}$$

$i=1,2\cdots n$, obviously, $\xi_i^k>0$ and $\eta_i^k>0$.

Compute the Jacobian matrix $V(s^k,t^k,x^k)$ of $H(s^k,t^k,x^k)$, we get

$$V(s^k,t^k,x^k)=\begin{pmatrix} I & 0 & -\nabla F(x^k) \\ 0 & I & -\nabla G(x^k) \\ diag(\xi_i^k) & diag(\eta_i^k) & 0 \end{pmatrix} \tag{8}$$

where I is identity matrix of $n \times n$, $\nabla F(x^k), \nabla G(x^k)$ are gradient matrix of $F(x^k)$ and $G(x^k)$ respectively, $diag(\xi_i^k)$ or $diag(\eta_i^k)$ denotes the diagonal matrix whose ith diagonal element is ξ_i^k or η_i^k respectively.

Algorithm 3.1

Step0 Initialization:

Given initial point $(s^0, t^0, x^0) \in R^{3n}$, $\tau \in (0,1)$, $\lambda \in (0,1)$, $0 < \theta, \overline{\theta} < 1$, $T_0 = B_0 = V(s^0, t^0, x^0)$, $k = 0$.

Step1 If $\Psi(s^k, t^k, x^k) = 0$ then stop. Otherwise, calculation of the search direction:

Calculate u^k, v^k and d^k by solving the following linear system in (u, v, d):

$$T_k \begin{pmatrix} u \\ v \\ d \end{pmatrix} = \begin{pmatrix} F(x^k) - s^k \\ G(x^k) - t^k \\ -\Phi(s^k, t^k) \end{pmatrix} \tag{9}$$

Step2 Nonmonotone line search.

Step2.1 if

$$\Psi(s^k + u^k, t^k + v^k, x^k + d^k) \leq \theta \Psi(s^k, t^k, x^k) \tag{10}$$

$$\|\Phi(s^k + u^k, t^k + v^k)\| \leq \theta \max_{0 \leq r \leq m(k)-1} \|\Phi^{k-r}\| \tag{11}$$

where $m(0) = 0$, $0 \leq m(k) \leq \min\{m(k-1) + 1, M\}$, M is a positive constant. then let

$$s^{k+1} = s^k + u^k, t^{k+1} = t^k + v^k, x^{k+1} = x^k + d^k \tag{12}$$

go to step 3, otherwise go to step 2.2.

Step2.2 let

$$s^{k+1} = s^k + \alpha_k u^k, t^{k+1} = t^k + \alpha_k v^k, x^{k+1} = x^k + \alpha_k d^k \tag{13}$$

where $\alpha_k = \tau^j$ $(0 < \tau < 1)$ and j is the smallest non-negative integer and satisfied:

$$\|\Phi(s^{k+1}, t^{k+1})\| \leq \theta \max_{0 \leq r \leq m(k)-1} \|\Phi^{k-r}\| \tag{14}$$

$$\|s^{k+1} - F(x^{k+1}) + t^{k+1} - G(x^{k+1})\| \leq \theta \max_{0 \leq r \leq m(k)-1} \|s^{k-r} - F(x^{k-r}) + t^{k-r} - G(x^{k-r})\| \tag{15}$$

Step3 Update B_k to get B_{k+1}:

$$B_{k+1} = B_k + \theta_k \frac{(y^k - B_k z^k)(z^k)^\top}{\|z^k\|^2} \tag{16}$$

where

$$z^k = \begin{pmatrix} s^{k+1} \\ t^{k+1} \\ x^{k+1} \end{pmatrix} - \begin{pmatrix} s^k \\ t^k \\ x^k \end{pmatrix}, y^k = H(s^{k+1}, t^{k+1}, x^{k+1}) - H(s^k, t^k, x^k). \tag{17}$$

select θ_k to satisfy $|\theta_k - 1| \leq \overline{\theta}$ and matrix B_{k+1} is nonsingular.

Step4 Get T_{k+1}:

$$T_{k+1} = \lambda V_{k+1} + (1-\lambda)B_{k+1} \tag{18}$$

Step5 Let k=k+1 and go to Step 1.

Remark 3.1 In the above algorithm, there are two types of successive iteration: (12) and (13).

4 Main Results

In this section, we discuss the global convergence properties of a modified nonmonotone algorithm with 3-1 piecewise NCP function. In order to achieve the convergence of the algorithm, we give some Assumptions as follows:

Assumption 4.1

(a): $F, G : R^n \to R^n$ is continuously differentiable P_0-function.

(b): For initial point $(s^0, t^0, x^0) \in R^{3n}$, denote the level set of $\Psi(s, t, x)$ by

$$L(s^0, t^0, x^0) = \{(s, t, x) \in R^{3n} | \Psi(s, t, x) \leq \Psi(s^0, t^0, x^0)\} \tag{19}$$

Defined by (19), the level set of $\Psi(s, t, x)$ is bounded.

Remark 4.1 $F(x)$, $G(x)$ are P_0-function, then $\nabla F(x)$, $\nabla G(x)$ is positive semidefinite.

Lemma 4.1 Suppose the Assumption 4.1 holds, if $H(s^k, t^k, x^k) \neq 0$ then V^k is nonsingular.

Proof: Assume $H(s^k, t^k, x^k) \neq 0$. If $V^k(u, v, d)^\top = 0$ for some $(u, v, d)^\top \in R^{3n}$, where $u = (u_1 \cdots u_n)^\top$, $v = (v_1 \cdots v_n)^\top$, $d = (d_1 \cdots d_n)^\top$ then

$$Iu - \nabla F(x^k)d = 0 \tag{20}$$

$$Iv - \nabla G(x^k)d = 0 \tag{21}$$

$$diag(\xi^k)u + diag(\eta^k)v = 0 \tag{22}$$

From the definitions of ξ_i^k and η_i^k ,we know that $\xi_i^k > 0$ and $\eta_i^k > 0$ for all i. So, $diag(\eta^k)$ is nonsingular. By (22) we have

$$v = -(diag(\eta^k))^{-1} diag(\xi^k)u \tag{23}$$

Substitute v in (23) by (21), we have

$$d = -(\nabla G(x^k))^{-1}(diag(\eta^k))^{-1} diag(\xi^k)u \tag{24}$$

Substitute d in (24) by (20), and multiplying by $u^\top$, we have

$$u^\top Iu + u^\top \nabla F(x^k)(\nabla G(x^k))^{-1}(diag(\eta^k))^{-1} diag(\xi^k)u = 0 \tag{25}$$

The fact that $F(x)$, $G(x)$ is the P_0-function, so all principal minor determinant of the $\nabla F(x)$, $\nabla G(x)$ is non-negative, that is to say, $\nabla F(x)$, $(\nabla G(x))^{-1}$ is positive semidefinite. And matrix $(diag(\eta^k))^{-1}diag(\xi^k)$ is positive definite. So $u = 0$. By (22),(23), we know $v = 0$, $d = 0$. Hence, V^k is nonsingular.

Lemma 4.2 Suppose the Assumption 4.1 holds, if $H(s^k, t^k, x^k) \neq 0$ then T_k is nonsingular.

Proof: By lemma 4.1, we know V_k is nonsingular, $B_0 = V_0$ is nonsingular. And B_k which is produced by iteration of Algorithm 3.1 is nonsingular . So $T_k = \lambda V_k + (1-\lambda)B_k$ is nonsingular.

Lemma 4.3 $\Phi(s^k, t^k) \longrightarrow 0$, as $k \longrightarrow \infty$.

Proof: In view of convenience, if for all sufficiently large k (11),(14) holds, define

$\|\Phi^{l(k)}\| = \max\limits_{0\le r\le m(k)-1} \|\Phi^{k-r}\|$, where $k - m(k) + 1 \le l(k) \le k$.

Since $m(k+1) \le m(k) + 1$, then

$$\begin{aligned}\|\Phi^{l(k+1)}\| &= \max_{0\le r\le m(k+1)-1} \|\Phi^{k+1-r}\| \\ &\le \max_{0\le r\le m(k)} \|\Phi^{k+1-r}\| \\ &= \max\,\{\|\Phi^{l(k)}\|, \|\Phi^{k+1}\|\} \\ &= \|\Phi^{l(k)}\|\end{aligned}$$

So, $\|\Phi^{l(k)}\|$ is monotone decreasing, which implies that the $\{\|\Phi^{l(k)}\|\}$ converges.
It follows from (11),(14) that $\|\Phi^{l(k)}\| \le \theta\|\Phi^{l(l(k)-1)}\|$.
Since $\theta \in (0,1)$, therefore $\{\|\Phi^{l(k)}\|\} \longrightarrow 0 (k \longrightarrow \infty)$.
Therefore $\|\Phi^{k+1}\| \le \theta\|\Phi^{l(k)}\| \longrightarrow 0 (k \longrightarrow \infty)$ holds by the Algorithm 3.1.
That is, $\lim\limits_{k\longrightarrow\infty} \|\Phi^k\| = 0$.

Lemma 4.4 $[s^k - F(x^k) + t^k - G(x^k)] \longrightarrow 0$, as $k \longrightarrow \infty$.

The proof is similar to Lemma 4.3.

Lemma 4.5 $u^k \longrightarrow 0$, $v^k \longrightarrow 0$, $d^k \longrightarrow 0$, $H(s^k, t^k, x^k) \longrightarrow 0$, as $k \longrightarrow \infty$.

Proof: For iteration (12), it satisfied (10). And $\Psi(s,t,x) = \|H(s,t,x)\| \ge 0$. So, $\Psi(s^k, t^k, x^k)$ is monotone decreasing and $\Psi(s^k, t^k, x^k) \to 0$, Therefore, $H(s^k, t^k, x^k) \to 0$, as $k \to \infty$.

For iteration (13), it satisfied (14) and (15). By Lemma 4.2 and 4.3, we have $\Phi(s^k, t^k) \longrightarrow 0$, $[s^k - F(x^k) + t^k - G(x^k)] \longrightarrow 0$, as $k \longrightarrow \infty$. So, $\|H(s^k, t^k, x^k)\| = \|\Phi(s^k, t^k)\| + \|s^k - F(x^k) + t^k - G(x^k)\| \longrightarrow 0$, as $k \longrightarrow \infty$.

Thus,

$$T_k \begin{pmatrix} u^k \\ v^k \\ d^k \end{pmatrix} = \begin{pmatrix} F(x^k) - s^k \\ G(x^k) - t^k \\ -\Phi(s^k, t^k) \end{pmatrix} = 0 \tag{26}$$

T_k is nonsingular by lemma 4.2. Therefore, $u^k \longrightarrow 0$, $v^k \longrightarrow 0$, $d^k \longrightarrow 0$, as $k \longrightarrow \infty$.

Theorem 4.1 Suppose the Assumption 4.1 holds, equation (9) is solvable, Algorithm 3.1 is well-defined.

Proof: T_k is nonsingular by lemma 4.2. Therefore the equation (9) has one and only one solution. And we know $H(s^k, t^k, x^k) \longrightarrow 0$, as $k \longrightarrow \infty$ by lemma 4.4. So, Algorithm 3.1 is well-defined.

Lemma 4.6 Suppose the Assumption 4.1 holds, let $\{(s^k, t^k, x^k)\}$ is generated sequence by Algorithm 3.1, then $\{(s^k, t^k, x^k)\} \subset L(s^0, t^0, x^0)$.

Proof: We prove it by induction, when k=0, it is clearly that $(s^0, t^0, x^0) \in L(s^0, t^0, x^0)$.
Assume $(s^k, t^k, x^k) \in L(s^0, t^0, x^0)$, then we have $\Psi(s^k, t^k, x^k) \leq \Psi(s^0, t^0, x^0)$.

For iteration (12), it satisfied (10). We get
$\Psi(s^{k+1}, t^{k+1}, x^{k+1}) \leq \theta\Psi(s^k, t^k, x^k) \leq \theta\Psi(s^0, t^0, x^0) \leq \Psi(s^0, t^0, x^0)$

For iteration (13), it satisfied (14) and (15). We get

$$\begin{aligned}\Psi(s^{k+1}, t^{k+1}, x^{k+1}) &= \|\Phi(s^{k+1}, t^{k+1})\| + \|s^{k+1} - F(x^{k+1}) + t^{k+1} - G(x^{k+1})\| \\ &\leq \theta \max_{0 \leq r \leq m(k)-1} (\|\Phi^{k-r}\| + \|s^{k-r} - F(x^{k-r}) + t^{k-r} - G(x^{k-r})\|) \\ &= \theta \max_{0 \leq r \leq m(k)-1} \Psi(s^{k-r}, t^{k-r}, x^{k-r}) \\ &\leq \Psi(s^0, t^0, x^0)\end{aligned}$$

So, $(s^{k+1}, t^{k+1}, x^{k+1}) \in L(s^0, t^0, x^0)$.
By the similar discuss as above, we obtain $\{(s^k, t^k, x^k)\} \subset L(s^0, t^0, x^0)$ for all k.

Theorem 4.2 Suppose the Assumption 4.1 holds, and $\{(s^k, t^k, x^k)\}$ is generated by Algorithm 3.1, Then there exists an accumulation point (s^*, t^*, x^*) of the sequence $\{(s^k, t^k, x^k)\}$, and x^* is the solution of GNCP(1).

Proof: From Lemma 4.6, we know $\{(s^k, t^k, x^k)\} \subset L(s^0, t^0, x^0)$ for all k. In Assumption 4.1 (b), we set $L(s^0, t^0, x^0)$ is bounded. So, $\{(s^k, t^k, x^k)\}$ has an accumulation point. Thus, suppose $\{(s^k, t^k, x^k)\}_{k \in K}$ convergence to (s^*, t^*, x^*). We need to prove $H(s^*, t^*, x^*) = 0$.

Suppose $\{(s^k, t^k, x^k)\}_{k \in K}$ is infinite sequence that generated by Algorithm 3.1. By Lemma 4.5, we know $H(s^k, t^k, x^k) \longrightarrow 0$, as $k \longrightarrow \infty$.

So, $H(s^*, t^*, x^*) = 0$, the conclusion is followed.

5 Numerical Results

In this section, we will report some numerical results. All experiments were performed on a personal computer with 2.0 GB memory and Intel(R) Core(TM)2 Duo CPU 2.93 GHz. The operating system was Windows 7 and the computer codes were all written in Matlab 7.1. Throughout the experiments, the parameters used in Algorithm 3.1 were $\theta = 0.9$, $\tau = 0.8$, $\lambda = 0.2$, $\theta_k \equiv 1$. We used $\|H(s, t, x)\| \leq 10^{-6}$ as the stop criterion.

In the following tables, SP denotes the initial point. IN denotes the number of iterations. FV denotes the value of $\|H(s, t, x)\|^2$ when the algorithm terminates. We considered the following examples.

Example 5.1 : We consider an implicit complementarity problem: find $y \in R^n$ such that

$y - m(y) \geq 0, F(y) \geq 0, F(y)^\top (y - m(y)) = 0$

where $m_i : R^n \longrightarrow R, i = 1, \ldots, n$, and

$$F(y) = Ay + b = \begin{pmatrix} 2 & -1 & 0 & \cdots & 0 & 0 \\ -1 & 2 & -1 & \cdots & 0 & 0 \\ 0 & -1 & 2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2 & -1 \\ 0 & 0 & 0 & \cdots & -1 & 2 \end{pmatrix} y + \begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{pmatrix} \tag{27}$$

and $m(y) = \psi(Ay + b)$ with $\psi : R^n \longrightarrow R^n$ being twice continously differentiable. The following choices of function ψ define our test problems.

$$\begin{array}{ll} P1.\ \psi_i(x) = -0.5 - x_i, & i = 1, 2, \ldots, n. \\ P2.\ \psi_i(x) = -1.5x_i + 0.25x_i^2, & i = 1, 2, \ldots, n. \end{array} \tag{28}$$

For each problem, the following starting point were used, namely,

$(a)(-0.5, -0.5, \ldots, -0.5)^\top$

$(b)(1, 1, \ldots, 1)^\top$

$(c)(0.5, 0.5, \ldots, 0.5)^\top$

We test this problem by using $s^0 = (2, 2, \cdots, 2)^\top$, $t^0 = (0.8, 0.8, \cdots, 0.8)^\top$, and choose n=4, 12 as the dimension of the problem. The tested results are listed in Table 1 and Table 2.

Table 1

Computational results for Example 5.1 with n=4.

		Algorithm 3.1		Algorithm 3.1 in [12]	
P	SP	IN	FV	IN	FV
(1)	(a)	6	1.6143×10^{-14}	17	2.3634×10^{-20}
(2)	(a)	9	2.7567×10^{-13}	15	8.9799×10^{-15}
(1)	(b)	6	1.8070×10^{-14}	19	7.5432×10^{-14}
(2)	(b)	10	1.4636×10^{-13}	17	4.2448×10^{-14}
(1)	(c)	6	1.7023×10^{-14}	17	1.6306×10^{-14}
(2)	(c)	9	5.1280×10^{-13}	17	3.9887×10^{-13}

Table 2

Computational results for Example 5.1 with n=12.

		Algorithm 3.1		Algorithm 3.1 in [12]	
P	SP	IN	FV	IN	FV
(1)	(a)	6	2.5246×10^{-13}	43	3.2624×10^{-13}
(2)	(a)	10	2.0561×10^{-13}	39	8.2523×10^{-13}
(1)	(b)	6	3.3399×10^{-13}	61	2.3346×10^{-13}
(2)	(b)	13	4.0709×10^{-14}	39	6.8721×10^{-13}
(1)	(c)	6	3.0490×10^{-13}	60	6.3884×10^{-13}
(2)	(c)	11	2.3232×10^{-13}	42	7.3077×10^{-13}

These numerical results indicate that the presented the modified nonmonotone method with 3-1 piecewise NCP function for GNCP is effective.

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