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Almost contra semi generalized star b - continuous functions in Topological Spaces

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Abstract

In this paper, the authors introduce a new class of functions called almost contra semi generalized star b - continuous function (briefly almost contra sg^*b-continuous) in topological spaces. Some characterizations and several properties concerning almost contra sg^*b-continuous functions are obtained.

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1 Introduction

In 2002, Jafari and Noiri introduced and studied a new form of functions called contra-pre continuous functions. The purpose of this paper is to introduce and study almost contra sg^*b-continuous functions via the concept of sg^*b-closed sets. Also, properties of almost contra sg^*b-continuity are discussed. Moreover, we obtain basic properties and preservation theorems of almost contra sg^*b-continuous functions and relationships between almost contra sg^*b-continuity and sg^*b-regular graphs.

Through out this paper (X, τ) and (Y, σ) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. Let $A \subseteq X$, the closure of A and interior of A will be denoted by $cl(A)$ and $int(A)$ respectively, union of all sg^*b-open sets X contained in A is called sg^*b-interior of A and it is denoted by $sg^*bint(A)$, the intersection of all sg^*b-closed sets of X containing A is called sg^*b-closure of A and it is denoted by $sg^*bcl(A)$ [5].

2 Preliminaries

Definition 2.1. *Let a subset A of a topological space (X, τ), is called*
1) a pre-open set [9] if $A \subseteq int(cl(A))$.
2) a semi-open set [7] if $A \subseteq cl(int(A))$.
3) a α -open set [9] if $A \subseteq int(cl(int(A)))$. 4) a α generalized closed set (briefly αg- closed) [8] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
5) a generalized $*$ closed set (briefly g^*-closed)[14] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$ open in X.
6) a generalized b- closed set (briefly gb- closed) [1] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
7) a generalized semi-pre closed set (briefly gsp - closed) [3] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
8) a semi generalized closed set (briefly sg- closed) [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.
9) a generalized pre regular closed set (briefly gpr-closed) [5] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
10) a semi generalized b- closed set (briefly sgb- closed) [6] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.
11) a $\ddot{g}$ - closed set [12] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg open in X.
12) a semi generalized star b - closed set (briefly sg^*b - closed)[13] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg open in X.

Definition 2.2. *A function $f : (X, \tau) \to (Y, \sigma)$, is called*
1) almost contra continuous [1] if $f^{-1}(V)$ is closed in (X, τ) for every regular-open set V of (Y, σ).
2) almost contra α-continuous [11] if $f^{-1}(V)$ is b-closed in (X, τ) for every regular-open set V of (Y, σ).
3) almost contra pre-continuous [4] if $f^{-1}(V)$ is pre-closed in (X, τ) for every regular-open set V of (Y, σ).
4) almost contra semi-continuous [6] if $f^{-1}(V)$ is semi-closed in (X, τ) for every regular-open set V of (Y, σ).
5) almost contra gb-continuous [10] if $f^{-1}(V)$ is gb-closed in (X, τ) for every regular-open set V of (Y, σ).

3 Almost contra semi generalized star b - Continuous functions

In this section, we introduce almost contra semi generalized star b - continuous functions and investigate some of their properties.

Definition 3.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is called almost contra semi generalized star b - continuous if $f^{-1}(V)$ is sg^*b - closed in (X, τ) for every regular open set V in (Y, σ).*

Example 3.2. *Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \varphi, \ \{a\}, \{b\}, \{a, b\}.\{a, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$. Clearly f is almost contra sg^*b - continuous.*

Theorem 3.3. *If $f : X \to Y$ is contra sg^*b - continuous then it is almost contra sg^*b - continuous.*

Proof. Obvious, because every regular open set is open set. □

Remark 3.4. *Converse of the above theorem need not be true in general as seen from the following example.*

Example 3.5. *Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{c\}\}$ and $\sigma = \{Y, \varphi, \{a\}, \{b\}, \ \{a, b\}, \{b, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$. Then f is almost contra sg^*b - continuous function but not contra sg^*b - continuous, because for the open set $\{a\}$ in Y and $f^{-1}\{a\} = \{c\}$ is not sg^*b - closed in X.*

Theorem 3.6. *1) Every almost contra pre - continuous function is almost contra sg^*b - continuous function.*
2) Every almost contra semi continuous function is almost contra sg^*b - continuous function.
3) Every almost contra α - continuous function is almost contra sg^*b - continuous function.
4) Every almost contra αg - continuous function is almost contra sg^*b - continuous function.
5) Every almost contra sg^*b - continuous function is almost contra gsp - continuous function.
6) Every almost contra sg^*b - continuous function is almost contra gb - continuous function.
7) Every almost contra sg - continuous function is almost contra sg^*b - continuous function.

Remark 3.7. *Converse of the above statements is not true as shown in the following example.*

Example 3.8. *i) Let* $X = Y = \{a, b, c\}$ *with* $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$ *and* $\sigma = \{Y, \varphi, \{a\}, \{c\}, \{a, c\}\}$. *Define a function* $f : (X, \tau) \to (Y, \sigma)$ *by* $f(a) = b$, $f(b) = a$, $f(c) = c$. *Clearly* f *is almost contra* sg^*b *- continuous but* f *is not almost contra pre - continuous. Because* $f^{-1}(\{c\}) = \{b\}$ *is not pre - closed in* (X, τ) *where* $\{c\}$ *is regular - open in* (Y, σ).
ii) Let $X = Y = \{a, b, c\}$ *with* $\tau = \{X, \varphi, \{a, b\}\}$ *and* $\sigma = \{Y, \varphi, \{a\}, \{b\}, \{a, b\}\}$. *Define a function* $f : (X, \tau) \to (Y, \sigma)$ *by* $f(a) = c$, $f(b) = a$, $f(c) = b$. *Clearly* f *is almost contra* sg^*b *- continuous but* f *is not almost contra semi - continuous. Because* $f^{-1}(\{a\}) = \{b\}$ *is not semi - closed in* (X, τ) *where* $\{a\}$ *is regular - open in* (Y, σ).
iii) Let $X = Y = \{a, b, c\}$ *with* $\tau = \{X, \varphi, \{b\}, \{a, b\}\}$ *and* $\sigma = \{Y, \varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. *Define a function* $f : (X, \tau) \to (Y, \sigma)$ *by* $f(a) = b$, $f(b) = a$, $f(c) = c$. *Clearly* f *is almost contra* sg^*b *- continuous but* f *is not almost contra* α *- continuous. Because* $f^{-1}(\{a, c\}) = \{b, c\}$ *is not* α *- closed in* (X, τ) *where* $\{a, c\}$ *is regular - open in* (Y, σ).
iv) Let $X = Y = \{a, b, c\}$ *with* $\tau = \{X, \varphi, \{a, b\}\}$ *and* $\sigma = \{Y, \varphi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. *Define a function* $f : (X, \tau) \to (Y, \sigma)$ *by* $f(a) = c$, $f(b) = a$, $f(c) = b$. *Clearly* f *is almost contra* sg^*b *- continuous but* f *is not almost contra* αg *- continuous. Because* $f^{-1}(\{a\}) = \{b\}$ *is not* αg *- closed in* (X, τ) *where* $\{a\}$ *is regular - open in* (Y, σ).
v) Let $X = Y = \{a, b, c\}$ *with* $\tau = \{X, \varphi, \{a\}, \{a, c\}\}$ *and* $\sigma = \{Y, \varphi, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$. *Define a function* $f : (X, \tau) \to (Y, \sigma)$ *by* $f(a) = b$, $f(b) = c$, $f(c) = a$. *Clearly* f *is almost contra gsp - continuous but* f *is not almost contra* sg^*b *- continuous. Because* $f^{-1}(\{b, c\}) = \{a, b\}$ *is not* sg^*b *- closed in* (X, τ) *where* $\{b, c\}$ *is regular - open in* (Y, σ).
vi) Let $X = Y = \{a, b, c\}$ *with* $\tau = \{X, \varphi, \{a\}, \{a, b\}\}$ *and* $\sigma = \{Y, \varphi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. *Define a function* $f : (X, \tau) \to (Y, \sigma)$ *by* $f(a) = b$, $f(b) = a$, $f(c) = c$. *Clearly* f *is almost contra gb - continuous but* f *is not almost contra* sg^*b *- continuous. Because* $f^{-1}(\{b, c\}) = \{a, c\}$ *is not* sg^*b *- closed in* (X, τ) *where* $\{b, c\}$ *is regular - open in* (Y, σ).
vii) Let $X = Y = \{a, b, c\}$ *with* $\tau = \{X, \varphi, \{b\}, \{a, b\}\}$ *and* $\sigma = \{Y, \varphi, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$. *Define a function* $f : (X, \tau) \to (Y, \sigma)$ *by* $f(a) = b$, $f(b) = a$, $f(c) = c$. *Clearly* f *is almost contra* sg^*b *- continuous but* f *is not almost contra sg - continuous. Because* $f^{-1}(\{a, b\}) = \{a, b\}$ *is not sg - closed in* (X, τ) *where* $\{a, b\}$ *is regular - open in* (Y, σ).

Remark 3.9. *The concept of almost contra* sg^*b*-continuous and almost contra sgb -continuous are independent as shown in the following examples.*

Example 3.10. *Let* $X = Y = \{a, b, c\}$ *with* $\tau = \{X, \varphi, \{b\}, \{a, b\}\}$ *and* $\sigma = \{Y, \varphi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. *Define a function* $f : (X, \tau) \to (Y, \sigma)$ *by* $f(a) = c$, $f(b) = b$, $f(c) = a$. *Clearly* f *is almost contra* sg^*b *- continuous but* f *is not almost contra sgb - continuous. Because* $f^{-1}(\{b, c\}) = \{a, b\}$ *is not sgb - closed in* (X, τ) *where* $\{b, c\}$ *is regular - open in* (Y, σ).

Example 3.11. *Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$ and $\sigma = \{Y, \varphi, \{b\}, \{c\}, \{b, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = a$, $f(c) = b$. Clearly f is almost contra sgb - continuous but f is not almost contra sg^*b - continuous. Because $f^{-1}(\{b\}) = \{c\}$ is not sg^*b - closed in (X, τ) where $\{b\}$ is regular - open in (Y, σ).*

Theorem 3.12. *The following are equivalent for a function $f : X \to Y$,*
(1) f is almost contra sg^*b - continuous.
(2) for every regular closed set F of Y, $f^{-1}(F)$ is sg^*b - open set of X.
(3) for each $x \in X$ and each regular closed set F of Y containing $f(x)$, there exists sg^*b - open U containing x such that $f(U) \subset F$.
(4) for each $x \in X$ and each regular open set V of Y not containing $f(x)$, there exists sg^*b - closed set K not containing x such that $f^{-1}(V) \subset K$.

Proof. (1) ⇒ (2) : Let F be a regular closed set in Y, then $Y - F$ is a regular open set in Y. By (1), $f^{-1}(Y - F) = X - f^{-1}(F)$ is sg^*b - closed set in X. This implies $f^{-1}(F)$ is sg^*b - open set in X. Therefore, (2) holds.
(2) ⇒ (1) : Let G be a regular open set of Y. Then $Y - G$ is a regular closed set in Y. By (2), $f^{-1}(Y - G)$ is sg^*b - open set in X. This implies $X - f^{-1}(G)$ is sg^*b - open set in X, which implies $f^{-1}(G)$ is sg^*b - closed set in X. Therefore, (1) hold.
(2) ⇒ (3) : Let F be a regular closed set in Y containing $f(x)$, which implies $x \in f^{-1}(F)$. By (2), $f^{-1}(F)$ is sg^*b - open in X containing x. Set $U = f^{-1}(F)$, which implies U is sg^*b - open in X containing x and $f(U) = f(f^{-1}(F)) \subset F$. Therefore (3) holds.
(3) ⇒ (2) : Let F be a regular closed set in Y containing $f(x)$, which implies $x \in f^{-1}(F)$. From (3), there exists sg^*b - open U_x in X containing x such that $f(U_x) \subset F$. That is $U_x \subset f^{-1}(F)$. Thus $f^{-1}(F) = \{\cup U_x : x \in f^{-1}(F)$, which is union of sg^*b - open sets. Therefore, $f^{-1}(F)$ is sg^*b - open set of X.
(3) ⇒ (4) : Let V be a regular open set in Y not containing $f(x)$. Then $Y - V$ is a regular closed set in Y containing $f(x)$. From (3), there exists a sg^*b - open set U in X containing x such that $f(U) \subset Y - V$ .This implies $U \subset f^{-1}(Y - V) = X - f^{-1}(V)$. Hence, $f^{-1}(V) \subset X - U$. Set $K = X - V$, then K is sg^*b - closed set not containing x in X such that $f^{-1}(V) \subset K$.
(4) ⇒ (3) : Let F be a regular closed set in Y containing $f(x)$. Then $Y - F$ is a regular open set in Y not containing $f(x)$. From (4), there exists sg^*b - closed set K in X not containing x such that $f^{-1}(Y - F) \subset K$. This implies $X - f^{-1}(F) \subset K$. Hence, $X - K \subset f^{-1}(F)$, that is $f(X - K) \subset F$. Set $U = X - K$, then U is sg^*b - open set containing x in X such that $f(U) \subset F$. □

Theorem 3.13. *The following are equivalent for a function $f : X \to Y$,*
(1) f is almost contra sg^*b - continuous.

(2) $f^{-1}(Int(Cl(G)))$ *is* sg^*b *- closed set in* X *for every open subset* G *of* Y*.*
(3) $f^{-1}(Cl(Int(F)))$ *is* sg^*b *- open set in* X *for every closed subset* F *of* Y*.*

Proof. (1) $\Rightarrow$ (2) : Let G be an open set in Y. Then $Int(Cl(G))$ is regular open set in Y. By (1), $f^{-1}(Int(Cl(G)) \in sg^*b - C(X)$.
(2) $\Rightarrow$ (1) : Proof is obvious.
(1) $\Rightarrow$ (3) : Let F be a closed set in Y. Then $Cl(Int(G))$ is regular closed set in Y. By (1), $f^{-1}(Cl(Int(G)) \in sg^*b - O(X)$.
(3) $\Rightarrow$ (1) : Proof is obvious. □

Definition 3.14. *A function* $f : X \to Y$ *is said to be* R *- map if* $f^{-1}(V)$ *is regular open in* X *for each regular open set* V *of* Y*.*

Definition 3.15. *A function* $f : X \to Y$ *is said to be perfectly continuous if* $f^{-1}(V)$ *is clopen in* X *for each open set* V *of* Y*.*

Theorem 3.16. *For two functions* $f : X \to Y$ *and* $g : Y \to Z$*, let* $g \circ f : X \to Z$ *be a composition function. Then, the following properties hold.*
(1) If f *is almost contra* sg^*b *- continuous and* g *is an* R *- map, then* $g \circ f$ *is almost contra* sg^*b *- continuous.*
(2) If f *is almost contra* sg^*b *- continuous and* g *is perfectly continuous, then* $g \circ f$ *is contra* sg^*b *- continuous.*
(3) If f *is contra* sg^*b *- continuous and* g *is almost continuous, then* $g \circ f$ *is almost contra* sg^*b *- continuous.*

Proof. (1) Let V be any regular open set in Z. Since g is an R - map, $g^{-1}(V)$ is regular open in Y. Since f is almost contra sg^*b - continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is sg^*b - closed set in X. Therefore $g \circ f$ is almost contra sg^*b - continuous.
(2) Let V be any regular open set in Z. Since g is perfectly continuous, $g^{-1}(V)$ is clopen in Y. Since f is almost contra sg^*b - continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is sg^*b - open and sg^*b - closed set in X. Therefore $g \circ f$ is sg^*b continuous and contra sg^*b - continuous.
(3) Let V be any regular open set in Z. Since g is almost continuous, $g^{-1}(V)$ is open in Y. Since f is almost contra sg^*b - continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is sg^*b - closed set in X. Therefore $g \circ f$ is almost contra sg^*b - continuous. □

Theorem 3.17. *Let* $f : X \to Y$ *be a contra* sg^*b *- continuous and* $g : Y \to Z$ *be* sg^*b *- continuous. If* Y *is* Tsg^*b *- space, then* $g \circ f : X \to Z$ *is an almost contra* sg^*b *- continuous.*

Proof. Let V be any regular open and hence open set in Z. Since g is sg^*b - continuous $g^{-1}(V)$ is sg^*b - open in Y and Y is Tsg^*b - space implies $g^{-1}(V)$ open in Y. Since f is contra sg^*b - continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is sg^*b - closed set in X. Therefore, $g \circ f$ is an almost contra sg^*b - continuous. □

Theorem 3.18. *If $f : X \to Y$ is surjective strongly sg^*b - open (or strongly sg^*b - closed) and $g : Y \to Z$ is a function such that $g \circ f : X \to Z$ is an almost contra sg^*b - continuous, then g is an almost contra sg^*b - continuous.*

Proof. Let V be any regular closed (resp. regular open) set in Z. Since $g \circ f$ is an almost contra sg^*b - continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is sg^*b - open (resp. sg^*b - closed) in X. Since f is surjective and strongly sg^*b - open (or strongly sg^*b - closed), $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is sg^*b - open(or sg^*b - closed). Therefore g is an almost contra sg^*b - continuous. □

Definition 3.19. *A function $f : X \to Y$ is called weakly sg^*b - continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in sg^*b - O(X; x)$ such that $f(U) \subset cl(V)$.*

Theorem 3.20. *If a function $f : X \to Y$ is an almost contra sg^*b - continuous, then f is weakly sg^*b - continuous function.*

Proof. Let $x \in X$ and V be an open set in Y containing $f(x)$. Then $cl(V)$ is regular closed in Y containing $f(x)$. Since f is an almost contra sg^*b - continuous function by Theorem 3 (2), $f^{-1}(cl(V))$ is sg^*b - open set in X containing x. Set $U = f^{-1}(cl(V))$, then $f(U) \subset f(f^{-1}(Cl(V))) \subset cl(V)$. This shows that f is weakly sg^*b - continuous function. □

Definition 3.21. *A space X is called locally sg^*b - indiscrete if every sg^*b - open set is closed in X.*

Theorem 3.22. *If a function $f : X \to Y$ is almost contra sg^*b - continuous and X is locally sg^*b - indiscrete space, then f is almost continuous.*

Proof. Let U be a regular open set in Y. Since f is almost contra sg^*b - continuous $f^{-1}(U)$ is sg^*b - closed set in X and X is locally sg^*b - indiscrete space, which implies $f^{-1}(U)$ is an open set in X. Therefore f is almost continuous. □

Lemma 3.23. *Let A and X_0 be subsets of a space X. If $A \in sg^*b - O(X)$ and $X_0 \in \tau^\alpha$, then $A \cap X_0 \in sg^*b - O(X_0)$.*

Theorem 3.24. *If $f : X \to Y$ is almost contra sg^*b - continuous and $X_0 \in \tau^\alpha$ then the restriction $f/X_0 : X_0 \to Y$ is almost contra sg^*b - continuous.*

Proof. Let V be any regular open set of Y. By Theorem, we have $f^{-1}(V) \in sg^*b - O(X)$ and hence $(f/X_0)^{-1}(V) = f^{-1}(V) \cap X_0 \in sg^*b - O(X_0)$. By Lemma 1, it follows that f/X_0 is almost contra sg^*b - continuous. □

Theorem 3.25. *If $f : X \to \prod Y_\lambda$ is almost contra sg^*b - continuous, then $P_\lambda \circ f : X \to Y_\lambda$ is almost contra sg^*b - continuous for each $\lambda \in \nabla$, where P_λ is the projection of $\prod Y_\lambda$ onto Y_λ.*

Proof. Let Y_λ be any regular open set of Y. Since P_λ is continuous open, it is an R - map and hence $(P_\lambda)^{-1} \in RO(\prod Y_\lambda)$.
By theorem, $f^{-1}(P_\lambda^{-1}(V)) = (P_\lambda \circ f)^{-1} \in sg^*b - O(X)$. Hence $P_\lambda \circ f$ is almost contra sg^*b - continuous. □

4 Semi generalized star b - regular graphs and strongly contra semi generalized star b - closed graphs

Definition 4.1. *A graph G_f of a function $f : X \to Y$ is said to be sg^*b - regular (strongly contra sg^*b - closed) if for each $(x, y) \in (X \times Y)\backslash G_f$, there exist a sg^*b - closed set U in X containing x and $V \in R - O(Y)$ such that $(U \times V) \cap G_f = \varphi$.*

Theorem 4.2. *If $f : X \to Y$ is almost contra sg^*b - continuous and Y is T_2, then G_f is sg^*b - regular in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y)\backslash G_f$). It is obvious that $f(x) \neq y$. Since Y is T_2, there exists $V, W \in RO(Y)$ such that $f(x) \in V$, $y \in W$ and $V \cap W = \varphi$. Since f is almost contra sg^*b - continuous, $f^{-1}(V)$ is a sg^*b - closed set in X containing x. If we take $U = f^{-1}(V)$, we have $f(U) \subset V$. Hence, $f(U) \cap W = \varphi$ and G_f is sg^*b - regular. □

Theorem 4.3. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function and $g : (X, \tau) \to (X \times Y, \tau \times \sigma)$ the graph function defined by $g(x) = (x, f(x))$ for every $x \in X$. Then f is almost sg^*b - continuous if and only if g is almost sg^*b - continuous.*

Proof. **Necessary :** Let $x \in X$ and $V \in sg^*b - O(Y)$ containing $f(x)$. Then, we have $g(x) = (x, f(x)) \in R - O(X \times Y)$. Since f is almost sg^*b - continuous, there exists a sg^*b - open set U of X containing x such that $g(U) \subset X \times Y$. Therefore, we obtain $f(U) \subset V$. Hence f is almost sg^*b continuous.
Sufficiency : Let $x \in X$ and w be a regular open set of $X \times Y$ containing $g(x)$. There exists $U_1 \in RO(X, \tau)$ and $V \in RO(Y, \sigma)$ such that $(x, f(x)) \in (U_1 \times V) \subset W$. Since f is almost sg^*b - continuous, there exists $U_2 \in sg^*b - O(X, \tau)$ such that $x \in U_2$ and $f(U_2) \subset V$. Set $U = U_1 \cap U_2$. We have $x \in U_x \in sg^*b - O(X, \tau)$ and $g(U) \subset (U_1 \times V) \subset W$. This shows that g is almost sg^*b - continuous. □

Theorem 4.4. *If a function $f : X \to Y$ be a almost contra sg^*b - continuous and almost continuous, then f is regular set - connected.*

Proof. Let $V \in RO(Y)$. Since f is almost contra sg^*b - continuous and almost continuous, $f^{-1}(V)$ is sg^*b - closed and open. So $f^{-1}(V)$ is clopen. It turns out that f is regular set - connected. □

5 Connectedness

Definition 5.1. *A space X is called sg^*b - connected if X cannot be written as a disjoint union of two non - empty sg^*b - open sets.*

Theorem 5.2. *If $f : X \to Y$ is an almost contra sg^*b - continuous surjection and X is sg^*b - connected, then Y is connected.*

Proof. Suppose that Y is not a connected space. Then Y can be written as $Y = U_0 \cup V_0$ such that U_0 and V_0 are disjoint non - empty open sets. Let $U = int(cl(U_0))$ and $V = int(cl(V_0))$. Then U and V are disjoint nonempty regular open sets such that $Y = U \cup V$. Since f is almost contra sg^*b - continuous, then $f^{-1}(U)$ and $f^{-1}(V)$ are sg^*b - open sets of X. We have $X = f^{-1}(U) \cup f^{-1}(V)$ such that $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint. Since f is surjective, this shows that X is not sg^*b - connected. Hence Y is connected. □

Theorem 5.3. *The almost contra sg^*b - continuous image of sg^*b - connected space is connected.*

Proof. Let $f : X \to Y$ be an almost contra sg^*b - continuous function of a sg^*b - connected space X onto a topological space Y. Suppose that Y is not a connected space. There exist non - empty disjoint open sets V_1 and V_2 such that $Y = V_1 \cup V_2$. Therefore, V_1 and V_2 are clopen in Y. Since f is almost contra sg^*b - continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are sg^*b - open in X. Moreover, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are non - empty disjoint and $X = f^{-1}(V_1) \cup f^{-1}(V_2)$. This shows that X is not sg^*b - connected. This is a contradiction and hence Y is connected. □

Definition 5.4. *A topological space X is said to be sg^*b - ultra connected if every two non - empty sg^*b - closed subsets of X intersect.*

A topological space X is said to be hyper connected if every open set is dense.

Theorem 5.5. *If X is sg^*b - ultra connected and $f : X \to Y$ is an almost contra sg^*b - continuous surjection, then Y is hyper connected.*

Proof. Suppose that Y is not hyperconnected. Then, there exists an open set V such that V is not dense in Y. So, there exist non - empty regular open subsets $B_1 = int(cl(V))$ and $B_2 = Y - cl(V)$ in Y. Since f is almost contra sg^*b - continuous, $f^{-1}(B_1)$ and $f^{-1}(B_2)$ are disjoint sg^*b - closed. This is contrary to the sg^*b - ultra - connectedness of X. Therefore, Y is hyperconnected. □

6 Separation axioms

Definition 6.1. *A topological space X is said to be $sg^*b - T_1$ space if for any pair of distinct points x and y, there exist a sg^*b - open sets G and H such that $x \in G$, $y \notin G$ and $x \notin H$, $y \in H$.*

Theorem 6.2. *If $f : X \to Y$ is an almost contra sg^*b - continuous injection and Y is weakly Hausdorff, then X is $sg^*b - T_1$.*

Proof. Suppose Y is weakly Hausdorff. For any distinct points x and y in X, there exist V and W regular closed sets in Y such that $f(x) \in V$, $f(y) \notin V$, $f(y) \in W$ and $f(x) \notin W$. Since f is almost contra sg^*b - continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are sg^*b - open subsets of X such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $y \in f^{-1}(W)$ and $x \notin f^{-1}(W)$. This shows that X is $sg^*b - T_1$. □

Corollary 6.3. *If $f : X \to Y$ is a contra sg^*b - continuous injection and Y is weakly Hausdorff, then X is $sg^*b - T_1$.*

Definition 6.4. *A topological space X is called Ultra Hausdorff space, if for every pair of distinct points x and y in X, there exist disjoint clopen sets U and V in X containing x and y, respectively.*

Definition 6.5. *A topological space X is said to be $sg^*b - T_2$ space if for any pair of distinct points x and y, there exist disjoint sg^*b - open sets G and H such that $x \in G$ and $y \in H$.*

Theorem 6.6. *If $f : X \to Y$ is an almost contra sg^*b - continuous injective function from space X into a Ultra Hausdorff space Y, then X is $sg^*b - T_2$.*

Proof. Let x and y be any two distinct points in X. Since f is an injective $f(x) \neq f(y)$ and Y is Ultra Hausdorff space, there exist disjoint clopen sets U and V of Y containing $f(x)$ and $f(y)$ respectively. Then $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$, where $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint sg^*b - open sets in X. Therefore X is $sg^*b - T_2$. □

Definition 6.7. *A topological space X is called Ultra normal space, if each pair of disjoint closed sets can be separated by disjoint clopen sets.*

Definition 6.8. *A topological space X is said to be sg^*b - normal if each pair of disjoint closed sets can be separated by disjoint sg^*b - open sets.*

Theorem 6.9. *If $f : X \to Y$ is an almost contra sg^*b - continuous closed injection and Y is ultra normal, then X is sg^*b - normal.*

Proof. Let E and F be disjoint closed subsets of X. Since f is closed and injective $f(E)$ and $f(F)$ are disjoint closed sets in Y . Since Y is ultra normal there exists disjoint clopen sets U and V in Y such that $f(E) \subset U$ and $f(F) \subset V$. This implies $E \subset f^{-1}(U)$ and $F \subset f^{-1}(V)$. Since f is an almost contra sg^*b - continuous injection, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint sg^*b - open sets in X. This shows X is sg^*b - normal. □

Theorem 6.10. *If $f : X \to Y$ is an almost contra sg^*b - continuous and Y is semi - regular, then f is sg^*b - continuous.*

Proof. Let $x \in X$ and V be an open set of Y containing $f(x)$. By definition of semi - regularity of Y, there exists a regular open set G of Y such that $f(x) \in G \subset V$. Since f is almost contra sg^*b - continuous, there exists $U \in sg^*b - O(X, x)$ such that $f(U) \subset G$. Hence we have $f(U) \subset G \subset V$. This shows that f is sg^*b - continuous function. □

7 Compactness

Definition 7.1. *A space X is said to be:*
(1) sg^*b - compact if every sg^*b - open cover of X has a finite subcover.
(2) sg^*b - closed compact if every sg^*b - closed cover of X has a finite subcover.
(3) Nearly compact if every regular open cover of X has a finite subcover.
(4) Countably sg^*b - compact if every countable cover of X by sg^*b - open sets has a finite subcover.
(5) Countably sg^*b - closed compact if every countable cover of X by sg^*b - closed sets has a finite sub cover.
(6) Nearly countably compact if every countable cover of X by regular open sets has a finite sub cover.
(7) sg^*b - Lindelof if every sg^*b - open cover of X has a countable sub cover.
(8) sg^*b - Lindelof if every sg^*b - closed cover of X has a countable sub cover.
(9) Nearly Lindelof if every regular open cover of X has a countable sub cover.
(10) S - Lindelof if every cover of X by regular closed sets has a countable sub cover.
(11) Countably S - closed if every countable cover of X by regular closed sets has a finite sub - cover.
(12) S - closed if every regular closed cover of x has a finite sub cover.

Theorem 7.2. *Let $f : X \to Y$ be an almost contra sg^*b - continuous surjection. Then, the following properties hold:*
(1) If X is sg^*b - closed compact, then Y is nearly compact.
(2) If X is countably sg^*b - closed compact, then Y is nearly countably compact.
(3) If X is sg^*b - Lindelof, then Y is nearly Lindelof.

Proof. (1) Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. Since f is almost contra sg^*b - continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is sg^*b - closed cover of X. Since X is sg^*b - closed compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{(V_\alpha) : \alpha \in I_0\}$ which is finite sub cover of Y, therefore Y is nearly compact.
(2) Let $\{V_\alpha : \alpha \in I\}$ be any countable regular open cover of Y . Since f is almost contra sg^*b - continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is countable sg^*b - closed cover of X. Since X is countably sg^*b - closed compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{(V_\alpha) : \alpha \in I_0\}$ is finite subcover for Y . Hence Y is nearly countably compact.
(3) Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. Since f is almost contra sg^*b - continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is sg^*b - closed cover of X. Since X is sg^*b - Lindelof, there exists a countable subset I_0 of I such that $X = \{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{(V_\alpha) : \alpha \in I_0\}$ is finite sub cover for Y . Therefore, Y is nearly Lindelof. □

Theorem 7.3. *Let $f : X \to Y$ be an almost contra sg^*b - continuous surjection. Then, the following properties hold:*
(1) If X is sg^*b - compact, then Y is S - closed.
(2) If X is countably sg^*b - closed, then Y is is countably S - closed.
(3) If X is sg^*b - Lindelof, then Y is S - Lindelof.

Proof. (1) Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of Y. Since f is almost contra sg^*b - continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is sg^*b - open cover of X. Since X is sg^*b - compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite sub cover for Y. Therefore, Y is S - closed.
(2) Let $\{V_\alpha : \alpha \in I\}$ be any countable regular closed cover of Y. Since f is almost contra sg^*b - continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is countable sg^*b - open cover of X. Since X is countably sg^*b - compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite sub cover for Y. Hence, Y is countably S - closed.
(3) Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of Y. Since f is almost contra sg^*b - continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is sg^*b - open cover of X. Since X is sg^*bsg^*b - Lindelof, there exists a countable sub - set I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite sub cover for Y. Hence, Y is S - Lindelof. □

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