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Contra Pre Generalized b - Continuous Functions in Topological Spaces

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Abstract

In this paper, the authors introduce a new class of functions called contra pre generalized b - continuous function (briefly contra pgb - continuous) in topological spaces. Some characterizations and several properties concerning contra pre generalized b - continuous functions are obtained.

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1 Introduction

In 1970, Dontchev [7, 15] introduced the notions of contra continuous function. A new class of function called contra b-continuous function introduced by Nasef [6]. In 2009, Omari and Noorani [1] have studied further properties of contra b-continuous functions. In this paper, we introduce the concept of contra pgb-continuous function via the notion of pgb-open set and study some of the applications of this function. We also introduce and study two new spaces called pgb-Hausdorff spaces, pgb-normal spaces and obtain some new results.

Through out this paper (X, τ) and (Y, σ) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. Let $A \subseteq X$, the closure of A and interior of A will be denoted by $cl(A)$ and $int(A)$ respectively, union of all pgb-open sets X contained in A is called pgb-interior of A and it is denoted by pgb-$int(A)$, the intersection of all pgb-closed sets of X containing A is called pgb-closure of A and it is denoted by pgb-$cl(A)$.

2 Preliminaries

Definition 2.1. *Let a subset A of a topological space (X, τ), is called*

1) *a pre-open set [12] if $A \subseteq int(cl(A))$.*

2) *a semi-open set [8] if $A \subseteq cl(int(A))$.*

3) *a α -open set [14] if $A \subseteq int(cl(int(A)))$.*

4) *a b -open set [2] if $A \subseteq cl(int(A)) \cup int(cl(A))$.*

5) *a generalized closed set (briefly g-closed)[4] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.*

6) *a generalized α- closed set (briefly $g\alpha$- closed) [8] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α open in X.*

7) *a generalized b- closed set (briefly gb- closed) [1] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.*

8) *a generalized $\alpha*$ closed set (briefly $g\alpha*$-closed)[11] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α open in X.*

9) *a pre generalized closed set (briefly pg- closed) [16] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is pre open in X.*

10) *a semi generalized closed set (briefly sg- closed) [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.*

11) *a generalized αb closed set (briefly $g\alpha b$- closed) [10] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is α open in X.*

12) *a regular generalized b -closed set (briefly rgb-closed)[12] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.*

13) *a pre generalized b -closed set (briefly pgb- closed) [17] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is pre open in X.*

Definition 2.2. *A function $f : (X, \tau) \to (Y, \sigma)$, is called*

1) *a contra continuous [5] if $f^{-1}(V)$ is closed in (X, τ) for every open set V of (Y, σ).*

2) *a contra b-continuous [13] if $f^{-1}(V)$ is b-closed in (X, τ) for every open set V of (Y, σ).*

3) *a contra $g\alpha$-continuous [8] if $f^{-1}(V)$ is $g\alpha$-closed in (X, τ) for every open set V of (Y, σ).*

4) *a contra $g\alpha*$-continuous [11] if $f^{-1}(V)$ is $g\alpha*$-closed in (X, τ) for every open set V of (Y, σ).*

5) *a contra g-continuous [9] if $f^{-1}(V)$ is g-closed in (X, τ) for every open set V of (Y, σ).*

6) *a contra $g\alpha b$-continuous [18] if $f^{-1}(V)$ is $g\alpha b$-closed in (X, τ) for every open set V of (Y, σ).*

7) *a contra rgb-continuous [11] if $f^{-1}(V)$ is rgb-closed in (X, τ) for every open set V of (Y, σ).*

3 On Contra Pre Generalized b - Continuous Functions

In this section, we introduce contra pre generalized b - continuous functions and investigate some of their properties.

Definition 3.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is called contra pre generalized b - continuous if $f^{-1}(V)$ is pgb - closed in (X, τ) for every open set V in (Y, σ).*

Example 3.2. *Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma = \{Y, \varphi, \{a, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$. Clearly f is contra pgb - continuous.*

Definition 3.3. *Let A be a subset of a space (X, τ).*

(i) *The set $\cap\{F \subset X : A \subset F, F \text{ is } pgb - closed\}$ is called the pgb - closure of A and it is denoted by $pgb - cl(A)$.*

(ii) *The set $\cup\{G \subset X : G \subset A, G \text{ is } pgb - open\}$ is called the pgb - interior of A and it is denoted by $pgb - int(A)$.*

Lemma 3.4. *For $x \in X$, $x \in pgb - cl(A)$ if and only if $U \cap A \neq \phi$ for every pgb - open set U containing x.*

Proof. **Necessary part :** Suppose there exists a pgb - open set U containing x such that $U \cup A = \varphi$. Since $A \subset X - U$, $pgb - cl(A) \subset X - U$. This implies $x \notin pgb - cl(A)$. This is a contradiction.
Sufficiency part : Suppose that $x \notin pgb - cl(A)$. Then $\exists$ a pgb - closed subset F containing A such that $x \notin F$. Then $x \in X - F$ is pgb - open, $(X - F) \cap A = \varphi$. This is contradiction. □

Lemma 3.5. *The following properties hold for subsets A, B of a space X:*

(i) $x \in ker(A)$ if and only if $A \cap F \neq \phi$ for any $F \in (X, x)$.

(ii) $A \subset ker(A)$ and $A = ker(A)$ if A is open in X.

(iii) If $A \subset B$, then $ker(A) \subset ker(B)$.

Theorem 3.6. *Let $f : (X, \tau) \to (Y, \sigma)$ be a map. The following conditions are equivalent:*

(i) f is contra pgb - continuous,

(ii) The inverse image of each closed in (Y, σ) is pgb - open in (X, τ),

(iii) For each $x \in X$ and each $F \in C(Y, f(x))$, there exists $U \in pgb - O(X)$, such that $f(U) \subset F$,

(iv) $f(pgb - cl(X)) \subset ker(f(A))$, for every subset A of X,

(v) $pgb - cl(f^{-1}(B)) \subset f^{-1}(ker(B))$, for every subset B of Y.

Proof. (i) $\Leftrightarrow$ (ii) and (ii) $\Rightarrow$ (iii) are obvious.
(iii $\to$ (ii) : Let F be any closed set of Y and $x \in f^{-1}(F)$. Then $f(x) \in F$ and there exists $U_x \in pgb - O(X, x)$ such that $f(U_x) \subset F$.
Hence we obtain $f^{-1}(F) = \bigcup \{U_x \backslash x \in f^{-1}(F)\} \in pgb - O(X, x)$. Thus the inverse of each closed set in (Y, σ) is pgb - open in (X, τ).
(ii) $\Rightarrow$ (iv) : Let A be any subset of X. Suppose that $y \notin ker f(A))$. By lemma there exists $F \in C(Y, y)$ such that $f(A) \cap F = \varphi$. Then, we have $A \cap f^{-1}(F) = \varphi$ and $pgb - cl(A) \cap f^{-1}(F) = \varphi$.
Therefore, we obtain $f\big(pgb - cl(A)\big) \cap F = \varphi$ and $y \notin f\big(pgb - cl(A)\big)$. Hence we have $f\big(pgb - cl(X)\big) \subset ker\big(f(A)\big)$.
(iv) $\Rightarrow$ (v): Let B be any subset of Y. By (iv) and Lemma, We have
$f\Big(pgb - cl\big(f^{-1}(B)\big)\Big) \subset \Bigg(ker\Big(f\big(f^{-1}(B)\big)\Big)\Bigg) \subset ker(B)$ and
$pgb - cl\big(f^{-1}(B)\big) \subset f^{-1}\big(ker(B)\big)$.

(v) $\Rightarrow$ (i): Let V be any open set of Y. By lemma we have $pgb-cl\big(f^{-1}(V)\big) \subset f^{-1}\big(ker(V)\big) = f^{-1}(V)$ and $pgb-cl\big(f^{-1}(V)\big) = f^{-1}(V)$. It follows that $f^{-1}(V)$ is pgb - closed in X. We have f is contra pgb - continuous. □

Definition 3.7. *A function $f : (X, \tau) \to (Y, \sigma)$ is called pgb - continuous if the preimage of every open set of Y is pgb - open in X.*

Remark 3.8. *The following two examples will show that the concept of pgb - continuity and contra pgb - continuity are independent from each other.*

Example 3.9. *Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{b\}, \{a, b\}, \{b, c\}\}$ and $\sigma = \{Y, \varphi, \{a, b\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$. Clearly f is contra pgb - continuous but f is not pgb - continuous. Because $f^{-1}(\{a, b\}) = \{a, c\}$ is not pgb - open in (X, τ) where $\{a, b\}$ is open in (Y, σ).*

Example 3.10. *Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ and $\sigma = \{Y, \varphi, \{b\}, \{b, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by identity function. Clearly f is pgb - continuous but f is not contra pgb - continuous. Because $f^{-1}(\{b, c\}) = \{b, c\}$ is not contra pgb - closed in (X, τ) where $\{a, b\}$ is open in (Y, σ).*

Theorem 3.11. *If a function $f : (X, \tau) \to (Y, \sigma)$ is contra pgb - continuous and (Y, σ) is pre then f is pgb - continuous.*

Proof. Let x be an arbitrary point of (X, τ) and V be an open set of (Y, σ) containing $f(x)$. Since (Y, σ) is regular, there exists an open set W of (Y, σ) containing $f(x)$ such that $cl(W) \subset V$. Since f is contra pgb - continuous, by Theorem there exists $U \in pgb - O(X, x)$ such that $f(U) \subset cl(W)$. Then $f(U) \subset cl(W) \subset V$. Hence f is pgb - continuous. □

Theorem 3.12. *Every contra - continuous function is contra pgb - continuous function.*

Proof. Let V be an open set in (Y, σ). Since f is contra - continuous function, $f^{-1}(V)$ is b - closed in (X, τ). Every b-closed set is pgb - closed. Hence $f^{-1}(V)$ is pgb - closed in (X, τ). Thus f is contra pgb - continuous function. □

Remark 3.13. *The converse of theorem need not be true as shown in the following example.*

Example 3.14. *Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma = \{Y, \varphi, \{a, b\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$. Clearly f is contra pgb - continuous but f is not contra b-continuous. Because $f^{-1}(\{a, b\}) = \{a, c\}$ is not b-closed in (X, τ) where $\{a, b\}$ is open in (Y, σ).*

Theorem 3.15. *(i) Every contra gα-continuous function is contra pgb-continuous function.*

(ii) Every contra g$\alpha *$-continuous function is contra pgb-continuous function.

(iii) Every contra g - continuous function is contra pgb-continuous function.

(iv) Every contra gαb-continuous function is contra pgb-continuous function.

(v) Every contra rgb-continuous function is contra pgb-continuous function.

(vi) Every contra pgb-continuous function is contra pg-continuous function.

Remark 3.16. *Converse of the above statements is not true as shown in the following example.*

Example 3.17. *(i) Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma = \{Y, \varphi, \{a\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$. Clearly f is contra pgb - continuous but f is not contra gα-continuous. Because $f^{-1}(\{a\}) = \{c\}$ is not gα-closed in (X, τ) where $\{a\}$ is open in (Y, σ).*

(ii) Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \varphi, \{c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = a$, $f(c) = b$. Clearly f is contra pgb-continuous but f is not contra g$\alpha *$-continuous. Because $f^{-1}(\{c\}) = \{a\}$ is not g$\alpha *$-closed in (X, τ) where $\{c\}$ is open in (Y, σ).

(iii) Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \varphi, \{b\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$. Clearly f is contra pgb-continuous but f is not contra g-continuous. Because $f^{-1}(\{b\}) = \{b\}$ is not g-closed in (X, τ) where $\{b\}$ is open in (Y, σ).

(iv) Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and $\sigma = \{Y, \varphi, \{c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = a$, $f(c) = b$. Clearly f is contra pgb - continuous but f is not contra gαb - continuous. Because $f^{-1}(\{c\}) = \{b\}$ is not gαb- closed in (X, τ) where $\{c\}$ is open in (Y, σ).

(v) Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \varphi, \{b\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$. Clearly f is contra pgb - continuous but f is not contra rgb-continuous. Because $f^{-1}(\{b\}) = \{a\}$ is not rgb-closed in (X, τ) where $\{b\}$ is open in (Y, σ).

(vi) *Let* $X = Y = \{a, b, c\}$ *with* $\tau = \{X, \varphi, \{a, b\}\}$ *and* $\sigma = \{Y, \varphi, \{b\}, \{a, b\}\}$. *Define a function* $f : (X, \tau) \to (Y, \sigma)$ *by* $f(a) = b$, $f(b) = a$, $f(c) = c$. *Clearly* f *is contra pgb - continuous but* f *is not contra pgb - continuous. Because* $f^{-1}(\{a, b\}) = \{a, b\}$ *is not pgb - closed in* (X, τ) *where* $\{a, b\}$ *is open in* (Y, σ).

Definition 3.18. *A space* (X, τ) *is said to be (i) pgb - space if every pgb - open set of* X *is open in* X, *(ii) locally pgb - indiscrete if every pgb - open set of* X *is closed in* X.

Theorem 3.19. *If a function* $f : X \to Y$ *is contra pgb - continuous and* X *is pgb - space then* f *is contra continuous.*

Proof. Let $V \in O(Y)$. Then $f^{-1}(V)$ is *pgb* - closed in X. Since X is *pgb* - space, $f^{-1}(V)$ is closed in X. Hence f is contra continuous. □

Theorem 3.20. *Let* X *be locally pgb - indiscrete. If* $f : X \to Y$ *is contra pgb - continuous, then it is continuous.*

Proof. Let $V \in O(Y)$. Then $f^{-1}(V)$ is *pgb* - closed in X. Since X is locally *pgb* - indiscrete space, $f^{-1}(V)$ is open in X. Hence f is continuous. □

Definition 3.21. *A function* $f : X \to Y$, *the subset* $\{(x, f(x)) : x \in X\} \subset X \times Y$ *is called the graph of* f *and is denoted by* G_f.

Definition 3.22. *The graph* G_f *of a function* $f : X \to Y$ *is said to be contra pgb - closed if for each* $(x, y) \in (X \times Y) - G_f$ *there exists* $U \in pgb - O(X, x)$ *and* $V \in C(Y, y)$ *such that* $(U \times V) \cap G_f$.

Theorem 3.23. *If a function* $f : X \to Y$ *is contra pgb - continuous and* Y *is Urysohn, then* G_f *is contra pgb - closed in the product space* $X \times Y$.

Proof. Let $(x, y) \in (X \times Y) - G_f$. Then $y \neq f(x)$ and there exist open sets H_1, H_2 such that $f(x) \in H_1$, $y \in H_2$ and $cl(H_1) \cap cl(H_2) = \varphi$. From hypothesis, there exists $V \in pgb - O(X, x)$ such that $f(V) \subset cl(H_1)$. Therefore, we have $f(V) \cap cl(H_2) = \varphi$. This shows that G_f is contra *pgb* - closed in the product space $X \times Y$. □

Theorem 3.24. *If* $f : X \to Y$ *is pgb - continuous and* Y *is* T_1, *then* G_f *is contra pgb - closed in* $X \times Y$.

Proof. Let $(x, y) \in (X \times Y) - G_f$. Then $y \neq f(x)$ and there exist open set V of Y such that $f(x) \in V$ and $y \notin V$. Since f is *pgb* - continuous, there exists $U \in pgb - O(X, x)$ such that $f(U) \subset V$. Therefore, we have $f(U) \cap (Y - V) = \varphi$ and $(Y - V) \in pgb - C(Y, y)$. This shows that G_f is contra *pgb* - closed in $X \times Y$. □

Theorem 3.25. *Let $f : X \to Y$ be a function and $g : X \to X \times Y$, the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is contra pgb - continuous, then f is contra pgb - continuous.*

Proof. Let U be an open set in Y, then $X \times U$ is an open set in $X \times Y$. Since g is contra *pgb* - continuous. It follows that $f^{-1}(U) = g^{-1}(X \times U)$ is an *pgb* - closed in X. Hence f is *pgb* - continuous. □

Theorem 3.26. *If $f : X \to Y$ is a contra pgb - continuous function and $g : Y \to Z$ is a continuous function, then $g \circ f : X \to Z$ is contra pgb - continuous.*

Proof. Let $V \in O(Y)$. Then $g^{-1}(V)$ is open in Y. Since f is contra *pgb* - continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is *pgb* - closed in X. Therefore, $g \circ f : X \to Z$ is contra *pgb* - continuous. □

Theorem 3.27. *Let $p : X \times Y \to Y$ be a projection. If A is pgb - closed subset of X, then $p^{-1}(A) = A \times Y$ is pgb - closed subset of $X \times Y$.*

Proof. Let $A \times Y \subset U$ and U be a regular open set of $X \times Y$. Then $U = V \times Y$ for some regular open set V of X. Since A is *pgb* - closed in X, $bcl(A)$ and so $bcl(A) \times Y \subset V \times Y = U$. Therefore $bcl(A \times Y) \subset U$. Hence $A \times Y$ is *pgb* - closed sub set of $X \times Y$. □

4 Applications

Definition 4.1. *A topological space (X, τ) is said to be pgb - Hausdorff space if for each pair of distinct points x and y in X there exists $U \in pgb - O(X, x)$ and $V \in pgb - O(X, y)$ such that $U \cap V = \varphi$.*

Example 4.2. *Let $X = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. Let x and y be two distinct points of X, there exists an pgb-open neighbourhood of x and y respectively such that $\{x\} \cap \{y\} = \varphi$. Hence (X, τ) is pgb-Hausdorff space.*

Theorem 4.3. *If X is a topological space and for each pair of distinct points x_1 and x_2 in X, there exixts a function f of X into Uryshon topological space Y such that $f(x_1) \neq f(x_2)$ and f is contra pgb-continuous at x_1 and x_2, then X is pgb-Hausdorff space.*

Proof. Let x_1 and x_2 be any distinct points in X. By hypothesis, there is a Uryshon space Y and a function $f : X \to Y$ such that $f(x_1) \neq f(x_2)$ and f is contra *pgb*-continuous at x_1 and x_2. Let $y_i = f(x_i)$ for $i = 1, 2$ then $y_1 \neq y_2$. Since Y is Uryshon, there exists open sets U_{y_1} and U_{y_2} containing y_1

and y_2 respectively in Y such that $cl(U_{y_1}) \cap cl(U_{y_2}) = \varphi$. Since f is contra *pgb*-continuous at x_1 and x_2, there exists and *pgb*-open sets V_{x_1} and V_{x_2} containing x_1 and x_2 respectively in X such that $f(V_{xi}) \subset cl(U_{yi})$ for $i = 1, 2$. Hence we have $(V_{x_1}) \cap (V_{x_2}) = \varphi$. Therefore X is *pgb* - Hausdorff space. □

Corollary 4.4. *If f is contra pgb - continuous injection of a topological space X into a Uryshon space Y then X is pgb-Hausdorff.*

Proof. Let x_1 and x_2 be any distinct points in X. By hypothesis, f is contra *pgb*-continuous function of X into a Uryshon space Y such that $f(x_1) \neq f(x_2)$, because f is injective. Hence by theorem, X is *pgb* - Hausdorff. □

Definition 4.5. *A topological space (X, τ) is said to be pgb - normal if each pair of non - empty disjoint closed sets in (X, τ) can be separated by disjoint pgb - open sets in (X, τ).*

Definition 4.6. *A topological space (X, τ) is said to be ultra normal if each pair of non - empty disjoint closed sets in (X, τ) can be separated by disjoint clopen sets in (X, τ).*

Theorem 4.7. *If $f : X \to Y$ is a contra pgb - continuous function, closed, injection and Y is Ultra normal, then X is pgb - normal.*

Proof. Let U and V be disjoint closed subsets of X. Since f is closed and injective, $f(U)$ and $f(V)$ are disjoint subsets of Y. Since Y is ultra normal, there exists disjoint clopen sets A and B such that $f(U) \subset A$ and $f(V) \subset B$. Hence $U \subset f^{-1}(A)$ and $V \subset f^{-1}(B)$. Since f is contra *pgb* - continuous and injective, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint *pgb* - open sets in X. Hence X is *pgb* - normal. □

Definition 4.8. *A topological space X is said to be pgb - connected if X is not the union of two disjoint non - empty pgb - open sets of X.*

Theorem 4.9. *A contra pgb - continuous image of a pgb - connected space is connected.*

Proof. Let $f : X \to Y$ be a contra *pgb* - continuous function of *pgb* - connected space X onto a topological space Y. If possible, let Y be disconnected. Let A and B form disconnectedness of Y. Then A and B are clopen and $Y = A \cup B$ where $A \cap B = \varphi$. Since f is contra *pgb* - continuous, $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are non - empty *pgb* - open sets in X. Also $f^{-1}(A) \cap f^{-1}(B) = \varphi$. Hence X is non - *pgb* - connected which is a contradiction. Therefore Y is connected. □

Theorem 4.10. *Let X be pgb - connected and Y be T_1. If $f : X \to Y$ is a contra pgb - continuous, then f is constant.*

Proof. Since Y is T_1 space $v = \{f^{-1}(y) : y \in Y\}$ is a disjoint *pgb* - open partition of X. If $|v| \geq 2$, then X is the union of two non empty *pgb* - open sets. Since X is *pgb* - connected, $|v| = 1$. Hence f is constant. □

Theorem 4.11. *If $f : X \to Y$ is a contra pgb - continuous function from pgb - connected space X onto space Y, then Y is not a discrete space.*

Proof. Suppose that Y is discrete. Let A be a proper non - empty open and closed subset of Y. Then $f^{-1}(A)$ is a proper non - empty *pgb* - clopen subset of X, which is a contradiction to the fact X is *pgb* - connected. □

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