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Weakly Symmetric and Weakly Concircular Symmetric $N(k)$-Contact Metric Manifolds

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Abstract

The object of the present paper is to study weakly symmetric, weakly Ricci symmetric, weakly concircular symmetric and weakly concircular Ricci symmetric $N(k)$-contact metric manifolds.

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1 Introduction

Tamassy and Binh in their articles [21] and [22] respectively introduced the notion of weakly symmetric manifolds and weakly Ricci symmetric manifolds. There after many geometers studied these conditions on different manifolds [[6], [9], [19], [22], [27]].

The notion of weakly concircular symmetric manifold was introduced by Shaikh and Hui [18]. Recently, several authors investigated the concircular symmetries on Kenmotsu manifolds [14], Trans-Sasakian manifolds [[15], [16]], Lorentzian concircular structure manifolds [13], generalized Sasakian space forms [25], (ϵ)-trans Sasakian manifolds [20], etc.

The concircular curvature tensor is the most important curvature tensor from the Riemannian point of view. A transformation of an n-dimensional Riemannian manifold M, which transforms every geodesic circle of M into a geodesic circle, is called a concircular transformation [26]. A concircular transformation is always a conformal transformation. Here geodesic circle means a curve in M whose first curvature is constant and whose second curvature is identically zero. An interesting invariant of a concircular transformation is the concircular curvature tensor C [26].

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A concircular curvature tensor of $(2n+1)$-dimensional $N(k)$-contact metric manifolds is given by

$$\begin{aligned} C(Y,Z,U,V) &= R(Y,Z,U,V) \\ &- \frac{r}{2n(2n+1)}\{g(Z,U)g(Y,V)-g(Y,U)g(Z,V)\}, \end{aligned} \tag{1}$$

where r is the scalar curvature of the manifold.

If $\{e_i : i = 1, 2, \cdots, 2n+1\}$ is an orthonormal basis of the tangent space at each point of the manifold and if

$$\bar{C}(Y,V) = \sum_{i=1}^{2n+1} C(Y,e_i,e_i,V), \tag{2}$$

then in view of (1), we obtain

$$\bar{C}(Z,U) = S(Z,U) - \frac{r}{2n+1}g(Z,U). \tag{3}$$

The organization of this paper is as follows: In section 2, we recall some necessary notations and terminologies. Sections 3, 4, 5 and 6 are respectively devoted to the study of weakly symmetric, weakly Ricci symmetric, weakly concircular symmetric and weakly concircular Ricci symmetric $N(k)$-contact metric manifolds.

2 Preliminaries

A $(2n+1)$-dimensional smooth manifold M^{2n+1} is said to have an almost Contact structure (ϕ,ξ,η) if it carries tensor field ϕ of type $(1,1)$, a characteristic vector field ξ and a global 1-form η satisfying

$$\phi^2 = -I + \eta\otimes\xi, \quad \eta(\xi)=1, \quad \phi\xi = 0, \quad \eta\circ\phi = 0. \tag{4}$$

An almost Contact structure is said to be normal if the induced almost complex structure J on the product manifold $M^{2n+1}\times R$ defined by

$$J\left(X,\lambda\frac{d}{dt}\right) = \left(\phi X - \lambda\xi, \eta(X)\frac{d}{dt}\right),$$

is integrable, where X is tangent to M^{2n+1}, t is the coordinate of R and λ a smooth function on $M^{2n+1}\times R$. The condition of almost contact metric structure being normal is equivalent to vanishing of the torsion tensor $[\phi,\phi] + 2d\eta\otimes\xi$, where $[\phi,\phi]$ is the Nijenhuis tensor of ϕ.

Let g be the compatible Riemannian metric with almost Contact structure (ϕ, ξ, η) such that,

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y),\ g(X,\xi) = \eta(X),\ g(\phi X, Y) = g(X, \phi Y),\ (5)$$

where X, Y are vector fields defined on M^{2n+1}. Then the structure (ϕ, ξ, η, g) on M^{2n+1} is said to have an almost contact metric structure. A manifold M^{2n+1} together with this almost Contact metric structure is said to be almost Contact metric manifold and it is denoted by $M^{2n+1}(\phi, \xi, \eta, g)$. An almost Contact metric structure becomes a contact metric structure if $g(X, \phi Y) = d\eta(X,Y)$, for all vector fields X, Y.

Given a contact metric manifold M^{2n+1}, we define a $(1,1)$-tensor field h by $h = \frac{1}{2}\mathscr{L}_\xi \phi$, where $\mathscr{L}$ denotes the Lie differentiation. Then the tensor h is symmetric and satisfies

$$h\xi = 0, \qquad h\phi + \phi h = 0, \qquad \nabla_X \xi = -\phi X - \phi h X, \tag{6}$$

where ∇ denotes the Riemannian connection of g.

Blair et al [4] introduced the (k, μ)-nullity distribution on a Contact metric manifold. The (k,μ)-nullity distribution $N(k,\mu)$ of a contact metric manifold M is defined by $N(k,\mu) : p \to N_p(k,\mu)$

$$N_p(k,\mu) = \{U \in T_pM \mid R(X,Y)U = (kI + \mu h)g(Y,U)X - g(X,U)Y\},$$

for all $X, Y \in TM$, where $(k,\mu) \in R^2$. A Contact metric manifold with $\xi \in N(k,\mu)$ is called a (k,μ)-contact metric manifold. In particular on a (k,μ)-contact metric manifold, we have

$$R(X,Y)\xi = k[\eta(Y)X - \eta(X)Y] + \mu[\eta(Y)hX - \eta(X)hY]. \tag{7}$$

On a (k,μ)-contact manifold ($k \leq 1$) if $k = 1$, the structure is Sasakian ($h = 0$ and μ is indeterminant) and if $k < 1$, then the (k,μ)-nullity condition determines the curvature of M completely.

If $\mu = 0$, the (k,μ)-nullity distribution reduces to k-nullity distribution. The k-nullity distribution $N(k)$ of a Riemannian manifold is defined by [23]

$$N(k) : p \to N_p(k) = \{U \in T_pM \mid R(X,Y)U = k[g(Y,U)X - g(X,U)Y]\},$$

k being a constant. If the characteristic vector field $\xi \in N(k)$, then we call a contact metric manifold as $N(k)$-contact metric manifold [5]. If $k = 1$, then the manifold is Sasakian and if $k = 0$, then the manifold is locally isometric to the product $E^{n+1}(0) \times S^n(4)$ for $n > 1$ and flat for $n = 1$ [3].

However, for a $N(k)$-contact metric manifold M^{2n+1} of dimension $(2n+1)$, we have [5]:

$$\begin{aligned} h^2 &= (k-1)\phi^2, & (8)\\ (\nabla_X\phi)(Y) &= g(X+hX,Y)\xi - \eta(Y)(X+hX), & (9)\\ R(X,Y)\xi &= k[\eta(Y)X-\eta(X)Y], & (10)\\ R(\xi,X)Y &= k[g(X,Y)\xi-\eta(Y)X], & (11)\\ S(X,Y) &= 2(n-1)g(X,Y)+2(n-1)g(hX,Y) & \\ &+ \{2nk-2(n-1)\}\eta(X)\eta(Y), \quad n\geq 1 & (12)\\ S(\phi X,\phi Y) &= S(X,Y)-2nk\eta(X)\eta(Y)-4(n-1)g(hX,Y), & (13)\\ S(X,\xi) &= 2nk\eta(X), & (14)\\ (\nabla_X\eta)(Y) &= g(X+hX,\phi Y). & (15) \end{aligned}$$

3 Weakly symmetric $N(k)$-contact metric manifold

A non-flat Riemannian manifold $(M^n,g)(n>2)$ is called weakly symmetric [21] if its curvature tensor R of type $(0,4)$ satisfies the condition

$$(\nabla_X R)(Y,Z,U,V) = A(X)R(Y,Z,U,V)+B(Y)R(X,Z,U,V)$$
$$+H(Z)R(Y,X,U,V)+D(U)R(Y,Z,X,V)+E(V)R(Y,Z,U,X), \qquad (16)$$

where A,B,D,E,H are 1-forms and $U,V,X,Y,Z\in\chi(M^n)$.

In 1999, De and Bandyopadhyay [11] studied a weakly symmetric manifolds and proved that in such a manifold the associated 1-forms $B=H$ and $D=E$. So, equation (16) reduces to

$$(\nabla_X R)(Y,Z,U,V) = A(X)R(Y,Z,U,V)+B(Y)R(X,Z,U,V)$$
$$+B(Z)R(Y,X,U,V)+D(U)R(Y,Z,X,V)+D(V)R(Y,Z,U,X). \qquad (17)$$

Definition 3.1 *A $N(k)$-contact metric manifold M^{2n+1} is said to be weakly symmetric if its curvature tensor R of type (0, 4) satisfies (17).*

Suppose $N(k)$-contact metric manifold is weakly symmetric, then from (17) we have

$$(\nabla_X S)(Z,U) = A(X)S(Z,U)+B(Z)S(X,U)+B(R(X,Z)U)$$
$$+D(U)S(X,Z)+D(R(X,U)Z). \qquad (18)$$

Setting $U=\xi$ in (18) and using (10), (11), (14) in the resulting equation, one can get

$$(\nabla_X S)(Z,\xi) = 2nkA(X)\eta(Z)+2nkB(Z)\eta(X)+k\eta(Z)B(X)$$
$$-k\eta(X)B(Z)+D(\xi)S(Z,X)+k\eta(Z)D(X)-kg(X,Z)D(\xi). \qquad (19)$$

We know that

$$(\nabla_X S)(Z,\xi) = \nabla_X S(Z,\xi) - S(\nabla_X Z,\xi) - S(Z,\nabla_X \xi), \tag{20}$$

which by virtue of (6), (14) and (15) yields,

$$(\nabla_X S)(Z,\xi) = 2nkg(X+hX,\phi Z) + S(Z,\phi X+\phi hX). \tag{21}$$

In view of (19) and (21), we get

$$\begin{aligned} 2nkg(X+hX,\phi Z) + S(Z,\phi X+\phi hX) = 2nkA(X)\eta(Z) \\ +2nkB(Z)\eta(X) + k\eta(Z)B(X) - k\eta(X)B(Z) + D(\xi)S(Z,X) \\ +k\eta(Z)D(X) - kg(X,Z)D(\xi). \end{aligned} \tag{22}$$

Substituting $X = Z = \xi$ in (22) and then making use of equations (4) and (14), we have

$$2nk\{A(\xi) + B(\xi) + D(\xi)\} = 0. \tag{23}$$

Equation (23) implies that

$$\text{either } k = 0 \text{ or } A(\xi) + B(\xi) + D(\xi) = 0.$$

This leads to the following:

Theorem 3.2 *A weakly symmetric $N(k)$-contact metric manifold M^{2n+1} is either locally isometric to the product $E^{n+1} \times S^n(4)$ for $n > 1$ and flat for $n = 1$ or M^{2n+1} satisfies $A(\xi) + B(\xi) + D(\xi) = 0$.*

Now we look at the case when $k \neq 0$. From (23), we have

$$A(\xi) + B(\xi) + D(\xi) = 0. \tag{24}$$

Replacing X by ξ in (22) and then using (4), (14) and (24), we obtain

$$B(Z) = B(\xi)\eta(Z). \tag{25}$$

Taking an orthonormal frame field at any point of the manifold and then contracting over Z and U in (17) we get

$$\begin{aligned} (\nabla_X S)(Y,V) = A(X)S(Y,V) + B(Y)S(X,V) + B(R(X,Y)V) \\ +D(V)S(X,Y) + D(R(X,V)Y). \end{aligned} \tag{26}$$

Putting $X = Y = \xi$ in (26) and by virtue of (4), (10), (11), (14) and (24), above equations takes the form

$$D(V) = D(\xi)\eta(V). \tag{27}$$

In a similar manner we can obtain

$$A(X) = A(\xi)\eta(X). \tag{28}$$

Adding (25), (27) and (28) and in view of (24), one can get

$$A(X) + B(X) + D(X) = 0. \tag{29}$$

Thus we can state the following assertion:

Theorem 3.3 *There is no weakly symmetric $N(k)$-contact metric manifold M^{2n+1} for $k \neq 0$, unless $A + B + D$ is everywhere zero.*

As we know that $N(k)$-contact metric manifold reduces to Sasakian manifold for $k = 1 \neq 0$ (see [2]), in view of Theorem 3.3 we have the following:

Corollary 3.4 *There is no weakly symmetric Sasakian manifold, unless $A + B + D$ is everywhere zero.*

Above corollary has been proved by Tamassy and Binh [22] and De et al [8] for Sasakian manifold.

4 Weakly Ricci symmetric $N(k)$-contact metric manifold

A Riemannian manifold $(M^n, g)(n > 2)$ is called weakly Ricci symmetric manifold [22] if its Ricci tensor S of type $(0, 2)$ is not identically zero and satisfies

$$(\nabla_X S)(Y, Z) = A(X)S(Y, Z) + B(Y)S(X, Z) + D(Z)S(Y, X), \tag{30}$$

where A, B and D are three non-zero 1-forms, called the associated 1-forms of the manifold.

Definition 4.1 *A $N(k)$-contact metric manifold M^{2n+1} is said to be weakly Ricci symmetric if there exists 1-forms A, B, D, satisfying (30).*

Theorem 4.2 *A weakly Ricci symmetric $N(k)$-contact metric manifold M^{2n+1} is either locally isometric to the product $E^{n+1} \times S^n(4)$ for $n > 1$ and flat for $n = 1$ or M^{2n+1} satisfies $A(\xi) + B(\xi) + D(\xi) = 0$.*

Proof: Let us consider a weakly Ricci-symmetric $N(k)$-contact metric manifold M^{2n+1}.
Substituting $Z = \xi$ in (30) and in view of (14) and (21), (30) yields

$$\begin{aligned} 2nkg(X + hX, \phi Y) + S(Y, \phi X + \phi hX) = 2nkA(X)\eta(Y) \\ +2nkB(Y)\eta(X) + D(\xi)S(X, Y). \end{aligned} \tag{31}$$

Setting $X = Y = \xi$ in (31) and using (4) and (14), it follows that

$$2nk\{A(\xi) + B(\xi) + D(\xi)\} = 0. \tag{32}$$

From above equation we have, either

$$k = 0, \tag{33}$$

or

$$A(\xi) + B(\xi) + D(\xi) = 0. \tag{34}$$

These equations provides the proof of the theorem 4.2.

Further, we consider the case when $k \neq 0$. Replacing Y by ξ in (31) and by virtue of (14) and (34), (31) takes the form

$$A(X) = A(\xi)\eta(X). \tag{35}$$

Proceeding in a similar way one can get

$$B(X) = B(\xi)\eta(X), \tag{36}$$

$$D(X) = D(\xi)\eta(X). \tag{37}$$

Adding (35), (36) and (37) and using (34) we obtain

$$A(X) + B(X) + D(X) = 0. \tag{38}$$

This leads the following assertion:

Theorem 4.3 *There is no weakly Ricci symmetric $N(k)$-contact metric manifold M^{2n+1} for $k \neq 0$, unless $A + B + D$ is everywhere zero.*

Corollary 4.4 *There is no weakly Ricci symmetric Sasakian manifold, unless $A + B + D$ is everywhere zero.*

This result being proved in [[8], [22]].

5 Weakly Concircular symmetric $N(k)$-contact metric manifolds

A Riemannian manifold $(M^n, g)(n > 2)$ is called weakly concircular symmetric manifold [18] if its concircular curvature tensor C of type $(0, 4)$ is not identically zero and satisfies

$$\begin{aligned}(\nabla_X C)(Y, Z, U, V) &= A(X)C(Y, Z, U, V) + B(Y)C(X, Z, U, V)\\ &+H(Z)C(Y, X, U, V) + D(U)C(Y, Z, X, V) + E(V)C(Y, Z, U, X).\end{aligned} \tag{39}$$

In a weakly concircular symmetric manifold it is also known that $B = H$ and $D = E$ [18]. Hence condition (39) reduces to

$$\begin{aligned}(\nabla_X C)(Y,Z,U,V) &= A(X)C(Y,Z,U,V) + B(Y)C(X,Z,U,V)\\ +B(Z)C(Y,X,U,V) &+ D(U)C(Y,Z,X,V) + D(V)C(Y,Z,U,X). \qquad (40)\end{aligned}$$

Definition 5.1 *A $N(k)$-contact metric manifold M^{2n+1} is said to be weakly concircular symmetric if its concircular curvature tensor C of type $(0,4)$ satisfies (40).*

Suppose $N(k)$-contact metric manifold is weakly concircular symmetric.

Putting $Y = V = e_i$ in (40) and taking summation over i, $1 \leq i \leq 2n+1$, we get

$$\begin{aligned}&(\nabla_X S)(Z,U) - \frac{dr(X)}{(2n+1)}g(Z,U) = A(X)[S(Z,U) - \frac{r}{(2n+1)}g(Z,U)]\\ &+B(Z)[S(X,U) - \frac{r}{(2n+1)}g(X,U)] + D(U)[S(Z,X) - \frac{r}{(2n+1)}g(Z,X)]\\ &+B(R(X,Z)U) + D(R(X,U)Z) - \frac{r}{2n(2n+1)}[(B(X)+D(X))g(Z,U)\\ &-g(X,U)B(Z) - g(X,Z)D(U)]. \qquad (41)\end{aligned}$$

Again replacing $X = Z = U = \xi$ in (41) and taking in to account of (4), (10) and (14), (41) takes the form

$$A(\xi) + B(\xi) + D(\xi) = \frac{dr(\xi)}{r - 2nk(2n+1)}. \qquad (42)$$

Thus, we have

Theorem 5.2 *In a weakly concircular symmetric $N(k)$-contact metric manifold the relation (42) holds.*

Setting any two of the vector fields X, Z and U equated to ξ in (41) cyclically and using (4), (10), (14), (21) and (42), one can get

$$\begin{aligned}D(U) &= D(\xi)\eta(U), & (43)\\ B(Z) &= B(\xi)\eta(Z) \qquad \textit{and} & (44)\\ A(X) &= [\frac{dr(\xi)}{2nk(2n+1)-r} - A(\xi)]\eta(X) - \frac{dr(X)}{2nk(2n+1)-r}. & (45)\end{aligned}$$

Hence we can state the following:

Theorem 5.3 *In a weakly concircular symmetric $N(k)$-contact metric manifold M^{2n+1} the associated 1-forms are given by (43)-(45).*

6 Weakly Concircular Ricci symmetric $N(k)$-contact metric manifolds

A Riemannian manifold $(M^n, g)(n > 2)$ is said to be weakly concircular Ricci symmetric manifold [10] if its concircular Ricci curvature $\bar{C}$ of type (0, 2) is not identically zero and satisfies the condition:

$$(\nabla_X \bar{C})(Y, Z) = A(X)\bar{C}(Y, Z) + B(Y)\bar{C}(X, Z) + D(Z)\bar{C}(X, Y). \qquad (46)$$

Definition 6.1 *A $N(k)$-contact metric manifold M^{2n+1} is said to be weakly concircular Ricci symmetric if its concircular curvature tensor C of type (0, 2) satisfies (46).*

Theorem 6.2 *In a weakly concircular Ricci symmetric $N(k)$-contact metric manifold the relation*

$$A(X) + B(X) + D(X) = \frac{dr(X)}{r - 2nk(2n+1)}, \qquad (47)$$

holds.

Proof: Let $N(k)$-contact metric manifold be weakly concircular Ricci symmetric.
Then by virtue of (3), it follows from (46) that

$$(\nabla_X S)(Y, Z) - \frac{dr(X)}{(2n+1)} g(Y, Z) = A(X)[S(Y, Z) - \frac{r}{(2n+1)} g(Y, Z)] + B(Y)[S(X, Z) - \frac{r}{(2n+1)} g(X, Z)] + D(Z)[S(X, Y) - \frac{r}{(2n+1)} g(X, Y)]. \qquad (48)$$

Replacing $X = Y = Z = \xi$ in the above equation, we get

$$A(\xi) + B(\xi) + D(\xi) = \frac{dr(\xi)}{r - 2nk(2n+1)}. \qquad (49)$$

Equating any two of the vector fields X, Y and Z to ξ in (48) cyclically and using (4), (14), (21) and (49), we find

$$D(Z) = D(\xi)\eta(Z), \qquad (50)$$

$$B(Y) = B(\xi)\eta(Y), \qquad (51)$$

$$A(X) = [A(\xi) - \frac{dr(\xi)}{r - 2nk(2n+1)}]\eta(X) - \frac{dr(X)}{2nk(2n+1) - r}. \qquad (52)$$

Adding the equations (50)-(52) and then using (42), one can arrive at (47). This completes the proof of the theorem 6.2.

Thus, we have the following corollary:

Corollary 6.3 *In a weakly concircular Ricci symmetric $N(k)$-contact metric manifold M^{2n+1} the sum of the 1-forms A, B and D is zero everywhere if and only if the scalar curvature r of the manifod is constant.*

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