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SOME RESULTS ON D-CONFORMAL P-SASAKIAN MANIFOLDS

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Abstract:

In this paper a new type of tensor field P^h_{kji} is defined and established a few properties thereof. In the last, a tensor field P^h_{kji} is defined in termes of tensor fields U^h_{kji} and R^h_{kji} and established some important theorem.

Keywords: Differentiable manifold, Riemannian metric, almost para contact structure, Einstein manifold and Sasakian manifold.

1. INTRODUCTION:

Definition 1.1: Almost Para Contact Manifold:

Let M be a n-dimensional differentiable manifold which admits a (1-1) tensor field ψ, a vector field ξ and a 1-form η satisfying the conditions

(1.1) $\eta_a \xi^a = 1$

and

(1.2) $\psi^a_j \psi^i_a = \delta^i_j - \eta_j \xi^i$

then manifold M is said to be an almost para contact manifold with almost para contact structure (ψ, ξ, η).

Definition 1.2: Associated Riemannian Metric:

In an almost paracontact manifold, if there exists in such a manifold d M with a positive definite Riemannian metric g_{ji} satisfying the conditions

(1.3) $\xi^a \psi^i_a = 0$

(1.4) $\eta_a \psi^a_j = 0$

(1.5) $\text{rank } \psi^i_j = (n-1)$

(1.6) $\psi_{ji} = g_{ia} \psi^a_j$

(1.7) $\psi_{ji} = \psi_{ij}$

(1.8) $\eta_j = g_{ja} \xi^i$

(1.9) $g_{ab}\psi^a_j \psi^b_i = g_{ji} - \eta_j\eta_i$

Such a Riemannian metric g_{ji} is said to be a associated Riemannian metric with structure (ψ, ξ, η).

Definition 1.3: Para Contact Riemannian Manifold:

If in an almost paracontact Riemannian manifold M satisfying the condition

(1.10) $2\psi_{ji} = \nabla_j \eta_i + \nabla_i \eta_j$

Wherein ∇_j denotes the covariant differentiation with regard to the Riemannian metric g_{ji}, then M is called a para contact Riemannian manifold.

Definition 1.4: Special Para Contact Riemannian Manifold:

If in an almost para contact Riemannian manifold, the condition $\psi^i_j = \nabla_j \xi^i$ holds good, then M is termed as a special para contact Riemannian manifold.

Definition 1.5: Special Para Sasakian Manifold:

If in a para Sasakian manifold M the unit 1-form η satisfying the condition

(1.11) $\nabla_j \eta_i = - g_{ji} + \eta_j \eta_i$

then M is said to be a Special Para Sasakian manifold [2].

In a Para Sasakian manifold M, the Ricci tensor R_{ji} satisfies

(1.12) $R_{ji} = [\{R/(n-1)\}+1]\, g_{ji} – [\{R/(n-1) + n]\, \eta_j\eta_i$

then the manifold M is said to be η– Einstein manifold.

If the manifold M be Einstein then

(1.13) $R_{ji} = - (n-1)g_{ji}$

In this regard, new tensor fields U_{ji}, G_{ji} and P_{ji} are defined as follows

(1.14) $U_{ji} =R_{ji} - [\{R/(n-1)\}+1]\, g_{ji} + [\{R/(n-1) + n]\, \eta_j\eta_i$

(1.15) $G_{ji} = R_{ji} + (n-1)g_{ji}$

and

(1.16) $P_{ji} =U_{ji} - R_{ji} + [\{R/(n-1)\}+1]\, g_{ji} – [\{R/(n-1) + n]\, \eta_j\eta_i$

2. SOME CHANGES IN D-CONFORMAL CURVATURE TENSOR:

Let us consider the relation

(2.1) $B^h_{kji} = R^h_{kji} - \{1/(n-3)\}\{R_{ji}(\delta^h_k - \eta_k \xi^h) - \{R_{ki}(\delta^h_j - \eta_j \xi^h) + (g_{ji} - \eta_j\eta_i) R^h_k$
$- (g_{ki} - \eta_k\eta_i) R^h_j\}+ \{(R+2)/(n-2)(n-3)\}(g_{ji}\, \delta^h_k – g_{ki}\, \delta^h_j) +[\{R + 2(n-1)\}/(n-2)(n-3)]\, (g_{ki}\, \eta_j \xi^h – g_{ji}\, \eta_k \xi^h + \eta_k\eta_i\, \delta^h_j - \eta_j\eta_i\, \delta^h_k)$

Definition 2.1: D-Conformal Curvature Tensor:

The tensor field B^h_{kji} of the SP - Sasakian manifold satisfying the relation (2.1) is said to be D-Conformal Curvature tensor.

For the D-Conformal Curvature tensor, the following identities are obtained:

(2.2) $B^h_{kji} = - B^h_{jki}$

(2.3) $B_{kjih} = B_{ihkj}$

(2.4) $B^\alpha_{\alpha ji} = 0$

(2.5) $B^\alpha_{kji}\, \eta_\alpha = 0$

(2.6) $B^h_{kji} + B^h_{jik} + B^h_{ikj} = 0$

If we calculate $\nabla_h B^h_{kji}$, then it follows that

(2.7) $\nabla_h B^h_{kji} = \{(n-4)/(n-3)\}B_{kji}$

Where we take

(2.8) $B_{kji} = \nabla_k R_{ji} - \nabla_j R_{ki} – (\eta_k R_{ji} - \eta_j R_{ki}) + [\{R + 2(n-1)\}/(n-2)]\, (\eta_k g_{ji} - \eta_j g_{ki})$
$– \{1/2(n-2)\}\{\nabla_k R\, (g_{ji} - \eta_j\eta_i) - \nabla_j R\, (g_{ki} - \eta_k\eta_i)\}$

In a SP-Sasakian manifold, a tensor field U^h_{kji} is defined as

(2.9) $U^h_{kji} = R^h_{kji} – [\{R + 2(n-1)\}/(n-1)(n-2)](g_{ji}\, \delta^h_k - g_{ki}\, \delta^h_j)$
$- [\{R + n(n-1)\}/(n-1)(n-2)]\, (g_{ki}\, \eta_j \xi^h – g_{ji}\, \eta_k \xi^h + \eta_k\eta_i\, \delta^h_j - \eta_j\eta_i\, \delta^h_k)$

In this regard, the following identities are obtained:

(2.10) $U^h_{kji} = - U^h_{jki}$

(2.11) $U_{kjih} = U_{ihkj}$

(2.12) $U^{\alpha}_{\alpha ji} = 0$

(2.13) $U^{\alpha}_{kji}\, \eta_{\alpha} = 0$

(2.14) $U^{h}_{kji} + U^{h}_{jik} + U^{h}_{ikj} = 0$

In a SP - Sasakian manifold, a new type of tensor field P^{h}_{kji} in terms of tensor fields U^{h}_{kji} and R^{h}_{kji} is defined as

(2.15) $P^{h}_{kji} = U^{h}_{kji} - R^{h}_{kji} + [\{R + 2(n-1)\}/(n-1)(n-2)](g_{ji}\, \delta^{h}_{k} - g_{ki}\, \delta^{h}_{j})$
$+ [\{R + n(n-1)\}/(n-1)(n-2)]\, (g_{ki}\, \eta_{j}\, \xi^{h} - g_{ji}\, \eta_{k}\, \xi^{h} + \eta_{k}\eta_{i}\, \delta^{h}_{j} - \eta_{j}\eta_{i}\, \delta^{h}_{k})$

Theorem 2.1:

In a Special Para-Sasakian manifold, the tensor P^{h}_{kji} is skew-symmetric in the first two covariant indices i.e.

$P^{h}_{kji} = - P^{h}_{jki}$

Proof:

Interchanging the indices j and k in equation (2.15), we obtain

(2.16) $P^{h}_{jki} = U^{h}_{jki} - R^{h}_{jki} + [\{R + 2(n-1)\}/(n-1)(n-2)](g_{ki}\, \delta^{h}_{j} - g_{ji}\, \delta^{h}_{k})$
$+ [\{R + n(n-1)\}/(n-1)(n-2)]\, (g_{ji}\, \eta_{k}\, \xi^{h} - g_{ki}\, \eta_{j}\, \xi^{h} + \eta_{j}\eta_{i}\, \delta^{h}_{k} - \eta_{k}\eta_{i}\, \delta^{h}_{j})$

By virtue of equations (2.10) and (2.16), we get

(2.17) $P^{h}_{kji} = - P^{h}_{jki}$

Multiplying equation (2.15) by g_{hm} on both sides, we obtain

(2.18) $g_{hm}\, P^{h}_{kji} = g_{hm}\, U^{h}_{kji} - g_{hm}\, R^{h}_{kji} + [\{R + 2(n-1)\}/(n-1)(n-2)](g_{ji}\, g_{hm}\, \delta^{h}_{k}$
$- g_{ki}\, g_{hm}\, \delta^{h}_{j}) + [\{R + n(n-1)\}/(n-1)(n-2)]\, (g_{ki}\, g_{hm}\, \eta_{j}\, \xi^{h} - g_{ji}\, g_{hm}\eta_{k}\, \xi^{h}$
$+ g_{hm}\, \eta_{k}\eta_{i}\, \delta^{h}_{j} - g_{hm}\, \eta_{j}\eta_{i}\, \delta^{h}_{k})$

Then the equation (2.18) reduces in the form

(2.19) $P_{kjim} = U_{kjim} - R_{kjim} + [\{R + 2(n-1)\}/(n-1)(n-2)](g_{ji}\, g_{mk} - g_{ki}\, g_{mj})$
$+ [\{R + n(n-1)\}/(n-1)(n-2)]\, (g_{ki}\, \eta_{j}\, \eta_{m} - g_{ji}\, \eta_{k}\, \eta_{m} + g_{mj}\, \eta_{k}\eta_{i} - g_{mk}\, \eta_{j}\eta_{i})$

Theorem 2.2:

In SP-Sasakian manifold,a tensor P_{kjim} is symmetric in two pairs of indices i.e. $P_{kjim} = P_{imkj}$

Proof:

Interchanging the indices i and k as well as j and m in equation (2.19), we obtain

(2.20) $P_{imkj} = U_{imkj} - R_{imkj} + [\{R + 2(n-1)\}/(n-1)(n-2)](g_{mk}\, g_{ji} - g_{ik}\, g_{jm})$
$+ [\{R + n(n-1)\}/(n-1)(n-2)]\, (g_{ik}\, \eta_{m}\, \eta_{j} - g_{mk}\, \eta_{i}\, \eta_{j} + g_{jm}\, \eta_{i}\eta_{k} - g_{ji}\, \eta_{m}\eta_{k})$

By virtue of equations (2.11) and (2.19), the above equation reduces in the form

(2.21) $P_{kjim} = P_{imkj}$

Further, multiplying equation (2.15) by η_{h} on both sides, we obtain

(2.22) $\eta_{h}\, P^{h}_{kji} = \eta_{h}\, U^{h}_{kji} - \eta_{h}\, R^{h}_{kji} + [\{R + 2(n-1)\}/(n-1)(n-2)](g_{ji}\, \eta_{h}\, \delta^{h}_{k} - g_{ki}\, \eta_{h}\, \delta^{h}_{j})$
$+[\{R + n(n-1)\}/(n-1)(n-2)](g_{ki}\eta_{h}\eta_{j}\, \xi^{h} - g_{ji}\, \eta_{h}\eta_{k}\, \xi^{h} + \eta_{h}\, \eta_{k}\eta_{i}\, \delta^{h}_{j} - \eta_{h}\eta_{j}\eta_{i}\, \delta^{h}_{k})$

By virtue of equations (2.13) and (2.22), we get

(2.23) $\eta_{h}\, P^{h}_{kji} = 0$

Theorem 2.3:

For the tensor P^{h}_{kji} of the SP-Sasakian manifold, the relation

$P^{h}_{kji} + P^{h}_{jik} + P^{h}_{ikj} = 0$

i.e.

$P^{h}_{[kji]} = 0$,

holds good.

Proof:

On interchanging i,j,k cyclically, then equation (2.15) reduces in the forms

(2.24) $P^h_{jik} = U^h_{jik} - R^h_{jik} + [\{R + 2(n-1)\}/(n-1)(n-2)](g_{ik}\,\delta^h_j - g_{jk}\,\delta^h_i)$
$+ [\{R + n(n-1)\}/(n-1)(n-2)]\,(g_{jk}\,\eta_i\,\xi^h - g_{ik}\,\eta_j\,\xi^h + \eta_j\eta_k\,\delta^h_i - \eta_i\eta_k\,\delta^h_j)$

and

(2.25) $P^h_{ikj} = U^h_{ikj} - R^h_{ikj} + [\{R + 2(n-1)\}/(n-1)(n-2)](g_{kj}\,\delta^h_i - g_{ij}\,\delta^h_k)$
$+ [\{R + n(n-1)\}/(n-1)(n-2)]\,(g_{jj}\,\eta_k\,\xi^h - g_{kj}\,\eta_i\,\xi^h + \eta_i\eta_j\,\delta^h_k - \eta_k\eta_j\,\delta^h_i)$

Adding equations (2.15), (2.24) and (2.25), we obtain

(2.26) $P^h_{kji} + P^h_{jik} + P^h_{ikj} = 0$

i.e. $P^h_{[kji]} = 0$,

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