

Numerical modeling of generalized nonlinear system arising in thermoelasticity

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Abstract

In this paper, the Adomian decomposition method (ADM) is presented for finding numerical solution of fractional nonlinear system arising in thermoelasticity. The derivatives are understood in the Caputo sense. The reason of using fractional order differential equations (FOD) is that FOD are naturally related to systems with memory which exists in most biological systems. Also they are closely related to fractals which are abundant in biological systems. The given solutions comparer with the traveling wave solutions. The method provides the solutions in the form of a power series with easily computed terms. It has many advantages over the classical techniques mainly; it provides an efficient numerical solution with high accuracy, minimal calculations.

Mathematics Subject Classification: 35Qxx

Keywords: The decomposition method, Thermoelasticity, Numerical solution, Fractional Calculus.

1. Introduction

In recent more than 20 years, there have been a lot of works on local well-posedness, global well-posedness in time and asymptotic behavior of solutions to some initial boundary value problems in thermoelasticity (see [1-15]). The subject of nonlinear thermoelasticity has recently gained a considerable interest for its importance in understanding the nature of interaction between the elastic and thermal fields, as well as for applications.

In this paper, we present a solution of a more general model of nonlinear

system arising in Thermoelasticity. We implemented the Adomian decomposition method (ADM) [16-17] to the nonlinear system, the governing equations are (5-8):

$$\begin{aligned} \frac{\partial^{\alpha+1} u}{\partial t^{\alpha+1}} - a(u_x, v) \frac{\partial^2 u}{\partial x^2} + b(u_x, v) \frac{\partial v}{\partial x} &= \Phi(x, t), \quad 0 < \alpha \leq 1 \\ \frac{\partial^\alpha v}{\partial t^\alpha} - \frac{b(u_x, v)}{c(u_x, v)} \frac{\partial^2 v}{\partial x \partial t} + \frac{d(u_x, v)}{c(u_x, v)} \frac{\partial^2 v}{\partial x^2} &= \frac{\Psi(x, t)}{c(u_x, v)} \end{aligned} \quad (1.1)$$

under the following initial conditions:

$$u(x, 0) = f(x), \quad v(x, 0) = g(x)$$

where $u(x, t)$ is the body displacement from equilibrium, $v(x, t)$ is the difference of the body's temperature, and subscripts denote partial derivatives, a , b , c and d are given smooth functions.

The system is obtained from the standard diffusion-reaction systems (1.1) by replacing the first time derivative term by a fractional derivative of order. The concept of fractional or non-integer order derivation and integration can be traced back to the genesis of integer order calculus itself [18]. Almost most of the mathematical theory applicable to the study of non-integer order calculus was developed through the end of 19th century. However it is in the past hundred years that the most intriguing leaps in engineering and scientific application have been found. The calculation technique has in some cases had to change to meet the requirement of physical reality. The use of fractional differentiation for the mathematical modeling of real world physical problems has been widespread in recent years, e.g. the modeling of earthquake, the fluid dynamic traffic model with fractional derivatives, measurement of viscoelastic material properties, etc. Applications of fractional derivatives in other fields and related mathematical tools and techniques could be found in [19-29]. In fact, real world processes generally or most likely are fractional order systems

The reason of using fractional order differential equations (FOD) is that FOD are naturally related to systems with memory which exists in most biological systems. Also they are closely related to fractals which are abundant in biological systems. The results derived of the fractional system (1.1) are of a more general nature. Respectively, solutions to the fractional diffusion equation spread at a faster rate than the classical diffusion equation, and may exhibit asymmetry. However, the fundamental solutions of these equations still exhibit useful scaling properties that make them attractive for applications.

The derivatives are understood in the Caputo sense. The general response expression contains a parameter describing the order of the fractional derivative that can be varied to obtain various responses.

The Adomian's decomposition method will be applied for computing solutions to the systems of fractional partial differential equations considered in this paper. This method has been used to obtain approximate solutions of a large class of linear or nonlinear differential equations. It is also quite straightforward to write computer codes in any symbolic languages. The method provides solutions in the

