**Mathematica Aeterna, Vol. 1, 2011, no. 08, 529 - 536**

# A Study of Γ-Semigroups in terms of Anti Fuzzy Ideals

**Samit Kumar Majumder**

Tarangapur N.K High School, Tarangapur,
Uttar Dinajpur, West Bengal-733129, INDIA
samitfuzzy@gmail.com

**Abstract**

In this paper various relationships between anti fuzzy ideals of a Γ-semigroup and that of its operator semigroups have been obtained. An inclusion preserving bijection between the set of all anti fuzzy ideals of a Γ-semigroup(not necessarily with unities) and that of its operator semigroups has been obtained.

**Mathematics Subject Classification:** 20M99, 20N25
**Keywords:** Γ-Semigroup, Anti fuzzy ideal, Left and right operator semigroups.

# 1 Introduction

Uncertainty is an attribute of information and uncertain data are presented in various domains. The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets developed by Zadeh[21]. Semigroups are important in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics and mathematical analysis. A semigroup is an algebraic structure consisting of a non-empty set $S$ together with an associative binary operation[9]. The formal study of semigroups began in the early 20th century. The topic of investigations about fuzzy semigroups belongs to the theoretical soft computing (fuzzy structures). Indeed, it is well known that semigroups are basic structures in many applicative branches like automata and formal languages, coding theory, finite state machines and others. Due to these possibilities of applications, semigroups and related structures are presently extensively investigated in fuzzy settings. Azirel Rosenfeld[14] used the idea of fuzzy set to introduce the notions of fuzzy subgroups. Nobuaki Kuroki[11, 12, 13] is the pioneer of fuzzy ideal theory of semigroups. The idea of fuzzy subsemigroup was also introduced by Kuroki[11, 13]. In [12], Kuroki characterized several classes of semigroups in terms of fuzzy left, fuzzy right and fuzzy bi-ideals. Others who worked on fuzzy semigroup theory, such as X.Y. Xie[20], Y.B. Jun[10], are mentioned in the bibliography. X.Y. Xie[20] introduced the idea of extensions of fuzzy ideals in semigroups.

In 1981 M.K. Sen[16] introduced the notion of Γ-semigroup as a generalization of semigroup and ternary semigroup. We call this Γ-semigroup a both sided Γ-semigroup. In 1986 M.K. Sen and N.K. Saha[18] modified the definition of Sen's Γ-semigroup. This newly defined Γ-semigroup is known as one sided Γ-semigroup. Γ-semigroups have been analyzed by a lot of mathematicians, for instance by Chattopadhyay[2], Dutta and Adhikari[1, 4, 5, 6], Hila[7, 8], Chinram[3], Sen et al.[17, 19]. T.K. Dutta and N.C. Adhikari[1, 4] mostly worked on both sided Γ-semigroups. They defined operator semigroups of such type of Γ-semigroups and established many results and obtained many correspondences between a Γ-semigroup and its operator semigroups. In this paper we have considered both sided Γ-semigroups.

Many results of semigroups could be extended to Γ-semigroups directly and via operator semigroups[1] of a Γ-semigroup. In this paper in order to make operator semigroups of a Γ-semigroup work in the context of fuzzy sets as it worked in the study of Γ-semigroups[1, 4, 5], various relationships between anti fuzzy ideals of a Γ-semigroup and that of its operator semigroups have been obtained. Finally an inclusion preserving bijection between the set of all anti fuzzy ideals of a Γ-semigroup and that of its operator semigroups has been obtained. Then this bijection is applied it to give new proofs of its ideal analogue obtained in[4].

# 2 Preliminaries

**Definition 2.1** [5] *Let $S$ and $\Gamma$ be two non-empty sets. $S$ is called a $\Gamma$-semigroup if there exist mappings from $S\times\Gamma\times S$ to $S$, written as $(a,\alpha,b)\longrightarrow a\alpha b$, and from $\Gamma\times S\times\Gamma$ to $\Gamma$, written as $(\alpha,a,\beta)\longrightarrow \alpha a\beta$ satisfying the following associative laws $(a\alpha b)\beta c = a(\alpha b\beta)c = a\alpha(b\beta c)$ and $\alpha(a\beta b)\gamma = (\alpha a\beta)b\gamma = \alpha a(\beta b\gamma)$ for all $a,b,c\in S$ and for all $\alpha,\beta,\gamma\in\Gamma$.*

**Example 1** [5] Let $S$ be the set of all integers of the form $4n+1$ and $\Gamma$ be the set of all integers of the form $4n+3$ where $n$ is an integer. If $a\alpha b$ is $a+\alpha+b$ and $\alpha a\beta$ is $\alpha+a+\beta$(usual sum of integers) for all $a,b\in S$ and for all $\alpha,\beta\in\Gamma$, then $S$ is a $\Gamma$-semigroup.

**Definition 2.2** [5] *A left ideal(right ideal) of a $\Gamma$-semigroup $S$ is a non-empty subset $I$ of $S$ such that $S\Gamma I\subseteq I$ ($I\Gamma S\subseteq I$). If $I$ is both a left ideal and a right ideal of $S$, then we say that $I$ is an ideal of $S$.*

**Definition 2.3** [*21*] *A function $\mu$ from a non-empty set $S$ to the unit interval $[0,1]$ is called a fuzzy subset of $S$.*

**Definition 2.4** *Let $\mu$ be a fuzzy subset of a semigroup $S$ and let $t \in [0,1]$. Then the set $\mu_t := \{x \in S : \mu(x) \leq t\}$ is called the anti level subset of $\mu$.*

**Definition 2.5** [*15*] *A non-empty fuzzy subset $\mu$ of a $\Gamma$-semigroup $S$ is called an $AFLI(S)(AFRI(S))$*[1] *if $\mu(x\gamma y) \leq \mu(y)$(resp. $\mu(x\gamma y) \leq \mu(x)$)$\forall x, y \in S$ and $\forall \gamma \in \Gamma$.*

**Definition 2.6** [*15*] *A non-empty fuzzy subset $\mu$ of a $\Gamma$-semigroup $S$ is called an $AFI(S)$ if it is both an $AFLI(S)$ and an $AFRI(S)$.*

**Example 2** [15] Let $S$ be the set of all non-positive integers and $\Gamma$ be the set of all non-positive even integers. Then $S$ is a $\Gamma$-semigroup where $a\gamma b$ denote the usual multiplication of integers $a, \gamma, b$ with $a, b \in S$ and $\gamma \in \Gamma$. Let $\mu$ be a fuzzy subset of $S$, defined as follows

$$\mu(x) = \begin{cases} 0.2 & \text{if } x = 0 \\ 0.1 & \text{if } x = -1, -2 \\ 1 & \text{if } x < -2 \end{cases}$$

Then $\mu$ is an $AFI(S)$.

**Theorem 2.7** [*15*] *Let $I$ be a non-empty subset of a $\Gamma$-semigroup $S$ and $\chi$ be a fuzzy subset of $S$ such that*

$$\chi(x) = \begin{cases} 0 & \text{if } x \in I \\ 1 & \text{if } x \notin I \end{cases}$$

*Then $I$ is a left ideal(right ideal, ideal) of $S$ if and only if $\chi$ is an $AFLI(S)$(resp. $AFRI(S)$, $AFI(S)$).*

**Theorem 2.8** [*15*] *Let $S$ be a $\Gamma$-semigroup and $\mu$ be a non-empty fuzzy subset of $S$, then $\mu$ is an $AFLI(S)$(resp. $AFRI(S)$, $AFI(S)$) if and only if $\mu_t$'s are left ideals(resp. right ideals, ideals) of $S$ for all $t \in Im(\mu)$, where $\mu_t = \{x \in S : \mu(x) \leq t\}$.*

**Definition 2.9** [*5*] *Let $S$ be a $\Gamma$-semigroup. Let us define a relation $\rho$ on $S \times \Gamma$ as follows : $(x, \alpha)\rho(y, \beta)$ if and only if $x\alpha s = y\beta s$ for all $s \in S$ and $\gamma x\alpha = \gamma y\beta$ for all $\gamma \in \Gamma$. Then $\rho$ is an equivalence relation. Let $[x, \alpha]$ denote the equivalence class containing $(x, \alpha)$. Let $L = \{[x, \alpha] : x \in S, \alpha \in \Gamma\}$. Then $L$ is a semigroup with respect to the multiplication defined by $[x, \alpha][y, \beta] = [x\alpha y, \beta]$. This semigroup $L$ is called the left operator semigroup of the $\Gamma$-semigroup $S$. Dually the right operator semigroup $R$ of $\Gamma$-semigroup $S$ is defined where the multiplication is defined by $[\alpha, a][\beta, b] = [\alpha a\beta, b]$.*

---

[1] $AFLI(S), AFRI(S), AFI(S)$ stands for anti fuzzy left ideal of $S$, anti fuzzy right ideal of $S$, anti fuzzy ideal of $S$ and accordingly.

**Definition 2.10** [5] *If there exists an element $[e,\delta] \in L([\gamma, f] \in R)$ such that $e\delta s = s$(resp. $s\gamma f = s$) for all $s \in S$ then $[e,\delta]$(resp. $[\gamma, f]$) is called the left(resp. right) unity of $S$.*

Now we recall the following propositions which were proved therein for one sided ideals. But the results can be proved to be true for two sided ideals.

**Proposition 2.11** [5] *Let $S$ be a $\Gamma$-semigroup with unities and $L$ be its left operator semigroup. If $A$ is a (right) ideal of $L$ then $A^+$ is a (right)ideal of $S$.*

**Proposition 2.12** [5] *Let $S$ be a $\Gamma$-semigroup with unities and $L$ be its left operator semigroup. If $B$ is a (right)ideal of $S$ then $B^{+'}$ is a(right)ideal of $L$.*

**Proposition 2.13** [5]*Let $S$ be a $\Gamma$-semigroup with unities and $R$ be its right operator semigroup. If $A$ is a (left)ideal of $R$ then $A^*$ is a (left)ideal of $S$.*

**Proposition 2.14** [5]*Let $S$ be a $\Gamma$-semigroup with unities and $R$ be its right operator semigroup. If $B$ is a (left)ideal of $S$ then $B^{*'}$ is a (left)ideal of $R$.*

**Remark 1** For convenience of the readers, we may note that for a $\Gamma$-semigroup $S$ and its left, right operator semigroups $L, R$ respectively four mappings namely $()^+$, $()^{+'}$, $()^+$, $()^{*'}$ occur. They are defined as follows: For $I \subseteq R, I^* = \{s \in S, [\alpha, s] \in I \forall \alpha \in \Gamma\}$; for $P \subseteq S, P^{*'} = \{[\alpha, x] \in R : s\alpha x \in P \forall s \in S\}$; for $J \subseteq L, J^+ = \{s \in S, [s, \alpha] \in J \forall \alpha \in \Gamma\}$; for $Q \subseteq S, Q^{+'} = \{[x, \alpha] \in L : x\alpha s \in Q \forall s \in S\}$.

**Definition 2.15** *For a fuzzy subset $\mu$ of $R$ we define a corresponding fuzzy subset $\mu^*$ of $S$ by $\mu^*(a) = \sup_{\gamma \in \Gamma} \mu([\gamma, a])$, where $a \in S$. For a fuzzy subset $\sigma$ of $S$ we define a corresponding fuzzy subset $\sigma^{*'}$ of $R$ by $\sigma^{*'}([\alpha, a]) = \sup_{s \in S} \sigma(s\alpha a)$, where $[\alpha, a] \in R$. For a fuzzy subset $\delta$ of $L$, we define a corresponding fuzzy subset $\delta^+$ of $S$ by $\delta^+(a) = \sup_{\gamma \in \Gamma} \delta([a, \gamma])$ where $a \in S$. For a fuzzy subset $\eta$ of $S$ we define a corresponding fuzzy subset $\eta^{+'}$ of $L$ by $\eta^{+'}([a, \alpha]) = \sup_{s \in S} \eta(a\alpha s)$, where $[a, \alpha] \in L$.*

# 3 Main Results

**Proposition 3.1** *Let $\mu$ be a fuzzy subset of $R$(the right operator semigroup of the $\Gamma$-semigroup $S$). Then $(\mu_t)^* = (\mu^*)_t$ for all $t \in [0, 1]$ such that the sets are non-empty.*

**Proof:** Let $s \in S$. Then $s \in (\mu_t)^* \Leftrightarrow [\gamma, s] \in \mu_t \forall \gamma \in \Gamma \Leftrightarrow \mu([\gamma, s]) \leq t \forall \gamma \in \Gamma \Leftrightarrow \sup_{\gamma \in \Gamma} \mu([\gamma, s]) \leq t \Leftrightarrow \mu^*(s) \leq t \Leftrightarrow s \in (\mu^*)_t$. Hence $(\mu_t)^* = (\mu^*)_t$.

**Proposition 3.2** *Let $\sigma$ is a fuzzy subset of a Γ-semigroup $S$. Then $(\sigma_t)^{*'} = (\sigma^{*'})_t$ for all $t \in [0, 1]$ such that the sets under consideration are non-empty.*

**Proof:** Let $[\alpha, x] \in R$ and $t$ is as mentioned in the statement. Then $[\alpha, x] \in (\sigma_t)^{*'} \Leftrightarrow s\alpha x \subseteq \sigma_t \forall s \in S \Leftrightarrow \sigma(s\alpha x) \leq t \forall s \in S \Leftrightarrow \sup_{s \in S} \sigma(s\alpha x) \leq t \Leftrightarrow \sigma^{*'}([\alpha, x]) \leq t \Leftrightarrow [\alpha, x] \in (\sigma^{*'})_t$. Hence $(\sigma_t)^{*'} = (\sigma^{*'})_t$.

**Remark 2** In what follows $S$ denotes a Γ-semigroup with unities, $L$ , $R$ be its left and right operator semigroups respectively.

**Proposition 3.3** *If $\mu \in AFI(R)$ ( $AFLI(R)$) then $\mu^* \in AFI(S)$ (resp. $AFLI(S)$).*

**Proof:** Suppose $\mu \in AFI(R)$. Then $\mu_t$ is an ideal of $R$, $\forall t \in Im(\mu)$. Hence $(\mu_t)^*$ is an ideal of $S$, $\forall t \in Im(\mu)$(*cf.* Proposition 2.13). Now since $\mu$ is an $AFI(R)$, $\mu$ is a non-empty fuzzy subset of $R$. Hence for some $[\alpha, s] \in R, \mu([\alpha, s]) > 0$. Then $\mu_t \neq \phi$ where $t := \mu([\alpha, s])$. So by the same argument applied above $(\mu_t)^* \neq \phi$. Let $u \in (\mu_t)^*$. Then $[\beta, u] \in \mu_t$ for all $\beta \in \Gamma$. Hence $\mu([\beta, u]) \leq t$. This implies that $\sup_{\beta \in \Gamma} \mu([\beta, u]) \leq t$, *i.e.*, $\mu^*(u) \leq t$. Hence $u \in (\mu^*)_t$. Hence$(\mu^*)_t \neq \phi$. Consequently, $(\mu_t)^* = (\mu^*)_t$(*cf.* Proposition 3.1). It follows that $(\mu^*)_t$ is an ideal of $S$ for all $t \in Im(\mu)$. Hence $\mu^*$ is an $AFI(S)$(*cf.* Theorem 2.8). The proof for $AFLI$ follows similarly.

In a similar fashion by using Propositions 2.14, 3.2 and Theorem 2.8 we can deduce the following proposition.

**Proposition 3.4** *If $\sigma \in AFI(S)$( $AFLI(S)$) then $\sigma^{*'} \in AFI(R)$ (resp. $AFLI(R)$).*

We can also deduce the following left operator analogues of the above propositions.

**Proposition 3.5** *If $\delta \in AFI(L)$ ( $AFRI(L)$) then $\delta^+ \in AFI(S)$ (resp. $AFRI(S)$).*

**Proposition 3.6** *If $\eta \in AFI(S)$( $AFRI(S)$) then $\eta^{+'} \in AFI(L)$(resp, $AFRI(L)$).*

**Theorem 3.7** *Let $S$ be a $\Gamma$-semigroup with unities and $L$ be its left operator semigroup. Then there exist an inclusion preserving bijection $\sigma \mapsto \sigma^{+'}$ between the set of all $AFI(AFRI(S))$ and set of all $AFI$(resp. $AFRI(L)$), where $\sigma$ is an $AFI(AFRI(S))$.*

**Proof:** Let $\sigma \in AFI(S)(AFRI(S))$ and $x \in S$. Then $(\sigma^{+'})^{+}(x) = \sup_{\gamma \in \Gamma} \sigma^{+'}([x,\gamma]) = \sup_{\gamma \in \Gamma}[\sup_{s \in S} \sigma(x\gamma s)] \leq \sigma(x)$. Hence $\sigma \supseteq (\sigma^{+'})^{+}$. Let $[\gamma, f]$ be the right unity of $S$. Then $x\gamma f = x$ for all $x \in S$. Then $\sigma(x) = \sigma(x\gamma f) \leq \sup_{\alpha \in \Gamma}[\sup_{s \in S} \sigma(x\alpha s)]$ $= \sup_{\alpha \in \Gamma} \sigma^{+'}([x,\alpha]) = (\sigma^{+'})^{+}(x)$. So $\sigma \subseteq (\sigma^{+'})^{+}$. Hence $(\sigma^{+'})^{+} = \sigma$. Now let $\mu \in AFI(L)(AFRI(L))$. Then $(\mu^{+})^{+'}([x,\alpha]) = \sup_{s \in S} \mu^{+}(x\alpha s) = \sup_{s \in S}[\sup_{\gamma \in \Gamma} \mu([x\alpha s, \gamma]) = \sup_{s \in S}[\sup_{\gamma \in \Gamma} \mu([x,\alpha][s,\gamma])] \leq \mu([x,\alpha])$. So $\mu \supseteq (\mu^{+})^{+'}$. Let $[e,\delta]$ be the left unity of $L$. Then $\mu([x,\alpha]) = \mu([x,\alpha][e,\delta]) \leq \sup_{s \in S}[\sup_{\gamma \in \Gamma} \mu([x,\alpha][s,\gamma])] = (\mu^{+})^{+'}([x,\alpha])$. So $\mu \subseteq (\mu^{+})^{+'}$ and hence $\mu = (\mu^{+})^{+'}$. Thus the correspondence $\sigma \mapsto \sigma^{+'}$ is a bijection. Now let $\sigma_1, \sigma_2 \in AFI(S)(AFRI(S))$ be such that $\sigma_1 \subseteq \sigma_2$. Then for all $[x,\alpha] \in L, \sigma_1^{+'}([x,\alpha]) = \sup_{s \in S} \sigma_1(x\alpha s) \leq \sup_{s \in S} \sigma_2(x\alpha s) = \sigma_2^{+'}([x,\alpha])$. Thus $\sigma_1^{+'} \subseteq \sigma_2^{+'}$. Similarly we can show that if $\mu_1 \subseteq \mu_2$ where $\mu_1, \mu_2 \in AFI(L)(AFRI(L))$ then $\mu_1^{+} \subseteq \mu_2^{+}$. Hence $\sigma \mapsto \sigma^{+'}$ is an inclusion preserving bijection. The rest of the proof follows from Proposition 3.5 and Proposition 3.6.

In a similar way by using Proposition 3.3 and Proposition 3.4 we can deduce the following theorem.

**Theorem 3.8** *Let $S$ be a $\Gamma$-semigroup with unities and $R$ be its right operator semigroup. Then there exist an inclusion preserving bijection $\sigma \mapsto \sigma^{*'}$ between the set of all $AFI(AFLI(S))$ and set of all $AFI(AFLI(R))$, where $\sigma$ is an $AFI(AFLI(S))$.*

Now to apply the above theorem for giving a new proof of Theorem 4.6[5] and its two sided ideal analogue we obtain the following lemmas.

**Lemma 3.9** *Let $I$ be a (left) ideal of the right operator semigroup $R$ of a $\Gamma$-semigroup $S$. Then $(\lambda_I)^* = \lambda_{I^*}$, where $\lambda_I$ denotes the characteristic function of $I$.*

**Proof:** Suppose $s \in I^*$. Then $[\beta, s] \in I$ for all $\beta \in \Gamma$. This means $\sup_{\beta \in \Gamma}(\lambda_I([\beta, s])) = 0$. Also $\lambda_{I^*}(s) = 0$. Now suppose $s \notin I^*$. Then there exists

$\delta \in \Gamma$ such that $[\delta, s] \notin I$. Hence $\lambda_I([\delta, s]) = 1$ and so $\sup\limits_{\beta\in\Gamma}(\lambda_I([\beta, s])) = 1$. Hence $(\lambda_I)^*(s) = 1$. Again $(\lambda_{I^*})(s) = 1$. Thus $(\lambda_I)^* = \lambda_{I^*}$.

By applying similar argument as above we deduce the following lemma.

**Lemma 3.10** *Let $I$ be a (right) ideal of a $\Gamma$-semigroup $S$ and $R$ be the right operator semigroup of $S$. Then $(\lambda_I)^{*'} = \lambda_{I^{*'}}$, where $\lambda_I$ is the characteristic function of $I$.*

**Remark 3** By drawing an analogy we can deduce results similar to the above lemmas for left operator semigroup $L$ of the $\Gamma$-semigroup $S$.

Now we give a new proof of the following result which is originally due to Dutta and Adhikari[4].

**Theorem 3.11** [4] *Let $S$ be a $\Gamma$-semigroup with unities. Then there exists an inclusion preserving bijection between the set of all ideals (left ideals) of $S$ and that of its right operator semigroup $R$ via the mapping $I \to I^{*'}$.*

**Proof:** Let us denote the mapping $I \to I^{*'}$ by $\phi$. This is actually a mapping follows from Proposition 3.3. Now let $\phi(I_1) = \phi(I_2)$. Then $I_1^{*'} = I_2^{*'}$. This implies that $\lambda_{I_1^{*'}} = \lambda_{I_2^{*'}}$(where $\lambda_I$ is the characteristic function $I$). Hence by Lemma 3.10, $(\lambda_{I_1})^{*'} = (\lambda_{I_2})^{*'}$. This together with Theorem 3.8 gives $\lambda_{I_1} = \lambda_{I_2}$ whence $I_1 = I_2$. Consequently $\phi$ is one-one. Let $I$ be a (left) ideal of $R$. Then its characteristic function $\lambda_I$ is an $AFI(R)(AFLI(R))$. Hence by Theorem 3.8, $((\lambda_I)^*)^{*'} = \lambda_I$. This implies that $\lambda_{(I^*)^{*'}} = \lambda_I$ ($cf.$ Lemma 3.9 and Lemma 3.10). Hence $(I^*)^{*'} = I, i.e., \phi(I^*) = I$. Now since $I^*$ is a (left) ideal of $S$ ($cf.$ Proposition 2.13), it follows that $\phi$ is onto. Let $I_1, I_2$ be two (left) ideals of $S$ with $I_1 \subseteq I_2$. Then $\lambda_{I_1} \subseteq \lambda_{I_2}$. Hence by Theorem 3.8, we see that $(\lambda_{I_1})^{*'} \subseteq (\lambda_{I_2})^{*'}$ *i.e.*, $\lambda_{I_1^{*'}} \subseteq \lambda_{I_2^{*'}}$($cf.$ Lemma 3.10) which gives $I_1^{*'} \subseteq I_2^{*'}$.

**Remark 4** Now by using a similar argument as above and with the help of lemmas dual to the lemmas 3.9, 3.10 and Theorem 3.7 we can deduce that the mapping $()^{+'}$ is an inclusion preserving bijection(with $()^+$ as the inverse) between the set of all ideals(right ideals) of $S$ and that of its left operator semigroup $L$.

# References

[1] N.C. Adhikari; *Study of some problems associated with* $\Gamma$*-semigroups,* Ph.D. Dissertation.,( University of Calcutta).

[2] S. Chattopadhyay; *Right orthodox* $\Gamma$*-semigroup,* South East Asian Bull. Math., 29 (2005) 23-30.

[3] R. Chinram; *On quasi-*$\Gamma$*-ideals in* $\Gamma$*-semigroups,* ScienceAsia., 32 (2006) 351-353.

[4] T.K. Dutta and N.C. Adhikari; *On Prime Radical of* $\Gamma$*-semigroup,* Bull. Cal. math. Soc., 86 (5) (1994) 437-444.

[5] T.K. Dutta and N.C. Adhikari; *On* $\Gamma$*-semigroup with the right and left unities,* Soochow J. Math., 19 (4) (1993) 461-474.

[6] T.K. Dutta, S.K. Sardar and S.K. Majumder; *Fuzzy ideal extensions of* $\Gamma$*-semigroups via its operator semigroups,* Int. J. Contemp. Math. Sciences., 4(30) (2009) 1455-1463.

[7] K. Hila; *On regular, semiprime and quasi-reflexive* $\Gamma$*-semigroup and minimal quasi-ideals,* Lobachevski J. Math., 29 (2008) 141-152.

[8] K. Hila; *On some classes of le-* $\Gamma$*-semigroup and minimal quasi-ideals,* Algebras Groups Geom., 24 (2007) 485-495.

[9] J. Howie; *Fundamentals of semigroup theory,* London Mathematical Society Monographs. New Series, 12. Oxford Science Publications. The Clarendon Press, Oxford University Press, New York., 1995.

[10] Y.B. Jun, S.M. Hong and J. Meng; *Fuzzy interior ideals in semigroups,* Indian J. of Pure Appl. Math., 26(90 (1995) 859-863.

[11] N. Kuroki; *On fuzzy ideals and fuzzy bi-ideals in semigroups,* Fuzzy Sets and Systems., 5(1981) 203-215.

[12] N. Kuroki; *On fuzzy semigroups,* Information Sciences., 53(1991) 203-236.

[13] N. Kuroki; *Fuzzy semiprime quasi ideals in semigroups,* Inform. Sci., 75(3)(1993) 201-211.

[14] A. Rosenfeld; *Fuzzy groups,* J. Math. Anal. Appl., 35 (1971) 512-517.

[15] S.K. Sardar, S.K. Majumder and Pavel Pal; *Characterization of* $\Gamma$*-semigroups in terms of anti fuzzy ideals,* (Communicated).

[16] M.K. Sen; *On* $\Gamma$*-semigroups,* Proceedings of the International conference on Algebra and its application. Decker Publication, New York., 301(1981).

[17] M.K. Sen and S. Chattopadhyay; *Semidirect product of a monoid and a* $\Gamma$*-semigroup,* East-West J. Math., 6 (2004)131-138.

[18] M.K. Sen and N.K. Saha; *On* $\Gamma$*-semigroup I,* Bull. Cal. Math. Soc., 78 (1986) 180-186.

[19] M.K. Sen and A. Seth; *On po-*$\Gamma$*-semigroups,* Bull. Calcutta Math. Soc., 85 (1993) 445-450.

[20] X.Y. Xie; *Fuzzy ideal extensions of ordered semigroups,* Lobach Journal of Mathematics., 19 (2005) 29-40.

[21] L.A. Zadeh; *Fuzzy sets,* Information and Control., 8 (1965) 338-353.

**Received: October 29, 2011**