

**SOME PROBLEMS ON MORE GENERALISED RECURRENT
P-SASAKIAN MANIFOLD**

Dr. Indiwari Singh Chauhan

Deptt. of Mathematics, Ganjundwara (P.G.) College, Ganjundwara,
Kanshiram Nagar, U.P., India
e-mail: indiwarsingh.chauhan@gmail.com

Abstract:

The purpose of this paper is to study the theory of generalised recurrent P-Sasakian manifold. In section 1, we defined the birecurrent P-Sasakian manifold and generalised recurrent P-Sasakian manifold. Section 2 deals to more generalised recurrent P-Sasakian manifold. Section 3 is delineated to the associated recurrent vector and recurrent tensor fields in P-Sasakian manifold. In the end, we have discussed several cases and obtained few results.

Keywords: generalised recurrent P-Sasakian manifold, birecurrent P-Sasakian manifold, associated recurrent vector and recurrent tensor fields in P-Sasakian manifold.

1. GENERALISED RECURRENT P-SASAKIAN MANIFOLD:

Definition 1.1:

A P-Sasakian manifold M^n is called birecurrent with $a_{\nu\mu}$ as a non-zero recurrent tensor field if it satisfies the relation

$$(1.1) \quad \nabla_\nu \nabla_\mu R^\lambda_{\alpha\beta\gamma} = a_{\nu\mu} R^\lambda_{\alpha\beta\gamma},$$

wherein $a_{\nu\mu}$ is a non-zero tensor field.

Definition 1.2:

A P-Sasakian manifold M^n is said to be generalised recurrent P-Sasakian manifold if it satisfies the relation

$$(1.2) \quad \nabla_\nu \nabla_\mu R^\lambda_{\alpha\beta\gamma} = a_{\nu\mu} R^\lambda_{\alpha\beta\gamma} + b_\mu \nabla_\nu R^\lambda_{\alpha\beta\gamma},$$

wherein $a_{\nu\mu}$ and b_μ are non-zero tensor and associated vector fields respectively.

Hence, we have [7]:

Theorem 1.1:

For the generalised recurrent tensor of the P-Sasakian manifold, the condition

$$(1.3) \quad R^\lambda_{[\alpha\beta|\theta]R^\theta_{\gamma]\nu\mu} + R^\lambda_{\theta[\beta\gamma} R^\theta_{\alpha]\nu\mu} + R^\lambda_{[\gamma|\theta|\alpha} R^\theta_{\beta]\nu\mu} = 0$$

holds good.

Theorem 1.2:

If associated vector field vanishes then the generalised recurrent P-Sasakian manifold reduces to birecurrent P-Sasakian manifold.

Proof:

Since associated vector field vanishes i.e. $b_\mu = 0$. Then equation (1.2) reduces to equation (1.1).

2. MORE GENERALISED RECURRENT P-SASAKIAN MANIFOLD:

Let us consider the relation

$$(2.1) \quad \nabla_\nu \nabla_\mu R^\lambda_{\alpha\beta\gamma} = a_{\nu\mu} R^\lambda_{\alpha\beta\gamma} + b_\mu \nabla_\nu R^\lambda_{\alpha\beta\gamma} + c_\nu \nabla_\mu R^\lambda_{\alpha\beta\gamma},$$

wherein $a_{\nu\mu}$ is a non-zero tensor field and b_μ, c_ν are both non-zero associated vector fields.

Definition 2.1:

The curvature tensor $R^\lambda_{\alpha\beta\gamma}$ of the P-Sasakian manifold satisfying the relation (2.1) is termed as more generalised recurrent curvature tensor.

Definition 2.2:

The P-Sasakian manifold M^n equipped with the generalised recurrent curvature tensor $R^\lambda_{\alpha\beta\gamma}$ is said to be more generalised recurrent P-Sasakian manifold.

Hence, we have

Theorem 2.1:

If associated vector field vanishes then the more generalised recurrent P-Sasakian manifold becomes generalised recurrent P-Sasakian manifold.

Proof:

Since associated vector field b_μ vanishes. Then equation (2.1) reduces to equation (1.2).

Theorem 2.2:

If associated vector fields vanish then the more generalised recurrent P-Sasakian manifold reduces simply to birecurrent P-Sasakian manifold.

Proof:

Since associated vector fields b_μ and c_ν vanish. Then equation (2.1) reduces to equation (1.1).

3. ASSOCIATED RECURRENT VECTOR AND RECURRENT TENSOR FIELDS IN P-SASAKIAN MANIFOLD:

Without any loss of generality, we can write

$$(3.1) \quad \nabla_\nu \nabla_\mu R^\lambda_{\alpha\beta\gamma} = a'_{\nu\mu} R^\lambda_{\alpha\beta\gamma} + b'_\mu \nabla_\nu R^\lambda_{\alpha\beta\gamma} + c'_\nu \nabla_\mu R^\lambda_{\alpha\beta\gamma},$$

By virtue of equations (2.1) and (3.1), we get

$$(3.2) \quad a'_{\nu\mu} R^\lambda_{\alpha\beta\gamma} + b'_\mu \nabla_\nu R^\lambda_{\alpha\beta\gamma} + c'_\nu \nabla_\mu R^\lambda_{\alpha\beta\gamma} = 0,$$

Wherein

$$(3.3) \quad a^*_{\nu\mu} = a_{\nu\mu} - a'_{\nu\mu},$$

$$(3.4) \quad b^*_\mu = b_\mu - b'_\mu$$

and

$$(3.5) \quad c^*_\nu = c_\nu - c'_\nu.$$

Remark 3.1:

It is noteworthy that if we assume

$$(3.13) \quad b_{[\mu} \nabla_{\nu]} R^{\lambda}_{\alpha\beta\gamma} = 0.$$

Consequently, equation (3.12) takes the form

$$(3.14) \quad 2a_{[\nu\mu]} R^{\lambda}_{\alpha\beta\gamma} = R^{\theta}_{\alpha\beta\gamma} R^{\lambda}_{\theta\nu\mu} - R^{\lambda}_{\theta\beta\gamma} R^{\theta}_{\alpha\nu\mu} - R^{\lambda}_{\theta\gamma\alpha} R^{\theta}_{\beta\nu\mu} - R^{\lambda}_{\theta\alpha\beta} R^{\theta}_{\gamma\nu\mu},$$

Hence $a_{\nu\mu}$ is symmetric iff

$$(3.15) \quad R^{\theta}_{\alpha\beta\gamma} R^{\lambda}_{\theta\nu\mu} = R^{\lambda}_{\theta\beta\gamma} R^{\theta}_{\alpha\nu\mu} + R^{\lambda}_{\theta\gamma\alpha} R^{\theta}_{\beta\nu\mu} + R^{\lambda}_{\theta\alpha\beta} R^{\theta}_{\gamma\nu\mu}.$$

Hence follows the theorem:

Theorem 3.2:

The necessary and sufficient condition that for the P-Sasakian manifold M^n with $b_{[\mu} \nabla_{\nu]} R^{\lambda}_{\alpha\beta\gamma} = 0$, the associated recurrent tensor field $a_{\nu\mu}$ be symmetric is that the relation $R^{\theta}_{\alpha\beta\gamma} R^{\lambda}_{\theta\nu\mu} = R^{\lambda}_{\theta\beta\gamma} R^{\theta}_{\alpha\nu\mu} + R^{\lambda}_{\theta\gamma\alpha} R^{\theta}_{\beta\nu\mu} + R^{\lambda}_{\theta\alpha\beta} R^{\theta}_{\gamma\nu\mu}$ holds good.

REFERENCES:

- [1]. A.G. Walker : On Ruse's space of recurrent curvature, Proc. London, Math. Soc., (2)52, (1950).
- [2]. I. Sato and K. Matsumoto : On P-Sasakian manifolds satisfying certain conditions, Tensor, N.S., 33, P. (173-178), (1979).
- [3]. K. Yano : Differential geometry on complex and almost complex spaces, Pergamon Press, (1965).
- [4]. M. Matsumoto : On Riemannian spaces with recurrent projective curvature, Tensor, N.S., 19, P. (11-18), (1968).
- [5]. T. Adati and T. Miyazawa : Some properties of P-Sasakian manifolds, TRU Math., P. (33-42), 13, (1977).
- [6]. T.S. Chauhan : A note on Recurrent P-Sasakian manifolds, Tensor, N.S., Vol. 62, P. (215-218), (2000).
- [7]. T.S. Chauhan : On generalised recurrent P-Sasakian manifold, Acta Ciencia Indica, No. 3, Vol. XXVIII M, P. (289-290), (2002).
- [8]. T.S. Chauhan, I.S. Chauhan and Rajendra Kumar : On recurrent and birecurrent para-sasakian manifolds, Acta Ciencia Indica, Vol. XXIX M, No. 1, P. (145-148), (2003).